MAXIMAL CONDITIONAL EFFICIENCY SUCCESSIVE INTERFERENCE CANCELLATION

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ABSTRACT

Conditional asymptotic multi-user efficiency is introduced as a quantitative measure for comparing the performance of multi-user detectors that employ successive interference cancellation (SIC). For a given ordering of user signals, we derive the detector that achieves the maximum asymptotic conditional efficiency for each user among all possible SIC detectors. The optimal ordering that maximizes the asymptotic conditional efficiency at each stage of successive detection is also derived. We extend the concept of maximal asymptotic conditional efficiency detection to the case of joint successive interference cancellation (JSIC), where at each stage of successive detection, the corresponding bit is detected taking into account the interference of the "closest", or nearest, interferer in an ordered set of users. Both detection algorithms proposed are robust against strong correlation of user signals, e.g., in a multi-user system where the user signals are linearly dependent. Simulation results demonstrate that the maximum conditional efficiency approach significantly improves detector performance, particularly at high SNR.

1. INTRODUCTION

Multi-user detection in CDMA systems has been extensively studied for the past two decades. A detailed review of some of the popular approaches that have been proposed can be found in [1]. The maximum likelihood (ML), also known as the joint or optimal detector determines the most probable bits sent over the channel given the received signal and achieves the lowest probability of error achievable by any multi-user detector. However, the ML detector has a complexity that is exponential in the number of users and is not practical for present day CDMA systems. A number of sub-optimal approaches have been proposed with lower computational complexity. These include linear detection techniques such as the decorrelator detector [2], the maximum asymptotic efficiency linear detector [2], and the linear minimum mean square error (LMMSE) [3] among others. Aside from linear detection, a number of non-linear (i.e., multi-stage) detection techniques have been proposed such as successive interference cancellation (or SIC detection, [4], [5], [6]), parallel interference cancellation (or PIC detection, [7]), joint successive interference cancellation (or JSIC detection, [8]), and group detection (e.g., [9], [10]) among others. In this work, we have focused on successive interference cancellation, (alternatively known as successive decoding, or "onion peeling"), where a decision is made on a particular user's bit at each stage, and then the interference of that particular user is subtracted off from the received signal before moving on to the next stage.

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2. SYSTEM MODEL AND DEFINITIONS

We assume a synchronous K-user system in which the users transmit BPSK modulated signals $\{s_i(t)\}_{i=1}^{K}$ over an AWGN channel. The received signal can be written in equivalent discrete-time baseband form as

$$y[t] = \sum_{i=1}^{K} b_i s_i[t] + n[t], \;\; \{b_i\}_{i=1}^{K} \in \{-1,+1\}^K \tag{1}$$

In signal space notation, Equation (1) can be rewritten as

$$\mathbf{y} = \mathbf{S}\mathbf{b} + \mathbf{n} \tag{2}$$

where $\mathbf{y}$ is the received vector, $\mathbf{b}$ is the bit vector or K-tuple sent, $\mathbf{S} = [\mathbf{s}_1\mathbf{s}_2 \ldots \mathbf{s}_K]$ is the $(N \times K)$ signal matrix, $\{\mathbf{s}_i\}_{i=1}^{K}$ are the signal vectors, N is the dimension of signal space, and $\mathbf{n}$ is the noise vector modeled as a zero-mean Gaussian N-dimensional random vector with variance σ^2 per dimension. We denote the energy of the k^{th} user's signal as A_k^2, where $A_k^2 = \mathbf{s}^T\mathbf{s}$.

Let $P_k(\sigma)$ denote the bit-error-rate (BER) of the k^{th} user for a given multi-user detector in the CDMA system.

