AN EFFICIENT HEURISTIC APPROACH TO THE INFEASIBLE DOWNLINK POWER CONTROL PROBLEM

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ABSTRACT

This paper proposes a novel heuristic approach to the *infeasible* power control problem [i.e., no power allocation can provide users with specified quality of service (QoS)] in wireless communication systems. In such infeasible cases, it has been in great demand to increase the number of *accepted* users (i.e., users provided with the specified QoS) by certain *distributed algorithms*, which require no communication among base stations (BSs).

The number of accepted users is often suppressed by a small number of users in severe environments, hence removing such users obviously helps many other users be accepted. The proposed algorithm, which is fully distributed, detects and removes such users based on simple criteria including: (a) the QoS is below the necessary level, and (b) the BS is already transmitting with the maximum possible power to the user. Simulation results demonstrate that the proposed algorithm significantly increases the number of accepted users with low computational complexity.

1. INTRODUCTION

This paper addresses the *infeasible* power control problem in wireless communication systems; i.e., there is no power allocation that simultaneously achieves target quality of service (QoS) for multiple accessing users [1, 2]. The goal of this paper is to attain a novel heuristic approach to this challenging problem. We focus on the downlink power control without exchanging any information among base stations (BSs). In this case, the power control at each BS should be done with only local measurements, which is called *distributed (or decentralized)* power control [3, 4]. To further clarify our motivation, let us start with a brief introduction of power control schemes.

In wireless communication systems, such as code/space division multiple access (CDMA/SDMA), it is known that the conventional matched filter severely loses its efficiency in the presence of multiple access interference (MAI) [5]. A great deal of effort has been devoted for suppressing MAI at receiver sides to ensure high quality of service (QoS) (see, e.g., [6]). For successful MAI suppression, high level of signal-to-interference-plus-noise ratio (SINR) should be achieved at a receiver. An increase of transmitted power for a user improves the SINR at the user while it degrades those at the other users. This implies that the transmitted powers for all active users should be balanced, thus the power control is necessary. The power control is classified into two types: *centralized* and *distributed*. Recently, distributed power control has been playing a main role in both research and developments, since it is free from the practical problems such as additional infrastructure, latency, and network vulnerability [1, 4].

The power control easily becomes infeasible due to an increase of the number of users in important applications such as mobile communications and ad hoc networks [7], where the number of users is not very limited. The *call admission control* or *scheduling* are therefore performed to reduce the number of users before the power control is done [7, 8]. However, if they are imperfect, the power control can become infeasible. Strategic resolutions to the infeasible problem are thus important in power control schemes [3, 7]. One of the possible way is to temporarily *remove* some users during the power control in order to increase the number of users receiving the target SINR (which we refer to as *accepted* users). A reasonable removal algorithm that is almost distributed has been proposed in [3]. However, in practical situations, a *fully* distributed (see, Remark 1) removal algorithm has been in great demand.

In this paper, we first present a straightforward formulation for the infeasible power control problem; maximize the number of accepted users and minimize the total transmitted power. Unfortunately, it requires global information and a high computational complexity in order to compute the optimal solution to this problem. We then propose an efficient algorithm that increase the number of accepted users as much as possible by the user removal. The proposed algorithm is fully distributed and requires almost the same computational complexity as the one in [9]. Simulation results verify the efficiency of the proposed algorithm.

2. PRELIMINARIES

We consider a cellular wireless system with N base stations (BSs) and M cochannel users (see Fig. 1) [9, 10]. Assume that the channels are stationary and each BS updates its power allocation synchronously. Also assume the ith user is assigned to the b_ith BS, where $b_i \in \{1, \ldots, N\}$, and BS assignments for all users are fixed during considered a period.

Let $p_i \in [0, p_{\max}]$ be the transmitted power for the ith user, where $p_{\max}$ is the maximum transmitted power for each user, and $h_{ib_j} \geq 0$ the channel gain from the b_jth BS to the ith user. Then the received SINR at the ith user is defined as follows [9, 10]:

$$\Gamma_i(\boldsymbol{p}) := \frac{h_{ib_i} p_i}{\sum_{j \neq i}^{M} h_{ib_j} p_j + \sigma_i^2} =: \frac{h_{ib_i} p_i}{I_i(\boldsymbol{p})}, \tag{1}$$

where $\boldsymbol{p} := (p_1, \ldots, p_M)^t$ is the power vector, σ_i^2 and $I_i(\boldsymbol{p})$ denote the powers of additive noise and interference-plus-noise at the ith user, respectively.

