A SIMPLE AND EFFICIENT ANTENNA SUBSET SELECTION SCHEME FOR MIMO SYSTEMS

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ABSTRACT

The use of multiple antennas at both ends of a wireless link has been shown to provide significant improvements in terms of outage capacity and average probability of error as compared to systems that employ a single sensor transceiver. However, this improvement in system performance comes at the price of increased hardware cost and computational complexity. Transmitting and receiving over a subset of the available antennas, referred to as antenna selection, offers a solution by reducing the complexity of the system and cutting down on cost while still harnessing the advantages of a wireless multiple input multiple output (MIMO) system. In this paper, we propose a simple and efficient antenna selection algorithm based on the computation of a metric which depends on the correlation between antennas. We show analytically that our algorithm achieves almost the same outage capacity and bit error rate as the optimal selection technique while having lower computational complexity. We also validate our results through extensive simulations.

1. INTRODUCTION

Driven by the demand for increasingly sophisticated communication services available anytime, anywhere, wireless communications has emerged as one of the largest and most rapidly growing sectors of the global telecommunications industry. The increasing requirements on data rate and quality of service for wireless communication systems call for new techniques to increase spectrum efficiency and improve link reliability. One of the most significant technological developments of the last decade, that promises to play a key role in realizing this tremendous growth, is the use of multiple input multiple output (MIMO) antenna architectures.

The use of multiple antennas at both ends of a wireless link promises significant improvements in terms of spectral efficiency and link reliability. Analytical as well as simulation studies have verified that MIMO systems can increase the data rate by transmitting different data streams from different antenna elements (spatial multiplexing) or improve the quality of a single data stream by exploiting transmit and/or receive diversity [1]-[3]. In either case, a major drawback is the requirement for multiple RF chains (one for each antenna element), which leads to high implementation costs. For this reason, recent papers [4], [5], have proposed antenna selection schemes that choose a subset of available transmit and/or receive antennas, and process the signals associated with these antennas. Optimal selection of antenna elements requires an exhaustive search of all possible combinations for the one that gives the best *SNR* (for diversity) or capacity (for spatial multiplexing). However, for joint transmit and receive antenna selection, this requires a singular value decomposition (SVD) operation for each combination of antennas at the transmitter and receiver. The computation of this many singular value decompositions is computationally demanding, when the number of antennas is large.

We propose a simple and efficient antenna selection algorithm based on the correlation between antennas, which performs as well as the optimal selection. In section 2, the MIMO system model is formulated. A brief overview of existing antenna selection schemes is presented in section 3. In section 4, we present and analyze the new selection algorithm. Simulation plots validating our results are presented in section 5. We conclude with a summary of results in section 6.

2. SYSTEM MODEL DESCRIPTION

We consider a single user, point to point, flat fading wireless link with N_t transmit and N_r receive antennas. The channel is described by $\mathbf{H}$, the N_r x N_t matrix of complex fading coefficients, which are assumed to be stationary and ergodic. These fading coefficients are assumed to be constant over the duration of several bursts. We adopt the widely used channel model described in [6],[7]

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{W}\mathbf{T}^{1/2}, \tag{1}$$

where $\mathbf{W}$ is a matrix with i.i.d complex Gaussian entries ~N (0,1), and $\mathbf{R}$ and $\mathbf{T}$ are N_r x N_r and N_t x N_t matrices denoting receive and transmit correlation matrices, respectively. This gives rise to the Rayleigh fading channel model, which has been used extensively to model terrestrial wireless communication channels without a direct line-of-sight path [8]. Here we have assumed independent transmit and receive correlations. The discrete-time received signal in such a system can be written in matrix form as

$$\mathbf{y}(i) = \sqrt{\frac{E_s}{\mathbf{N}_t}}\,\mathbf{H}(i)\mathbf{x}(i) + \mathbf{n}(i) \tag{2}$$

