MMSE CRITERIA FOR DOWNLINK BEAMFORMING IN CDMA WIRELESS SYSTEMS

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ABSTRACT

In wireless systems with multiple antennas at the access points, the downlink power has to be spatially optimized to allow several simultaneous users. We consider two different MMSE criteria for a multi-user downlink CDMA system and determine the optimal transmit beamforming weights. The MMSE criteria with and without receiver gain control are compared for a system where the transmitter is equipped with an antenna array and the receivers have single antennas. The MMSE and SINR versus different number of users is shown and the effect of including the gain control in the MMSE criteria is clearly demonstrated.

1. INTRODUCTION

To improve the link quality in wireless systems in terms of the signal to interference plus noise ratio (SINR), transmitters may be equipped with antenna arrays [1]. When the transmit antenna array has a properly determined beam pattern, it directs the signal power towards the desired receiver, while suppressing the signal gain towards the other users in the system. These transmit beamformers affect not only the desired user but also the interference at the other receivers. Therefore the optimal beamformers need to take all other users into account. Since the receivers usually have to be low cost units employing only one receive antenna, precoding at the transmitter is very attractive for downlink systems. This precoding can be carried out with or without channel state information (CSI). Here, we study systems where perfect channel knowledge is available at the transmitters. Such CSI can be obtained either using feedback of estimates done at the receiver end or by exploiting channel reciprocity, using channel estimates obtained in the uplink.

Many different downlink beamforming strategies have been proposed in the literature, see e.g. [2] and references therein for an overview. One major issue is how to take the quality of service (QoS) for all the individual users into account. One possibility is to optimize with respect to the worst SINR among the users. However, this leads to a complicated optimization problem that has to be solved jointly for all users, see [2, 3]. In [4], it was recently proposed to combine the QoS constraints of all the users into a single minimum mean square error (MMSE) formulation. The advantage is that the problem decouples and the solution easily can be obtained for each user separately. An alternative approach that also leads to a simple decoupled solution is to maximize the harmonic mean of the SINRs, see [1, 2]. In [5] an alternative MMSE formulation is considered for the design of spatio-temporal pre-filters for CDMA, including a proper handling of transmit power constraints. A minimax solution allocating identical MSE to all users is derived in [6].

Here, we focus on the MMSE beamforming formulation and study an extension, also proposed in [7], which includes the fact that every receiver can compensate for differences in the absolute gain. This is clearly a more relevant formulation, but unfortunately, the computational complexity is increased. In comparison to [7], the solution is here formulated for a CDMA system and an alternative algorithm is proposed. Also, the performance of the different beamforming formulations is illustrated by numerical simulations.

2. SYSTEM MODEL

Consider a downlink code division multiple access (CDMA) system where the transmitter is equipped with multiple antenna elements and the receivers with single antennas [4]. In order to keep the notation easy, a single-cell system is considered.
Let the number of users be K and denote the base band signal transmitted for the kth user by

$$d_k(t) = \sum_{l=-\infty}^{\infty} c_k(t - Tl)s_k[l] \tag{1}$$

where $c_k(t)$ and $s_k[l]$ are the spreading waveform and data symbol sequence for the kth user, respectively. The symbols are normalized such that $E[|s_k[l]|^2] = 1$ and T denotes the length of the symbol interval.
The spreading waveform can be written as

$$c_k(t) = \sum_{n=0}^{N-1} p(t - T_c n)c_k[n] \tag{2}$$

where $p(t)$ is a shaping pulse, $c_k[n]$ is the spreading code and T_c is the chip duration such that $T = NT_c$. The spreading waveform has finite support over $0 \le t \le T$ and is normalized such that $\int_0^T |c_k(t)|^2 dt = 1$. When the number of transmit antenna elements is L, the total transmitted signal can be written as

$$\mathbf{z}(t) = \sum_{k=1}^{K} \mathbf{w}_k d_k(t) \tag{3}$$

where $\mathbf{w}_k$ is an $L \times 1$ beamforming vector for the kth user.
Assuming a flat fading environment, the channel from the transmitter to the qth user can be represented by the $L \times 1$ vector $\mathbf{g}_q$. The total received signal at the qth receiver is then given by