In order to suffice QoS requirements of all users, the SINR at the ith user must exceed at least a certain level γ_i, which we refer to as target SINR. The objective of the power control is to minimize the total power of all users under such a condition. The power control problem is formulated as follows [10]:

$$\begin{aligned} &\underset{p_i \in [0, p_{\max}]}{\text{minimize}} \quad \sum_{i=1}^{M} p_i \\ &\text{subject to} \quad \Gamma_i(\boldsymbol{p}) \geq \gamma_i, \quad \forall i = 1, \ldots, M. \end{aligned} \tag{2}$$

Let $\mathcal{P} = [0, p_{\max}]^M$ denote the set of all power vectors. Then,

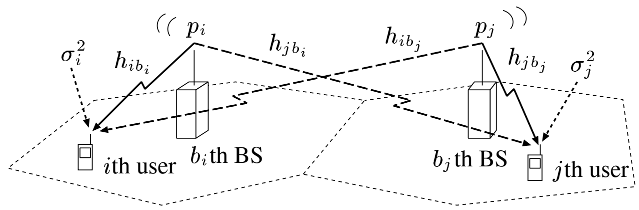

Fig. 1. A model of downlink transmission. Solid and dotted arrows are the paths for desired, interfering and noise signals, respectively.

we can rewrite the problem (2) in matrix form as follows [10]:

$$\begin{aligned} &\underset{\boldsymbol{p}\in\mathcal{P}}{\text{minimize}} \quad \boldsymbol{1}^t\boldsymbol{p} \\ &\text{subject to} \quad (\boldsymbol{I}-\boldsymbol{H})\boldsymbol{p} \geq \boldsymbol{u}, \end{aligned} \tag{3}$$

where $\boldsymbol{1} := (1,\ldots,1)^t$, $(\cdot)^t$ stands for the transposition,

$$[\boldsymbol{H}]_{ij} := \begin{cases} 0 & \text{if} \quad i=j; \\ \dfrac{\gamma_i h_{ib_j}}{h_{ib_i}} \geq 0 & \text{otherwise}, \end{cases} \tag{4}$$

and

$$[\boldsymbol{u}]_i := \frac{\gamma_i \sigma_i^2}{h_{ib_i}} > 0. \tag{5}$$

We assume that $\boldsymbol{H}$ is an irreducible matrix (see Appendix, Definition A) [10]. Let $\rho(\boldsymbol{H})$ denote the spectral radius (i.e. the maximum absolute eigenvalue) of $\boldsymbol{H}$. Most existing studies [10, 11] consider the case that $\rho(\boldsymbol{H}) < 1$. Hence, the matrix $(\boldsymbol{I}-\boldsymbol{H})$ is invertible and $(\boldsymbol{I}-\boldsymbol{H})^{-1}$ is positive (see Appendix, Definition B) [12]. In this case,

$$\boldsymbol{p}^* := (\boldsymbol{I}-\boldsymbol{H})^{-1}\boldsymbol{u} > \boldsymbol{0}, \tag{6}$$

and, if $\boldsymbol{p}^* \in \mathcal{P}$, the total transmitted power is minimized by $\boldsymbol{p}^*$, that is $\boldsymbol{p}^*$ is the optimal power vector.

In order to compute $\boldsymbol{p}^*$, the following distributed power control algorithm has been proposed [9].