Then the effective energy $e_k(\sigma)$ [1] is defined as the energy needed by the k^{th} user to achieve the same BER in a single-user channel, i.e., $P_k(\sigma) = Q\left(\sqrt{e_k(\sigma)}/\sigma\right)$. The multi-user efficiency [1] of the k^{th} user, $\eta_k(\sigma)$, is the ratio of the effective energy $e_k(\sigma)$ to the actual energy, of the k^{th} user. The asymptotic multi-user efficiency [1] of the k^{th} user, η_k, is the limit of $\eta_k(\sigma)$ as the noise variance σ^2 approaches zero, i.e.,

$$\eta_k = \lim_{\sigma \to 0} \frac{e_k(\sigma)}{A_k^2}. \tag{3}$$

The asymptotic multi-user efficiency can also be written as [1]

$$\eta_k = \frac{1}{A_k^2} \lim_{\sigma \to 0} 2\sigma^2 \log\left(\frac{1}{P_k(\sigma)}\right). \tag{4}$$

Without loss of generality let $\mathcal{O} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K\}$ be a given ordering of user signals for successive interference cancellation. Denote by $P_{i|\mathcal{O}}(\sigma)$ the conditional probability that the i^{th} user's bit is correctly detected at the i^{th} stage of successive detection, given that the users with signals $\{\mathbf{s}_1, \mathbf{s}_2, .., \mathbf{s}_{i-1}\}$ have been correctly detected in the first $(i-1)$ stages, i.e.,

$$P_{i|\mathcal{O}}(\sigma) = P\left(\hat{b}_i = b_i | \hat{b}_1 = b_1, \hat{b}_2 = b_2, \ldots, \hat{b}_{i-1} = b_{i-1}\right).$$

Definition 2.1 *Given an ordering $\mathcal{O}$, the asymptotic conditional multi-user efficiency for the i^{th} user, denoted as $\eta_{i|\mathcal{O}}$, is*

$$\eta_{i|\mathcal{O}} = \frac{1}{A_i^2} \lim_{\sigma \to 0} 2\sigma^2 \log\left(\frac{1}{P_{i|\mathcal{O}}(\sigma)}\right). \tag{5}$$

3. MAXIMUM CONDITIONAL EFFICIENCY SUCCESSIVE INTERFERENCE CANCELLATION

Any linear detector outputs decisions as $\hat{b}_k = \operatorname{sgn}\left(\mathbf{v}^T \mathbf{y}\right)$, $k = 1, 2, \ldots, K$ for some $\mathbf{v} \in \Re^N$. The linear detector that maximizes the asymptotic multiuser efficiency for the k^{th} user was derived in [2] for a CDMA model with $\mathcal{S} = \{s_i(t)\}_{i=1}^K$ as the basis of the signal space. Our model assumes an orthonormal basis. Incorporating this basis change in [2], (ref. Equation (4.4) in [2]), the maximum asymptotic efficiency for the k^{th} user achievable among all possible linear detectors, denoted as $\eta_{k,\max}$ is given by:

$$\eta_{k,\max} = \sup_{\mathbf{v} \in \Re^N} \max{}^2\left(0, \frac{\mathbf{s}_k^T \mathbf{v} - \sum_{j=1, j \neq k}^K |\mathbf{s}_j^T \mathbf{v}|}{\sqrt{\mathbf{s}^T \mathbf{s}}\sqrt{\mathbf{v}^T \mathbf{v}}}\right) \tag{6}$$

where $\max^2(0, f(x)) = \max\left(0, (f(x))^2\right)$ wherever applicable in the sequel. Any SIC detector outputs its bit estimates as $\hat{b}_i = \operatorname{sgn}\left(\mathbf{v}_i^T\left(\mathbf{y} - \sum_{j=1}^{i-1} \hat{b}_j \mathbf{s}_j\right)\right)$, i.e., at the i^{th} stage, it employs a linear detector on the received vector after subtracting off the interference of the user signals already detected in previous $(i-1)$ stages. Thus the maximum asymptotic conditional efficiency at the i^{th} stage of decoding is simply the maximum asymptotic efficiency achieved by a linear detector for the i^{th} user in the reduced system $\tilde{\mathcal{S}}_i = \{\mathbf{s}_j\}_{j=i}^K$. Therefore, given an ordering $\mathcal{O}$, the maximum asymptotic conditional efficiency achievable by a SIC detector at the i^{th} stage of decoding, denoted as $\eta_{i,\max|\mathcal{O}}$, will be given as:

$$\eta_{i,\max|\mathcal{O}} = \sup_{\mathbf{v}_i \in \Re^N} \max{}^2\left(0, \frac{\mathbf{s}_i^T \mathbf{v}_i - \sum_{j=i+1}^K |\mathbf{s}_j^T \mathbf{v}_i|}{\sqrt{\mathbf{s}^T \mathbf{s}}\sqrt{\mathbf{v}_i^T \mathbf{v}_i}}\right). \tag{7}$$