$$\begin{aligned} x_q[l] &= \int_{(l-1)T}^{lT} (\mathbf{g}_q^H \mathbf{z}(t) + n_q(t))c_q^*(t - lT)dt \\ &= \sum_{k=1}^{K} \rho_{q,k}\mathbf{g}_q^H \mathbf{w}_k s_k[l] + n_q[l] \end{aligned} \tag{4}$$

where $\rho_{q,k} = \int_0^T c_q^*(t)c_k(t)dt$ is the spreading code cross-correlation, $(\cdot)^H$ denotes the Hermitian transpose, $n_q(t)$ is the noise at the qth user and $n_q[l] = \int_{(l-1)T}^{lT} c_q^*(t-lT)n_q(t)dt$ is the noise projected on the spreading waveform. To keep a compact formulation, the received signals are re-written in vector form as

$$\mathbf{x}[l] = \begin{bmatrix} x_1[l] & x_2[l] & \dots & x_K[l] \end{bmatrix}^T \tag{5}$$
$$= \left[\mathbf{R} \odot \left(\begin{bmatrix} \mathbf{g}_1^H \\ \mathbf{g}_2^H \\ \vdots \\ \mathbf{g}_K^H \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \dots & \mathbf{w}_K \end{bmatrix} \right) \right] \mathbf{s}[l] + \begin{bmatrix} n_1[l] \\ n_2[l] \\ \vdots \\ n_K[l] \end{bmatrix}$$

where $[\mathbf{R}]_{q,k} = \rho_{q,k}$, $\mathbf{s}[l] = [s_1[l]\ s_2[l]\ \dots\ s_K[l]]^T$, and $\odot$, $(\cdot)^T$ denote the element-wise multiplication and transpose, respectively.

3. MMSE FORMULATIONS

This section presents two formulations of the MMSE criterion and the corresponding algorithms designed to obtain the optimal transmit beamformers. Also, we briefly review a couple of alternative criteria which are included for reference in the numerical examples below. The acronyms defined in the titles will be used in Section 4.

3.1. MMSE Without Gain Control (*MMSE NGC*)

When the transmit beamformers $\{\mathbf{w}_k\}$ are properly determined, the received signals represent estimates of the transmitted symbols. The transmitter weights are now chosen such that the MSE between the transmitted symbols and the estimated symbols is minimized [4], i.e.

$$\{\mathbf{w}_{\text{MMSE},k}\} = \arg\min_{\mathbf{W}} E[||\mathbf{s}[l] - ([\mathbf{R} \odot (\mathbf{G}^H\mathbf{W})]\mathbf{s}[l] + \mathbf{n}[l])||^2] \tag{6}$$

where $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2\ \dots\ \mathbf{w}_K]$, $\mathbf{G} = [\mathbf{g}_1\ \mathbf{g}_2\ \dots\ \mathbf{g}_K]$, and $\mathbf{n}[l] = [n_1[l]\ n_2[l]\ \dots\ n_K[l]]^T$. When there is no gain control at the receivers, a closed-form expression for the optimal transmit beamformers is available. Defining $\boldsymbol{\mathcal{R}}_k = \text{diag}(\mathbf{r}_k)$ and $\mathbf{r}_k$ as the kth column of $\mathbf{R}$, the optimal beamformers are given by [4]

$$\mathbf{w_k} = (\boldsymbol{\mathcal{R}}_k \mathbf{G}^H)^\dagger \mathbf{e}_k \tag{7}$$

where $(\cdot)^\dagger$ denotes the pseudo inverse and $\mathbf{e}_i$ is the ith column of the $K \times K$ identity matrix $\mathbf{I}_{K \times K}$. The total transmitted power is defined implicitly by the MMSE formulation. It can be observed from (7) that the optimal transmit beamformers can be determined individually. Note also that (7) reduces to zero forcing beamforming when $K \leq L$.