Algorithm 1 ([9])**.** *Generate a sequence of power vectors* $(\boldsymbol{p}^{(n)})_{n\in\mathbb{N}}$ *as [*$\mathbb{N}$*: the set of nonnegative integers]*

$$p_i^{(n+1)} = T_i(\boldsymbol{p}^{(n)}) := \min\left\{\frac{\gamma_i}{h_{ib_i}} I_i(\boldsymbol{p}^{(n)}), p_{\max}\right\}, \tag{7}$$

where $p_i^{(n)}$ *is the transmitted power for the ith user at the nth iteration. In matrix form,* (7) *can be written as*

$$\boldsymbol{p}^{(n+1)} = T(\boldsymbol{p}^{(n)}) := \min\left\{\boldsymbol{H}\boldsymbol{p}^{(n)} + \boldsymbol{u}, \boldsymbol{p}_{\max}\right\}, \tag{8}$$

where $\min\{\cdot,\cdot\}$ *is componentwise, and* $\boldsymbol{p}_{\max} = (p_{\max},\ldots,p_{\max})^t$.

Eq. (7) implies that each BS can update its transmitted power only by using γ_i, h_{ib_i} and $I_i(\boldsymbol{p}^{(n)})$.[1] No information is therefore exchanged among all BSs, i.e., Algorithm 1 is *distributed*. ¿From the results in [9], the following proposition is readily verified.

Proposition 1 ([9])**.** *Starting from an arbitrary* $\boldsymbol{p}^{(0)} \in \mathcal{P}$*, the sequence* $(T^n(\boldsymbol{p}^{(0)}))_{n\in\mathbb{N}}$ *converges to the unique fixed point of* $T : \mathcal{P} \to \mathcal{P}$*, i.e., it converges to* $\hat{\boldsymbol{p}} \in \mathcal{P}$ *such that* $T(\hat{\boldsymbol{p}}) = \hat{\boldsymbol{p}}$*. Furthermore,* $\hat{\boldsymbol{p}}$ *has the following properties: (i) if* $\rho(\boldsymbol{H}) < 1$ *[*$\Rightarrow \boldsymbol{p}^*$ *in* (6) *exists] and* $\boldsymbol{p}^* \in \mathcal{P}$ *then* $\hat{\boldsymbol{p}} = \boldsymbol{p}^*$*; and (ii) otherwise at least one component of* $\hat{\boldsymbol{p}}$ *is equal to* $p_{\max}$.

[1] h_{ib_i} and $I_i(\boldsymbol{p}^{(n)})$ are measured at the ith user and transmitted to the b_ith BS through feedback channels.

3. THE POWER CONTROL FOR INFEASIBLE CASES

We first reformulate the power control problem for infeasible cases. Then we propose an efficient heuristic power control algorithm to solve the reformulated problem.

3.1. A Mathematical Formulation for Infeasible Cases

Consider the case where the problem (2) is infeasible, i.e., there exists no power vector $\boldsymbol{p}$ satisfying $\Gamma_i(\boldsymbol{p}) \geq \gamma_i$, for all $i = 1,\ldots,M$. In this case, we need to find a *compromise* solution. A straightforward strategy is to maximize the number of active users. Define

$$\mathcal{M}(\boldsymbol{p}) := \{i \in \mathcal{M} \mid \Gamma_i(\boldsymbol{p}) \geq \gamma_i\}, \tag{9}$$

where $\mathcal{M} := \{1,\ldots,M\}$. Then we reformulate the problem (2) as follows:

$$\begin{aligned} &\text{minimize} \quad \boldsymbol{1}^t\boldsymbol{p} \\ &\text{subject to} \quad \boldsymbol{p} \in \arg\max_{\boldsymbol{p}\in\mathcal{P}} |\mathcal{M}(\boldsymbol{p})|, \end{aligned} \tag{10}$$

where $|\cdot|$ stands for the cardinality. It is obvious that, if the original problem (2) is feasible, then $\max_{\boldsymbol{p}\in\mathcal{P}}|\mathcal{M}(\boldsymbol{p})| = M$ and

$$\arg\max_{\boldsymbol{p}\in\mathcal{P}} |\mathcal{M}(\boldsymbol{p})| = \{\boldsymbol{p}\in\mathcal{P} \mid (\boldsymbol{I}-\boldsymbol{H})\boldsymbol{p} \geq \boldsymbol{u}\}. \tag{11}$$

The problem (10) is thus a natural extension of the original problem (2). Unfortunately, it is nonrealistic to directly solve this straightforward problem for the following reasons: (i) high computational complexity due to a combinatorial nature; and (ii) requirements for a distributed algorithm.