where $\mathbf{y}(i)$ is the complex N_r-dimensional vector representing the received signals at the receive antenna array at symbol time i, $\mathbf{x}(i)$ is the complex N_t-dimensional vector of transmit signals and $\mathbf{n}(i)$ is the N_r-dimensional, complex vector of receiver noise. The components of $\mathbf{n}(\boldsymbol{i})$ are zero mean, circularly symmetric, complex Gaussian with independent real and imaginary parts having equal variance. We assume that $\mathbf{n}(\boldsymbol{i})$ is a sequence of uncorrelated random vectors and thus, $\mathrm{E}\{\mathbf{n}(\boldsymbol{i})\mathbf{n}(\boldsymbol{i})^{\mathbf{H}}\} = N_0\mathbf{I}_{\mathrm{Nr}}$, where $\mathbf{I}_{\mathrm{N}r}$ denotes the $\mathrm{N_r}$ x $\mathrm{N_r}$ identity matrix and N_0 is the noise variance.

3. MIMO SYSTEMS WITH ANTENNA SELECTION

It is possible to select a subset of $\mathrm{L_r}$ ($\mathrm{L_t}$) receive (transmit) antennas, out of the possible $\mathrm{N_r}$ ($\mathrm{N_t}$) antennas, in order to reduce the hardware implementation cost without compromising on system performance. Without loss of generality, we assume that antenna selection is performed only at the receiver. Here, we have assumed perfect channel state information (CSI) at the receiver. One way to perform channel estimation at the receiver is to use a training preamble. But, the only way for the transmitter to have CSI is through a feedback channel from the receiver, which results in significant overhead.

Consider the channel matrix **H** introduced in section 2. The singular value decomposition of **H** can be written as

$$\mathbf{H} = \mathbf{UDV'}, \tag{3}$$

where **U** and **V** are unitary matrices and **D** is a real diagonal matrix. $\mathbf{V}'$ denotes the transpose conjugate of **V**. The SVD operation decomposes the channel matrix **H** into a number of independent orthogonal modes, which are referred to as the eigen modes of the channel. The MIMO channel is effectively decoupled into parallel single-input, single-output (SISO) channels with unequal gain. The selection criterion selects the "best" L active eigen modes for transmission and reception. However, such systems require knowledge of the CSI at the transmitter and receiver.

3.1 Condition for Optimizing Data Rate

The ergodic capacity of flat fading MIMO channel is given by [2]

$$\boldsymbol{C} = \mathbf{E}\left[\mathbf{log}_2\left\{\mathbf{det}\left(\mathbf{I}_{\mathbf{N}_t} + \frac{\boldsymbol{\rho}}{\mathbf{N}_t}\mathbf{HH'}\right)\right\}\right] \text{ bits/s/Hz} \quad , \tag{4}$$

subject to a constant total transmitted power of ρN_0, where N_0 is the noise variance. ρ is the nominal signal to noise ratio.

Decomposing **H** using SVD and assuming large *SNR*, the above expression can be equivalently written as

$$C = \boldsymbol{E}\left[\sum_{i=1}^{N_r}\log_2\left(1+\frac{\rho}{\boldsymbol{N_t}}\lambda_i\right)\right] \text{ bits/s/Hz} \tag{5}$$

or,

$$C \approx \boldsymbol{N_r}\log_2\frac{\rho}{\boldsymbol{N_t}} + \log_2\left[\prod_{i=1}^{N_r}\lambda_i\right] \text{ bits/s/Hz} \tag{6}$$

where $\sqrt{\lambda_i}$ is the *i*-th singular value of **H,** and the expectation is over the independent channel realizations. Thus, the ergodic capacity of a wireless MIMO channel is the sum of capacities of $\mathrm{N_r}$ virtual SISO channels defined by the spatial eigenmodes of the matrix product $\mathbf{HH'}$ ($\mathrm{N_r} \geq \mathrm{N_t}$).

Clearly from (6) for maximal capacity, the product of the eigenvalues of the matrix $\mathbf{HH'}$ has to be a maximum, *i.e.* the singular values should all be large and almost equal because the product is maximum when the multiplicands are large and almost the same.