3.2. MMSE With Gain Control (*MMSE GC*)

The above criterion attempts to enforce the same signal power at all receivers. This is unnecessarily restrictive since the equalizers at the receivers can scale the signal and the decoding performance only depends on the SINR, not the absolute scaling. This fact can be included in the MMSE criterion by introducing scaling gains at the receivers. Letting $\boldsymbol{\alpha} = [\alpha_1\ \dots\ \alpha_K]$ be the vector containing scaling gains for the K users, the optimization problem can be written as

$$\begin{aligned} \{\mathbf{w}_{\text{MMSE},k}, \boldsymbol{\alpha}\} &= \arg\min_{\mathbf{W},\boldsymbol{\alpha}} \\ &E[||\mathbf{s}[l] - \boldsymbol{\mathcal{A}}([\mathbf{R} \odot (\mathbf{G}^H\mathbf{W})]\mathbf{s}[l] + \mathbf{n}[l])||^2] \\ \text{s.t.}\,&\text{Tr}(\mathbf{W}^H\mathbf{W}) \leq P_0 \end{aligned} \tag{8}$$

where $\boldsymbol{\mathcal{A}} = \text{diag}(\boldsymbol{\alpha})$. The constraint on the total transmit power is introduced to avoid a solution where the scaling gains go to zero and the beamformers to infinity. Note that this constraint will be fulfilled with equality at the optimum.

Since this cost function is not jointly convex in $\mathbf{W}$ and $\boldsymbol{\alpha}$, we propose an iterative algorithm, where problem (8) is divided into two parts. First, the optimal beamformers are determined while the receiver gains are fixed and then the optimal receiver gains are determined while the beamformers are kept fixed. The complete solution is obtained by iterating between these two steps and is guaranteed to converge since the MSE is minimized in each step. However, the iteration may get stuck in a local optimum, and there is no guarantee to find a global optimum. An alternative solution is proposed in [7], using a reformulation into an equivalent virtual uplink problem which can be solved by a semidefinite program.

For fixed receiver gains $\boldsymbol{\alpha}$, the transmit beamformers $\{\mathbf{w}_i\}$ are determined by solving

$$\begin{aligned} &\{\mathbf{w}_{\text{MMSE},k}\} = \arg\min_{\mathbf{W}} E[||\mathbf{s}[l] - \boldsymbol{\mathcal{A}}\hat{\mathbf{s}}[l]||^2] \\ &\text{s.t. } \text{Tr}(\mathbf{W}^H\mathbf{W}) \leq P_0 \end{aligned} \tag{9}$$

where $\hat{\mathbf{s}}[l] = ([\mathbf{R} \odot (\mathbf{G}^H\mathbf{W})]\mathbf{s}[l] + \mathbf{n}[l])$.

Let $\overrightarrow{\mathbf{r}} = \text{vec}(\mathbf{R})$, $\overrightarrow{\mathbf{i}} = \text{vec}(\mathbf{I})$, $\overrightarrow{\mathbf{w}} = \text{vec}(\mathbf{W})$, where $\text{vec}(\cdot)$ is the vectorization operator and $\mathbf{I}$ is the identity matrix, and introduce $\boldsymbol{\mathcal{A}}_d = \mathbf{I}_{K \times K} \otimes \boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{R}} = \text{diag}(\overrightarrow{\mathbf{r}})$ where $\otimes$ denotes the Kronecker product. By substituting $\mathbf{M} = \boldsymbol{\mathcal{A}}_d \boldsymbol{\mathcal{R}}(\mathbf{I} \otimes \mathbf{G^H})$ and $\mathbf{b} = -\mathbf{M}^H \overrightarrow{\mathbf{i}}$ in (9), the optimal beamformers are obtained by solving

$$\begin{aligned} \{\mathbf{w}_{\text{MMSE},k}\} = \quad & \arg\min_{\overrightarrow{\mathbf{w}}}\ \overrightarrow{\mathbf{w}}^H \mathbf{M}^H \mathbf{M} \overrightarrow{\mathbf{w}} + \\ & \mathbf{b}^H \overrightarrow{\mathbf{w}} + \overrightarrow{\mathbf{w}}^H \mathbf{b} + \text{const} \\ \text{s.t. } \overrightarrow{\mathbf{w}}^H \overrightarrow{\mathbf{w}} \leq P_0 & \end{aligned} \tag{10}$$

which is a least squares minimization problem over the volume of a sphere. This problem can be solved by using Lagrange multipliers and the solution is given by [8]

$$\overrightarrow{\mathbf{w}} = -(\mathbf{M}^H\mathbf{M} + \mathbf{I}\lambda)^{-1}\mathbf{b} \tag{11}$$

where λ is chosen such that $\overrightarrow{\mathbf{w}}^H \overrightarrow{\mathbf{w}} = P_0$, using standard line search [9].