3.2. An Efficient Heuristic Power Control

We present an efficient algorithm by devising the Algorithm 1 to increase $|\mathcal{M}(\boldsymbol{p})|$ as much as possible within a practical computational cost. In order to increase $|\mathcal{M}(\boldsymbol{p})|$, we *remove* some users from the system, i.e., reduce the number of active users (although a similar policy has been proposed in [3, 7], its removal criteria differs; see Remark 1). If some users are removed then the interference is reduced, resulting in an increase of $|\mathcal{M}(\boldsymbol{p})|$. The key is how to select the users to be removed. Here, we define the *bottleneck-user* as follows:

Definition 1 (bottleneck-user)**.** *The bottleneck-user is characterized as a user satisfying the following three conditions at each iteration in* (7)*:*

$$\textstyle\sum_{k=n-\kappa+1}^{n} |p_i^{(k)} - p_i^{(k-1)}| \leq \varepsilon; \tag{12}$$

$$p_i^{(n)} = p_{\max}; \tag{13}$$

$$\Gamma_i(\boldsymbol{p}^{(n)}) \leq \alpha\gamma_i, \tag{14}$$

where a positive integer κ*, positive real values* ε *and* $\alpha \in (0,1)$ *are chosen appropriately.*

We propose the following algorithm that removes the bottleneck-users by priority.

Algorithm 2. *Starting from an arbitrary* $\boldsymbol{p}^{(0)} \in \mathcal{P}$*, do the followings for all* $i \in \mathcal{M}$ *at the nth* ($n \in \mathbb{N}$) *iteration.*

1. *Update* $p_i^{(n)}$ *by* (7)*, and* $n := n+1$.
2. *The ith user is removed if bottleneck, i.e.,* $p_i^{(\ell)} = 0$ *for all* $\ell \geq n$.
3. *If* (12) *holds for all* $i \in \mathcal{M}$ *then exit. Otherwise return to Step 1.*

Table 1. Comparison of power allocations of the proposed and conventional algorithms.

	Algorithm 1			Algorithm 2		
n	p_1	p_2	p_3	p_1	p_2	p_3
0	0	0	0	0	0	0
$\vdots$		$\vdots$			$\vdots$	
12	0.784	0.784	1.000	0.784	0.784	1.000
*13	0.917	0.917	1.000	0.917	0.917	0.000
14	0.984	0.984	1.000	0.484	0.484	0.000
$\vdots$		$\vdots$			$\vdots$	
25	1.000	1.000	1.000	0.050	0.050	0.000

Let us give an essential idea of Algorithm 2. Consider the situation where the power for some user sufficiently converges (12). In such a situation, a user allocated $p_{\max}$ is intuitively considered to be a factor deteriorating the SINRs of the other users (13). Therefore, if such a user does not receive the target SINR (14), it should be removed by priority.

The existence of a user allocated $p_{\max}$ is guaranteed in infeasible cases thanks to Proposition 1, which ensures that the bottleneck-users are removed successfully. Note that Algorithm 2 enjoys as low computational complexity as Algorithm 1.

Remark 1. In [3], a similar distributed algorithm that is called Limited Information Stepwise Removal Algorithm (LI-SRA) has been proposed, which is used in [7]. Compared with LI-SRA, Algorithm 2 has the following advantages:

1. Algorithm 2 is fully distributed, while LI-SRA is not [3]. This is because the removal stage of LI-SRA requires all values $\varGamma_i(\boldsymbol{p}^{(n)})$ $(i \in \mathcal{M})$.
2. Algorithm 2 can track the time-varying optimal power vector, in a practical situation, without reinitializing the power control, whereas LI-SRA cannot. This is because Algorithm 2 can start with an arbitrary $\boldsymbol{p}^{(0)} \in \mathcal{P}$, whereas LI-SRA must start with $\boldsymbol{p}^{(0)} = \mathbf{1}$.
3. LI-SRA cannot remove a user until the specified number of iterations are made. On the other hand, Algorithm 2 can remove a user as soon as a bottleneck-user is detected because the removal criteria for the ith user is independent of the states of the other users.