3.2 Condition for Optimizing Bit Error Rate

For optimizing BER, we need to maximize the received *SNR*. Assuming CSI at the transmitter, the optimum BER can be achieved by maximum ratio transmission (MRT) and maximum ratio reception (MRC), *i.e.*, **U** and **V** are the singular vectors corresponding to the largest singular value of **H**. The effective *SNR* is then given by $\rho\lambda_1$ [7], where $\sqrt{\lambda_1}$ is the largest singular value of **H**. When there is no CSI, using diversity at the transmitter requires that one of the singular values of **H** be large. For our simulations we assume QPSK modulation and the bit error rate is given by

$$P_e = Q\left(\sqrt{\frac{E_S}{N_0}}\right) \quad , \tag{7}$$

where $\boldsymbol{E_s = 2E_b}$ is the symbol energy, $\boldsymbol{E_b}$ is the energy per bit and $\boldsymbol{Q(.)}$ is the well known Q-function.

3.3 Optimum Selection Criterion

The optimum criterion for receiver antennas selection, in terms of capacity, is to choose the subset of antennas, which maximizes the capacity of the resulting sub-channel $\mathbf{H_s}$. This selection criterion requires the computation of the capacities of all $\binom{N_r}{L_r}$ possible subsets of selected antennas and thus becomes prohibitively complex for a large number of antennas. For the diversity case, we need to compute the *SNR* of all possible combinations before selection. Optimum selection provides an upper bound on the performance of more practical selection schemes.

3.4 Maximum Frobenius Norm Criterion

In case of joint transmit and receive antenna selection, this algorithm involves selecting the columns of **H** with L_t highest Frobenius norms and forming the matrix $\mathbf{H_{st}}$. Then the rows of $\mathbf{H_{st}}$ with L_r highest Frobenius norms are selected to form the matrix $\mathbf{H_s}$. For receive side selection ($N_t = L_t$) the algorithm selects L_r out of N_r rows with the highest Frobenius norm [4]. The drawback of this algorithm is that it may produce sub-optimum data rates in correlated channels.

4. PROPOSED ALGORITHM

Consider the case of a complex, flat-fading channel with the receiver having perfect knowledge of the channel state. Assume that 2 antennas are being selected at the receiver from N_r receiver antennas. Also, for each possible pair of receiving antennas, define the sub-matrix product as

$$\mathbf{H}_{\boldsymbol{s}}\mathbf{H}_{\boldsymbol{s}}^{\mathrm{H}}(i,j) = \begin{bmatrix} \boldsymbol{R}(i,i) & \boldsymbol{R}(i,j) \\ \boldsymbol{R}(j,i) & \boldsymbol{R}(j,j) \end{bmatrix}, \tag{8}$$

where $\mathbf{R} = \mathbf{H}\mathbf{H}^{\mathrm{H}}$ and i is the row number and j the column number for that pair. If λ_1 and λ_2 are the two eigenvalues of the Hermitian product $\mathbf{H}\mathbf{H}^{\mathrm{H}}$, then we can write the characteristic equation as:

$$\{\lambda - R(i,i)\}\{\lambda - R(j,j)\} - |R(i,j)|^2 = 0$$

$$\Rightarrow \lambda^2 - \{R(i,i) + R(j,j)\}\lambda + R(i,i)R(j,j) - |R(i,j)|^2 = 0$$

$$\Rightarrow \lambda_1 + \lambda_2 = R(i,i) + R(j,j) \tag{9}$$

$$\textit{and} \quad \lambda_1\lambda_2 = R(i,i)R(j,j) - |R(i,j)|^2 \tag{10}$$

As discussed in section 3.1, the condition for maximizing capacity is to maximize the product of the eigenvalues of $\mathbf{H}\mathbf{H}^{\mathrm{H}}$. Towards this goal, we propose to select antennas using the following metric which is a direct result from (10),

$$\boldsymbol{\gamma}_C(\boldsymbol{i},\boldsymbol{j}) = \mathbf{R}(\boldsymbol{i},\boldsymbol{i})\mathbf{R}(\boldsymbol{j},\boldsymbol{j}) - |\mathbf{R}(\boldsymbol{i},\boldsymbol{j})|^2, \tag{11}$$

R(*i,i*) and **R(*j,j*)** are the auto-correlation terms denoting signal power and **R(*i,j*)** is the cross-correlation term as defined in (8). By selecting antenna pairs having largest γ_c, we ensure large signal power as well as almost equal distribution of power between modes.