Note that Equation (7) represents a convex optimization problem whose computational complexity will be $\mathcal{O}(2^{K-i})$. Thus, the maximum asymptotic conditional efficiency successive interference canceller, given an ordering $\mathcal{O}$, henceforth referred to as the MACE-SIC($\mathcal{O}$) detector, will output its bit estimates as

$$\hat{b}_i = \operatorname{sgn}\left(\mathbf{v}_{i,\mathrm{opt}|\mathcal{O}}^T\left(\mathbf{y} - \sum_{j=1}^{i-1} \hat{b}_j \mathbf{s}_j\right)\right) \quad i = 1, 2, \ldots, K, \tag{8}$$

where

$$\mathbf{v}_{i,\mathrm{opt}|\mathcal{O}} = \arg \sup_{\mathbf{v}_i \in \Re^N} \max{}^2\left(0, \frac{\mathbf{s}_i^T \mathbf{v}_i - \sum_{j=i+1}^K |\mathbf{s}_j^T \mathbf{v}_i|}{\sqrt{\mathbf{s}^T \mathbf{s}}\sqrt{\mathbf{v}_i^T \mathbf{v}_i}}\right). \tag{9}$$

3.1. Optimal ordering of users

The maximum asymptotic conditional efficiency ordering $\mathcal{O}_{\mathrm{opt}}$, henceforth referred to as the optimal ordering, is the ordering that maximizes $\eta_{i,\max|\mathcal{O}}$, defined in Equation (7) among all possible orderings for each $i = 1, \ldots, K$. Let $\eta_{\max}\left(\tilde{\mathcal{S}}, \mathbf{s}_k\right)$ denote the maxmimized asymptotic efficiency for a linear detector for the user with signal $\mathbf{s}_k$ (as given in Equation (6)) in a multi-user system with $\tilde{\mathcal{S}}$ as the set of user signals. We can derive $\mathcal{O}_{\mathrm{opt}}$ according to the following greedy algorithm:

1. Among K possible choices for the user to be decoded first, choose the user signal $\mathbf{s}_{m_1}$ from the set $\tilde{\mathcal{S}}_1 = \mathcal{S}$ such that

$$\eta_{\max}(\tilde{\mathcal{S}}_1, \mathbf{s}_{m_1}) = \max_{k=1,2,\ldots,K} \eta_{\max}(\tilde{\mathcal{S}}_1, \mathbf{s}_k)$$

2. Choose the second user signal $\mathbf{s}_{m_2}$ among the $(K-1)$ choices for the reduced system $\tilde{\mathcal{S}}_2 = \tilde{\mathcal{S}}_1 - \mathbf{s}_{m_1}$ such that

$$\eta_{\max}\left(\tilde{\mathcal{S}}_2, \mathbf{s}_{m_2}\right) = \max_{k=1,2,\ldots,K, k \neq m_1} \eta_{k,\max}\left(\tilde{\mathcal{S}}_2, \mathbf{s}_{m_2}\right)$$

3. Continue in this fashion such that at the i^{th} stage we choose the user signal $\mathbf{s}_{m_i}$ among the $(K-i)$ choices left in $\tilde{\mathcal{S}}_i = \tilde{\mathcal{S}}_{i-1} - \mathbf{s}_{m_{i-1}}$ such that

$$\eta_{\max}\left(\tilde{\mathcal{S}}_i, \mathbf{s}_{m_i}\right) = \max_{k=1,2,\ldots,K,\{k\neq m_l\}_{l=1}^{i-1}} \tilde{\eta}_{\max}\left(\tilde{\mathcal{S}}_i, \mathbf{s}_k\right)$$