To clarify the advantages of the proposed algorithm, let us demonstrate how the proposed algorithm performs in an infeasible case.

Example 1. Consider the case with the number of users $M = 3$ and the following settings:

$$p_{\max} = 1, \quad \gamma_1 = \gamma_2 = \gamma_3 = 2, \quad \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 0.01,$$
$$h_{1b_1} = 0.8, \quad h_{1b_2} = 0.2, \quad h_{1b_3} = 0.2,$$
$$h_{2b_1} = 0.2, \quad h_{2b_2} = 0.8, \quad h_{2b_3} = 0.2,$$
$$h_{3b_1} = 0.2, \quad h_{3b_2} = 0.2, \quad h_{3b_3} = 0.5.$$

In this case, we have $\rho(\boldsymbol{H}) \approx 1.18$ (≥ 1) and $(\boldsymbol{I} - \boldsymbol{H})^{-1}\boldsymbol{u} = (-1.5, -1.5, -2.0)^t$, which implies that the problem is infeasible. Starting from $(0,0,0)^t$, Algorithms 1 and 2 respectively update the power vectors as in Table 1. At the 13th iteration, Algorithm 2 sets p_3 to 0 because the 3rd user is bottleneck. Finally, Algorithm 2 achieves $\varGamma_1(\boldsymbol{p}^{(25)}) = \varGamma_2(\boldsymbol{p}^{(25)}) = 2$, $\varGamma_3(\boldsymbol{p}^{(25)}) = 0$ (the 1st and 2nd users receive the target SINRs). On the other hand, Algorithm 1 results in $\varGamma_1(\boldsymbol{p}^{(25)}) = \varGamma_2(\boldsymbol{p}^{(25)}) = 1.95$, $\varGamma_3(\boldsymbol{p}^{(25)}) = 1.22$ (no user receives the target SINR).

Table 2. Some comparisons of the proposed and conventional algorithms for $\gamma = 5$. Only the results for the accepted users are averaged.

	Algorithm 1	Algorithm 2
Averaged power	0.8840	9.97×10^{-4}
Maximum power	1.0000	3.86×10^{-3}
Minimum power	0.0594	0.0 (removed)
Averaged SINR	1.775 dB	1.990 dB
Maximum SINR	5.000 dB	5.000 dB
Minimum SINR	-4.395 dB	$-\infty$ (removed)

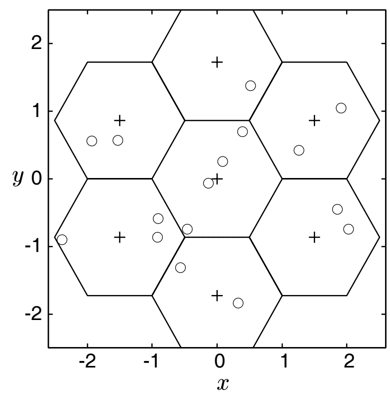

Fig. 2. Base stations and users distribution ($N = 7$ and $M = 16$). Base stations and users are indicated by $+$ and $\circ$, respectively.

4. SIMULATION RESULTS

We here consider a SDMA wireless system with $N = 7$ BSs illustrated in Fig. 2. It is known that the channel gain h_{ib_j} can be written as $h_{ib_j} = \boldsymbol{w}_j^H \boldsymbol{R}_{ib_j} \boldsymbol{w}_j$ [13], where $\boldsymbol{w}_j$ is the normalized (i.e., $\|\boldsymbol{w}_j\| = 1$) beamformer weight vector for the jth user, and $\boldsymbol{R}_{ib_j}$ is the downlink channel correlation (DCC, see [13]) matrix between the ith user and the b_jth BS. Each BS is equipped with a uniform circular array (UCA) of eight omnidirectional sensors. The channel gain is assumed to be proportional to $r_{ib_j}^{-4}$, where r_{ib_j} is the distance between the ith user and the b_jth BS. All users are randomly distributed in the area shown in Fig. 2 with a uniform distribution, and they are assigned to the nearest BS. We set to $p_{\max} = 1$. The beamformer weight vector $\boldsymbol{w}_i$ is computed by $\mathscr{P}\{[\sum_{i \neq j}^{M} \boldsymbol{R}_{i,b_j}]^{-1} \boldsymbol{R}_{ib_i}\}$ [14], where $\mathscr{P}\{\cdot\}$ denotes the principal eigenvector. We assume that the target SINR is common to all users. The noise power is randomly distributed between 0 and 0.01 with a uniform distribution. For the proposed algorithm, we set $\varepsilon = 10^{-9}$, $\kappa = 1$, and $\alpha = 0.8$. All the results are averaged over 1000 times independent runs.