For achieving optimum bit error rates, we need to maximize the received *SNR*. As discussed in section 3.2, the received *SNR* would be maximized when one of the eigenvalues of $\mathbf{H}\mathbf{H}^{\mathrm{H}}$ is large. Using (9) and (10), we can write

$$(\lambda_1 - \lambda_2)^2 = (\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2$$

$$\Rightarrow \lambda_1 - \lambda_2 = \sqrt{(\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2} \tag{12}$$

Adding (9) and (12) we get

$$\lambda_1 = \frac{1}{2}\left[R(i,i) + R(j,j) + \sqrt{\{R(i,i) - R(j,j)\}^2 + 4|R(i,j)|^2} \right] \tag{13}$$

Assuming equal power distribution between the two modes of excitation as would be expected in the case of an uninformed transmitter, (13) reduces to

$$\lambda_1 \approx \frac{1}{2}\left[R(i,i) + R(j,j) + 2|R(i,j)| \right] \tag{14}$$

Thus, we define our metric as

$$\boldsymbol{\gamma}_{\mathrm{B}}(\boldsymbol{i},\boldsymbol{j}) = \mathbf{R}(\boldsymbol{i},\boldsymbol{i}) + \mathbf{R}(\boldsymbol{j},\boldsymbol{j}) + 2|\mathbf{R}(\boldsymbol{i},\boldsymbol{j})|. \tag{15}$$

We pick the antenna pair with largest γ_{B}, ensuring large signal power. This criterion not only takes into account the power at each receiving antenna, but it also accounts for correlation between antennas, which will result in greater distribution of power to the largest eigenmode.

5. SIMULATION RESULTS

We study antenna selection at the receiver only. We compare the performance of the proposed algorithm with that of a system employing no antenna selection, *i.e.* system, which uses all its receiving antennas and is referred to as the full complexity system, and the optimal antenna selection technique, which uses the optimal singular values. We also evaluate the performance of the proposed metric in terms of accuracy and robustness to errors in channel estimation. All simulated points are obtained by averaging over 1000 independent channel realizations. We present BER plots as a function of increasing *SNR* and also plot capacity with 10% outage probability versus *SNR* for different channel characteristics. We notice that the proposed algorithm performs almost as well as the optimum selection, for all channel conditions.

5.1 Data Rate vs. *SNR*

Figures 1 shows the 10% outage capacity plotted against *SNR* for L_r=2 and N_r=4. We note that our metric performs as well as the optimum selection criterion and better than the Maximum Frobenius Norm Criterion, for all *SNR* values in uncorrelated channels. The outage capacity of these selection techniques is clearly less than that of the full complexity system utilizing all 4 antennas, as expected.

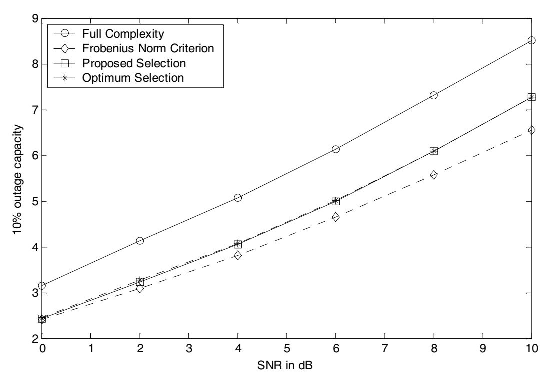

Figure 1 – 10% Outage capacity in bps/Hz versus *SNR* in dB for $N_t = 2$, $N_r = 4$ and $L_r = 2$ for uncorrelated channels.

5.2 Bit Error Rate vs. *SNR*

We plot bit error rate against increasing *SNR* for uncorrelated channel conditions in figure 2. Our metric performs as well as the optimum selection criterion for both correlated and uncorrelated channels (plot shown for uncorrelated channel only). It also performs better than the Maximum Frobenius Norm criterion for both cases. The BER from our metric at high *SNR* values is significantly better than that of the latter criterion.