The next step is to determine the optimal receiver weights $\boldsymbol{\alpha}$ while keeping the transmit beamformers fixed. This is done by minimizing $\text{MSE} = E[||\mathbf{s}[l] - \boldsymbol{\mathcal{A}}(\mathbf{H}\mathbf{s}[l] + \mathbf{n}[l])||^2]$, where $\mathbf{H} = \mathbf{R} \odot (\mathbf{G}^H\mathbf{W})$. Solving the least-squares problem yields

$$\alpha_i = \frac{\text{H}_{ii}^*}{||\mathbf{H}_i||^2 + \sigma_i^2} \tag{12}$$

where H_{ii} is the (i,i)th element of $\mathbf{H}$, $\mathbf{H}_i$ is the ith row of $\mathbf{H}$, $(\cdot)^*$ denotes the complex conjugate and $\sigma_i^2 = E[\text{n}_i[l]\text{n}_i^H[l]]$ is the noise power of the ith user.

It should be noted that each $\mathbf{w}_k$ only depends on the corresponding α_k, while α_k depends on all $\mathbf{w}$. The algorithm solving problem (8) is summarized in Table 1.

Note that (6) is a special case of (8) when all receive gains $\boldsymbol{\alpha}$ are set equal to one and the total power constraint is high enough.

3.3. Max Harmonic Mean SINR (*MHM SINR*)

An alternative formulation that leads to a decoupled closed form solution for each beamformer is obtained by maximizing the harmonic

1. Choose initial values for the receiver gains $\boldsymbol{\alpha}$.
2. Determine the optimal beamformers by using (11).
3. Determine the optimal receiver gains by using (12).
4. Stop when desired degree of convergence in obtained. Otherwise go back to step 2.

Table 1. Solution of problem (8).

mean of the SINR [1, 2]. From (4), the SINR for the ith link can be written as

$$\mathrm{SINR}_i = \frac{\mathbf{w}_i^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{w}_i}{\sum_{n \neq i} |\rho_{i,n}|^2 \mathbf{w}_n^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{w}_n + \sigma_i^2}. \tag{13}$$

Maximizing the harmonic mean is equivalent to minimizing

$$\sum_{i=1}^{K} \mathrm{SINR}_i^{-1} = \sum_{i=1}^{K} \frac{\sum_{n \neq i} |\rho_{i,n}|^2 \mathbf{w}_n^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{w}_n + \sigma_i^2}{\mathbf{w}_i^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{w}_i} =$$
$$\sum_{n=1}^{K} \mathbf{w}_n^H \Big(\sum_{i \neq n} \frac{|\rho_{i,n}|^2}{\beta_i} \mathbf{g}_i \mathbf{g}_i^H\Big) \mathbf{w}_n + \text{const} \tag{14}$$

where $\beta_i = \mathbf{w}_i^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{w}_i$ is a constraint on the desired power level for the received signal at each user.
The beamformers maximizing this harmonic mean are given by

$$\mathbf{w}_i = \arg\max \frac{\mathbf{w}_i^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{w}_i}{\mathbf{w}_i^H (\sum_{n \neq i} \frac{|\rho_{n,i}|^2}{\beta_n} \mathbf{g}_n \mathbf{g}_n^H + \kappa \mathbf{I}) \mathbf{w}_i} \tag{15}$$

where a regularization term κ has been added to keep the overall interference level low.

3.4. Max Min SINR (*Equal SINR*)

Absolute fairness between the users is obtained by the solution to

$$\begin{aligned} &\max_{\{\mathbf{w}_i\}} \min_{k} \ \mathrm{SINR}_k \\ &\text{s.t. } \sum_{k} \|\mathbf{w}_k\|^2 \leq P_0 \end{aligned} \tag{16}$$

which can be obtained using the algorithm in [3].