Fig. 3 shows the averaged number of accepted users. The target SINR ranges between -1 dB and 8 dB every 1 dB. We can see that the proposed algorithm always increases the number of accepted users more than the conventional one. Especially, approximately 5 times more users are accepted for $\gamma = 5$ (γ: target SINR) by the proposed algorithm compared to the conventional one. For $2 \leq \gamma \leq 6$, the proposed algorithm achieves more than twice as many accepted users as the conventional one.

Fig. 4 depicts how much the proposed algorithm reduces the total transmitted power. We observe that the proposed algorithm

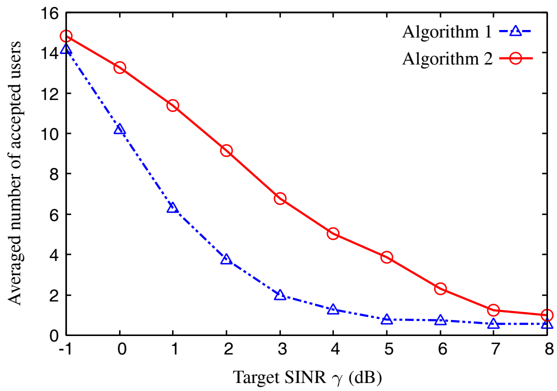

Fig. 3. Comparison of the averaged number of accepted users.

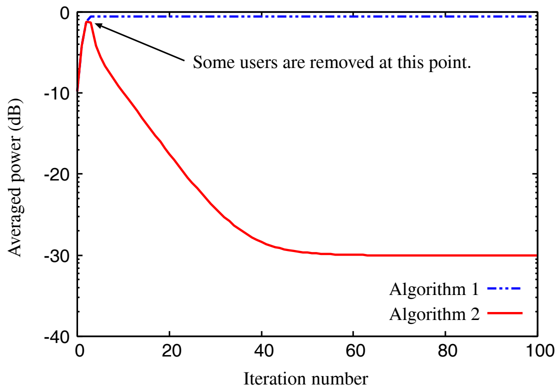

Fig. 4. An averaged power of users; $\sum_{i=1}^{M} p_i/M$ for $\gamma = 5$ dB.

attains approximately 30dB gain. Table 4 summaries the useful informations in Fig. 4.

5. CONCLUSIONS

We addressed the *infeasible* downlink power control problem and presented the efficient fully distributed algorithm. The proposed algorithm increases the number of accepted users in infeasible cases with the low computational complexity. We also verified the advantages of the proposed algorithm by simulations.

APPENDIX

IRREDUCIBLE AND NONNEGATIVE MATRICES

Definition A. *An $n \times n$ square matrix $\boldsymbol{A}$ is called reducible if there exists a permutation matrix $\boldsymbol{P}$ that puts $\boldsymbol{A}$ into the form*

$$\boldsymbol{P}\boldsymbol{A}\boldsymbol{P}^t = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{O} \\ \boldsymbol{C} & \boldsymbol{D} \end{pmatrix},$$

where $\boldsymbol{B}$ and $\boldsymbol{D}$ are square matrices. Otherwise $\boldsymbol{A}$ is called irreducible.

Definition B. *An $m \times n$ matrix $\boldsymbol{A}$ with real components is called non-negative or positive (notation: $\boldsymbol{A} \geq \boldsymbol{O}$ or $\boldsymbol{A} > \boldsymbol{O}$) if all the elements are non-negative or positive.*

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