4. At the K^{th} stage, the algorithm terminates yielding

$$\mathcal{O}_{\text{opt}} = \{\mathbf{s}_{m_1}, \mathbf{s}_{m_2}, \ldots, \mathbf{s}_{m_K}\}.$$

Note that $\mathcal{O}_{\text{opt}}$ maximizes the asymptotic conditional efficiency at each stage of successive detection, and does not necessarily maximize the overall asymptotic efficiency, or the asymptotic conditional efficiency for each (or any) user. Sufficiency conditions on the user signals for which $\mathcal{O}_{\text{opt}}$ does maximize η_k for all k can be derived and is outside the scope of this paper. It can also be shown that the sequence $\{\eta_{i,\max|\mathcal{O}_{\text{opt}}}(\tilde{\mathcal{S}}_i)\}$ is strictly increasing under certain conditions.

Also note that the computational complexity of deriving $\mathcal{O}_{\text{opt}}$ is $\mathcal{O}\left(K2^{K-1}\right)$. The motivation behind this work is to trade off the complexity of setting up the SIC detector with the performance gain achieved by optimizing the conditional efficiency at each stage of detection. The run-time complexity of the MACE-SIC detector is always linear in the number of users, i.e., $\mathcal{O}(K)$. The MACE-SIC detector is suitable for stable communication systems with good power control and a stable system of users, e.g., in satellite communications. In more dynamic multi-user systems, e.g., cellular networks, the high complexity of setting up the detector when the system changes may make it impractical for implementation. Sufficient conditions have been derived on the user signals for which $\mathcal{O}_{\text{opt}}$ and $\{v_{i,\text{opt}|\mathcal{O}_{\text{opt}}}\}_{i=1}^{K}$ for a dynamic system are less computationally expensive to compute, but are outside the scope of this paper.

4. MAXIMUM ASYMPTOTIC CONDITIONAL EFFICIENCY JSIC DETECTION

Joint successive interference cancellation (JSIC) was proposed in [8] and uses the geometric relationship between two consecutive users in an ordered set to achieve better bit estimates at each stage of successive interference cancellation. The kernel of the JSIC detector is a two-user ML detector that estimates b_i from the two-user system formed by $\{\mathbf{s}_i, \mathbf{s}_{i+1}\}$ ignoring the effect of the users $\{\mathbf{s}_{i+2}, \ldots, \mathbf{s}_K\}$ at the i^{th} stage of interference cancellation. The two-user JSIC kernel estimates b_i in two inner product operations using the vectors $\mathbf{s}_i$ and $\mathbf{p}_i = \arg\min\{||\mathbf{p}_{i_1}||^2, ||\mathbf{p}_{i_2}||^2\}$, where $\mathbf{p}_{i_1} = (\mathbf{s}_i - \mathbf{s}_{i+1})$, and $\mathbf{p}_{i_2} = (\mathbf{s}_i + \mathbf{s}_{i+1})$. We set up the maximum asymptotic conditional efficiency JSIC detector, henceforth referred to as the MACE-JSIC($\mathcal{O}$) detector, for a given ordering $\mathcal{O}$ as follows. We will continue the notation used in Section 3.1 and use $\eta_{\max}\left(\tilde{\mathcal{S}}, \mathbf{s}_k\right)$ to denote maximized asymptotic efficiency for a linear detector for the user with signal $\mathbf{s}_k$ in a reduced multi-user system $\tilde{\mathcal{S}}$.