4. SIMULATION RESULTS

The difference between the two proposed MMSE criteria is evaluated by considering a scenario with one multi-antenna transmitter and several single antenna receivers. The transmitter is equipped with 4 antenna elements and positioned at the origin. The receivers are randomly positioned in a circular area and the channel from the transmitter to one of the receivers is modeled as

$$\mathbf{g} = \gamma_{pl} \gamma_{sf} \mathbf{s} \tag{17}$$

where $\gamma_{pl} = \frac{1}{r^2}$ is the distance dependent path loss, r is the distance between the transmitter and the corresponding receiver, $\gamma_{sf} = 10^{-\Psi/10}$ is the shadow fading from large objects with Ψ modeled as a Gaussian distributed variable with zero mean and standard deviation 8 [dB] and $\mathbf{s}$ is a vector containing independent and identically distributed $CN(0, 1)$ elements representing multipath scattering in a rich scattering environment. This is a very simplified channel but good enough for our applications. The spreading codes are randomly generated sequences of ± 1. The same total transmitted power is used for all of the algorithms and the transmit power and noise power are chosen such that the mean SINR for one receiver on the cell boundary would be 20 [dB] if only a single antenna element was used at the transmitter. All the results have been averaged over 3000 different scenarios.

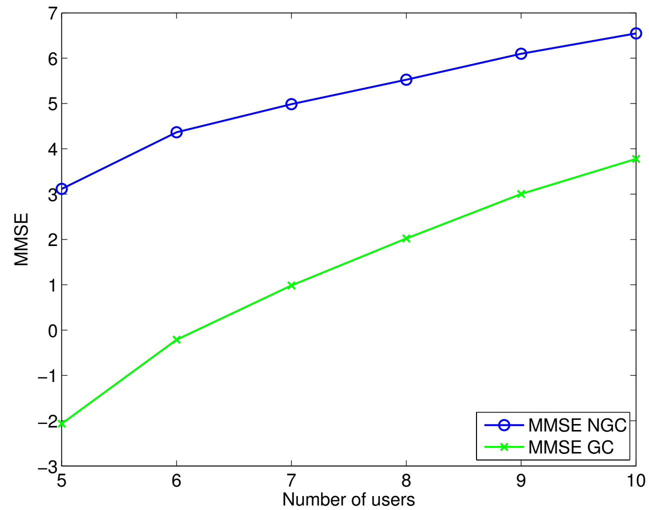

Fig. 1. MMSE with and without gain control at the receivers.

Fig. 1 shows the resulting MMSE of the optimal beamformers for the two MMSE criteria (calculated according to the cost function of (8), replacing $\boldsymbol{\mathcal{A}}$ by a scalar for the MMSE NGC solution). A significantly lower MMSE is obtained by including the gain control. To evaluate the fairness properties of the algorithms, the mean SINR, max SINR and min SINR among the users is plotted in Figures 2, 3 and 4, respectively. In the MHM SINR algorithm, the parameters were set to $\beta_i = \mathbf{g}_i^H \mathbf{g}_i$ and $\kappa = 0.01$ [1, 2]. Comparing the two MMSE criteria, a higher mean SINR and peak SINR is clearly obtained by including the receiver gain control into the MMSE criterion. However, this will often lead to a solution where no power at all is allocated to the weakest users. A related result can be found in [10], where the "water-filling" like solution found for minimization of the sum MSE of a single MIMO link may allocate zero power to some users. This is a problem in systems with strict delay requirements but may also provide multi-user diversity gains [11] if adaptive data rates and scheduling is used. Comparing with the other methods, we see that the maximum harmonic mean SINR method provides very similar performance at a lower computational complexity, and that it avoids the all-zero solution to the weakest users. Also, note that with the Equal SINR method, most users pay a high price for the increased fairness of the worst users.