5.3 Performance Metrics

Accuracy - Figure 3 shows a scatter plot of the outage capacity of the system selected using our metric plotted against the outage capacity of the optimum selection method for a correlated channel. It also shows the outage capacity of system selected using the Maximum Frobenius Norm criterion. Figure 3 shows almost 100% agreement between the system selected by our metric and that selected by the optimum selection criterion. The Maximum Frobenius Norm criterion appears to select pairs of antennas different from the optimum pair most of the time.

Figure 4 shows the scatter plot of BER from both metrics plotted against the BER of the optimum selection criterion. Again our metric selects the optimum pair of antennas almost 100% of the time, while the Maximum Frobenius Norm criterion fails to select the optimum pair most of the time.

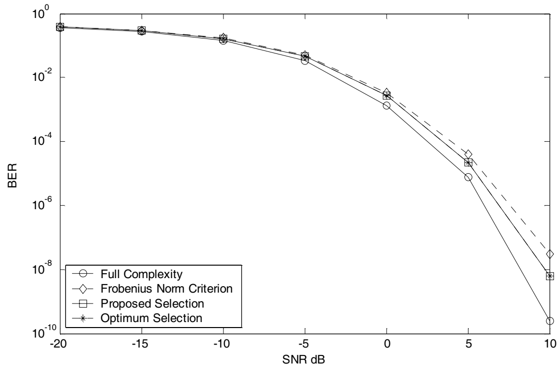

Figure 2 – BER versus *SNR* in dB for $N_t = 2$, $N_r = 4$ and $L_r = 2$ for uncorrelated channels.

Robustness – We investigated the effect of channel estimation errors on the performance of the proposed selection metric and the Frobenius norm criterion. The BER and outage capacity were plotted against *SNR* for the cases when the channel is assumed to be perfectly known as well as when there is a 20% error in channel estimation. We observed that the proposed selection scheme showed better robustness to errors in channel estimation than the Frobenius norm criterion both in terms of BER and outage capacity. Due to space constraints the plots have not been shown here.

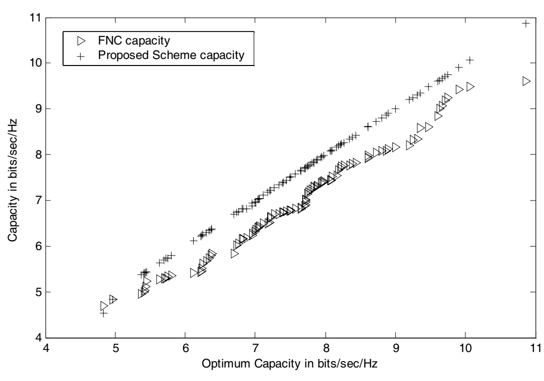

Figure 3 – Scatter plot of outage capacity for proposed scheme and Frobenius Norm criterion.

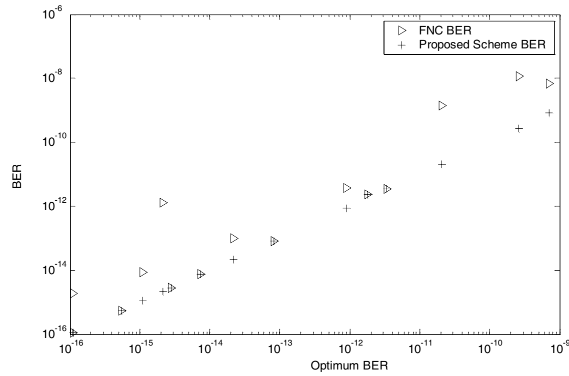

Figure 4 – Scatter plot of BER for proposed scheme and Frobenius Norm criterion.

6. CONCLUSIONS AND COMMENTS

We have shown that our algorithm performs as well as the optimum selection scheme albeit at a much lower computational complexity. We have also shown that our metrics for data rate and BER both perform better than the Maximum Frobenius Norm criterion. The performance enhancement is most significant at high *SNR* values.

We model the MIMO channel as i.i.d. This assumption is reasonable for an indoor environment, where the number of scatterers is large in the vicinity of the array and the antennas are separated by at least half a wavelength. However, the antenna separation requirement does place a limit on the size of the array especially for indoor applications.

7. REFERENCES

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