Consider the four sub-constellations $\mathcal{C}\left(\mathbf{s}_i^{\pm}, \mathbf{s}_{i+1}^{\pm}\right)$, where, e.g., $\mathcal{C}\left(\mathbf{s}_i^{+}, \mathbf{s}_{i+1}^{+}\right) = \{\mathbf{s}_i + \mathbf{s}_{i+1} + \sum_{j=i+2}^{K} b_j \mathbf{s}_j : b_j = \pm 1\}$, at the i^{th} stage of successive decoding. Let $\mathbf{v}_{i_1,\text{opt}|\mathcal{O}}$, $\mathbf{v}_{i_2,\text{opt}|\mathcal{O}}$ and $\mathbf{v}_{i_3,\text{opt}|\mathcal{O}}$ be the optimized vectors for linear detection that achieve $\eta_{\max}\left(\tilde{\mathcal{S}}_{i_1}, \mathbf{s}_i\right)$, $\eta_{\max}\left(\tilde{\mathcal{S}}_{i_1}, \mathbf{p}_{i_1}\right)$ and $\eta_{\max}\left(\tilde{\mathcal{S}}_{i_1}, \mathbf{p}_{i_2}\right)$ respectively, in the reduced systems $\tilde{\mathcal{S}}_{i_1} = \{\mathbf{s}_i, \mathbf{s}_{i+2}, \ldots, \mathbf{s}_K\}$, $\tilde{\mathcal{S}}_{i_2} = \{\mathbf{p}_{i_1}, \mathbf{s}_{i+2}, \ldots, \mathbf{s}_K\}$ and $\tilde{\mathcal{S}}_{i_3} = \{\mathbf{p}_{i_2}, \mathbf{s}_{i+2}, \ldots, \mathbf{s}_K\}$ respectively. The MACE-JSIC($\mathcal{O}$) detector uses the vectors $\mathbf{v}_{i_1,\text{opt}|\mathcal{O}}$, $\mathbf{v}_{i_2,\text{opt}|\mathcal{O}}$ and $\mathbf{v}_{i_3,\text{opt}|\mathcal{O}}$ instead of $\mathbf{s}_i$, $\mathbf{p}_{i_1}$ and $\mathbf{p}_{i_2}$ respectively at the i^{th} stage of interference cancellation. Since we are using sub-constellations, and not constellation points in the two-user JSIC kernel, we need to consider both $\mathbf{p}_{i_1}$ and $\mathbf{p}_{i_2}$, rather than $\mathbf{p}_i$ and therefore, have three, instead of two inner product operations in the JSIC kernel. Using large deviations theory ([11], [12], Theorem 5.2, page 77) it can be shown that the asymptotic conditional efficiency $\eta_{i,\max|\mathcal{O}}$ of the MACE-JSIC($\mathcal{O}$) detector, will be given by

$$\begin{aligned}\eta_{i,\max|\mathcal{O}} &= \min \left\{\eta_{\max}\left(\tilde{\mathcal{S}}_{i_1}, \mathbf{s}_i\right), \eta_{\max}\left(\tilde{\mathcal{S}}_{i_2}, \mathbf{p}_{i_1}\right),\right. \\ &\left.\qquad \eta_{\max}\left(\tilde{\mathcal{S}}_{i_3}, \mathbf{p}_{i_2}\right)\right\}. \end{aligned} \tag{10}$$

The optimal ordering $\mathcal{O}_{\text{opt}}$ that achieves the highest asymptotic conditional efficiency at each stage among all possible orderings can then be set up in a very similar algorithm as in Section 3.1. At each stage i, we choose the two user signals $\mathbf{s}_{m_{i_1}}$ and $\mathbf{s}_{m_{i_2}}$ over the C_2^{K-i+1} possible choices such that the quantity

$$\begin{aligned}\tilde{\eta}_i &= \min \left\{\eta_{\max}\left(\tilde{\mathcal{S}}_{l_{i_1}}, \mathbf{s}_{l_{i_1}}\right), \eta_{\max}\left(\tilde{\mathcal{S}}_{l_{i_2}}, \mathbf{p}_{l_{i_1}}\right),\right. \\ &\left.\qquad \eta_{\max}\left(\tilde{\mathcal{S}}_{l_{i_3}}, \mathbf{p}_{l_{i_2}}\right)\right\}\end{aligned}$$