5. CONCLUSIONS

Several strategies can be used to determine a set of beamformers to do spatial division multiple access. Algorithms that optimize the quality of service for the worst user have been proposed in [2, 3], however the computational complexity is fairly high. Combining the individual MSEs of each user into a single MMSE formulation for the full system was proposed in [4], which leads to a closed form solution where each beamformer can be determined separately. However, the performance can be improved significantly for most users

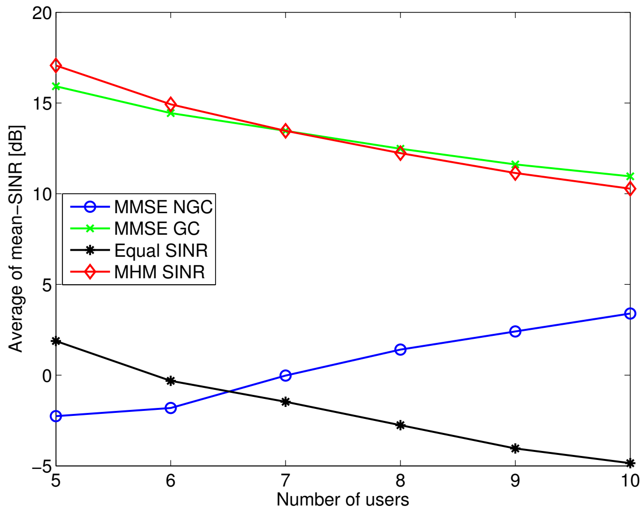

Fig. 2. Average of mean-SINR [dB] versus K.

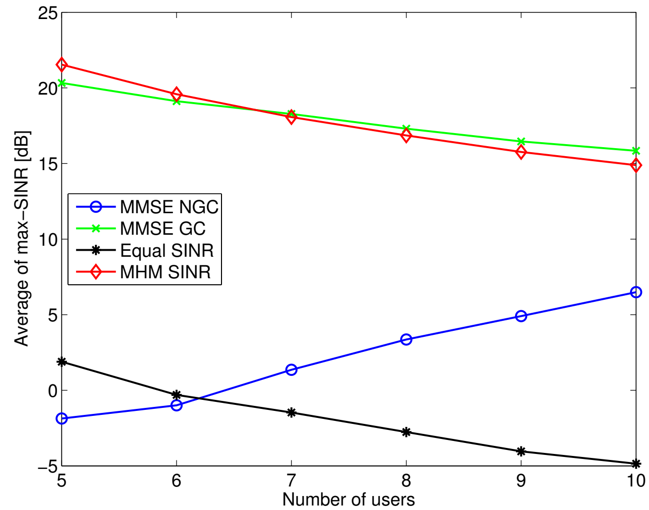

Fig. 3. Average of max-SINR [dB] versus K.

by incorporating a scale factor at each receiver into the MMSE criterion, at the expense of an increased computational complexity. If low computational complexity is a major issue, we have also shown that the maximum harmonic mean of the SINR [1, 2] is a good alternative.

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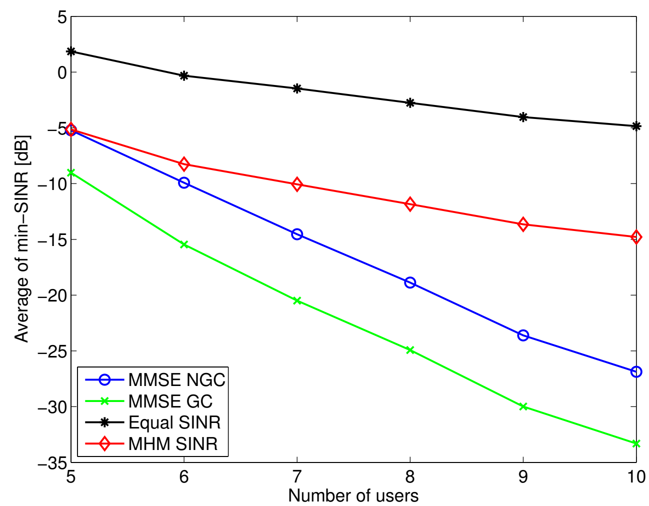

Fig. 4. Average of min-SINR [dB] versus K.

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