is maximized $\forall \mathbf{s}_{l_{i_1}}, \mathbf{s}_{l_{i_2}} \in \tilde{\mathcal{S}}_i = \mathcal{S} - \{\mathbf{s}_{m_j}\}_{j=1}^{i-1}$, where $\tilde{\mathcal{S}}_{l_{i_1}} = \tilde{\mathcal{S}}_i - \mathbf{s}_{l_{i_2}}$, $\tilde{\mathcal{S}}_{l_{i_2}} = \tilde{\mathcal{S}}_{l_{i_1}} - \mathbf{s}_{l_{i_1}} + \mathbf{p}_{l_{i_1}}$, and $\tilde{\mathcal{S}}_{l_{i_3}} = \tilde{\mathcal{S}}_{l_{i_1}} - \mathbf{s}_{l_{i_1}} + \mathbf{p}_{l_{i_2}}$.

5. SIMULATION RESULTS AND CONCLUSION

The performance of the MACE-SIC and MACE-JSIC detectors have been tested against the conventional SIC, and conventional JSIC detectors. Numerical simulations were run on several over-loaded systems, i.e., systems where the number of users, K, is greater than the number of dimensions N, and therefore, the user signals $\mathcal{S}$ form a linearly dependent set. The results were averaged over the multi-user systems considered and bitstreams. It is seen in Figure

1 that the MACE-SIC and MACE-JSIC consistently outperform the conventional SIC and JSIC detectors and are robust to strong correlation (linear dependence) among the user signals. Figure 2 shows the single user performance for a particular four-user system among the several multi-user systems considered, where the conventional SIC detector is not near-far resistant, but the MACE-SIC detector shows robustness against multi-user interference since the conditional multi-user efficiency has been optimized at each stage.

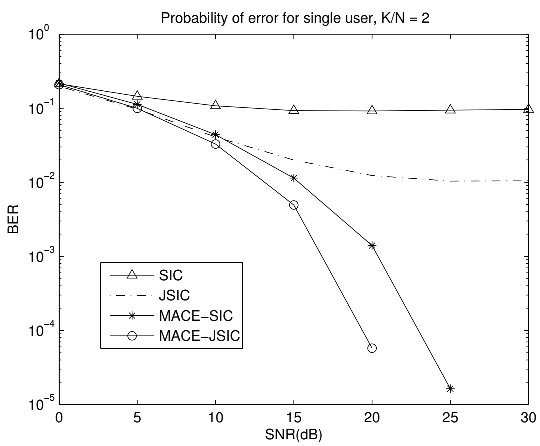

Fig. 1. Single-user BER in an average four-user system

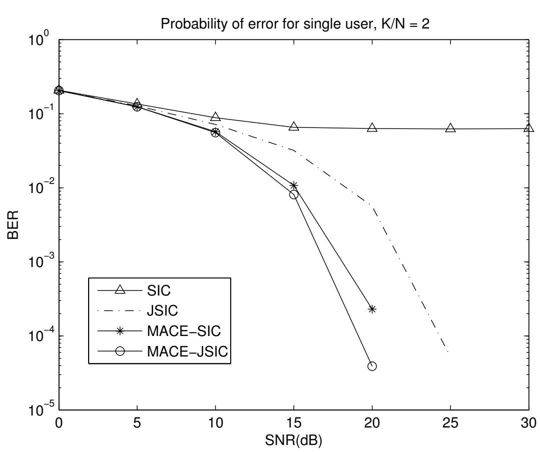

Fig. 2. Single-user BER in a representative four-user system

We have addressed the problem of user ordering and kernel detector choice in successive interference cancellation. Our focus has been the asymptotic multi-user efficiency, which rates detector performance in high SNR. We defined the conditional multi-user efficiency as a performance measure to test an existing ordering of users and the kernel detector in successive interference cancellation. We proposed an ordering of the user signals and derived the kernel to achieve the highest conditional asymptotic efficiency at each stage of successive decoding. Finally, we combined the maximal conditional efficiency (MACE) approach with joint successive interference cancellation (JSIC) achieving a lower probability of error in the high SNR situation.

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