BLIND MIMO CHANNEL ESTIMATION BASED ON STRUCTURED TRANSMIT DELAY

Qi Ling and Tongtong Li

Department of Electrical & Computer Engineering
Michigan State University, East Lansing, Michigan 48824, USA
Email: lingqi,tongli@egr.msu.edu

ABSTRACT

This paper aims to relax the restrictive conditions on blind MIMO channel identification by exploiting structured transmitter design. First, a simple structured transmit delay scheme is proposed, in which each antenna transmits an independent zero-padded data stream. Secondly, a second-order statistics based blind channel estimation algorithm is developed. Simulation results demonstrate that the proposed approach is robust in MIMO environment, even if when there are common zeros among the sub-channels.

1. INTRODUCTION

Due to the promised spectral efficiency, blind signal detection has attracted more and more research attention in recent years, especially in MIMO systems. It has been well understood that in SIMO systems, blind channel identification based on second order statistics generally requires that all SIMO sub-channels share no common zeros [1]. For MIMO systems, the existing direct blind identification algorithms generally require the matrix transfer function to be irreducible and column reduced, see [2–4] for example.

However, these restrictive conditions could be relaxed for MIMO systems by exploiting space-time diversity at the transmitter end. The approach in [5] guarantees channel identifiability without restrictions on channel zero locations, however, the data rate of the whole system is drastically decreased due to the Alamouti's block codes applied on each subcarrier in space time OFDM transmissions. Similarly, in [6], with reduced data rate, blind channel identification is achieved by exploiting the time reversed technique.

In this paper, we aim to relax the conditions on blind MIMO channel identification while achieving a higher data rate. First, a simple transmit delay scheme is proposed to exploit the guard intervals for blind channel estimation. Unlike existing transmit diversity schemes [5–8], in which different antennas transmit the delayed, zero-padded, or time reversed versions of the same signal, the proposed transmit delay scheme promises higher data rate since each antenna transmits an independent data stream. Secondly, a second-order statistics based blind channel estimation algorithm is developed. The proposed approach involves no pre-equalization and has no limitations on channel zero locations. The size of data block can be chosen large enough to make the data rate arbitrarily close to the maximum transmission rate.

The rest of this paper is organized as follows. The structured transmit delay scheme is proposed in Section 2. In Section 3, the blind channel estimation algorithm is developed. In Section 4, structured transmit delay scheme and blind channel estimation algorithm are extended to general MIMO systems. Simulation results are provided in Section 5 and we conclude in Section 6.

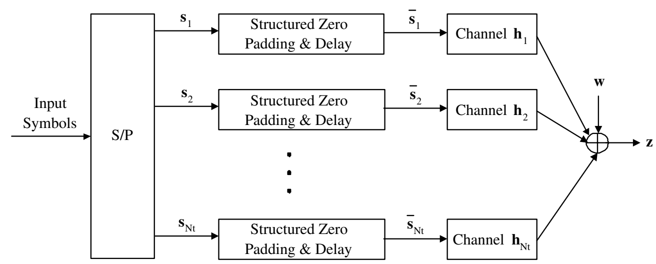


Fig. 1. A structured transmit delay scheme for MIMO systems.

2. THE PROPOSED TRANSMIT DELAY SCHEME

The block diagram of the proposed transmit delay scheme is illustrated in Fig. 1. Take the 2×2 MIMO system as an example. The input symbols are first split by a serial-to-parallel converter (S/P) into parallel data streams. Each data stream is then packed into N-symbol blocks. Denote the lth block from the 1st antenna and the 2nd antenna by $\mathbf{s}_1(l) = [s_1(lN), s_1(lN+1), \cdots, s_1(lN+N-1)]^T$ and $\mathbf{s}_2(l) = [s_2(lN), s_2(lN+1), \cdots, s_2(lN+N-1)]^T$, respectively. Assume that there are $2K$ blocks in a frame. Zero-padding is performed according to the following structure:
when $l \in [1, K]$,

$$\begin{cases} \bar{\mathbf{s}}_1(l) \triangleq [s_1(lN), \cdots, s_1(lN+N-1), \underbrace{0, \cdots, 0}_{L+1}]^T \\ \bar{\mathbf{s}}_2(l) \triangleq [0, s_2(lN), \cdots, s_2(lN+N-1), \underbrace{0, \cdots, 0}_{L}]^T \end{cases}$$

when $l \in [K+1, 2K]$,

$$\begin{cases} \bar{\mathbf{s}}_1(l) \triangleq [0, s_1(lN), \cdots, s_1(lN+N-1), \underbrace{0, \cdots, 0}_{L}]^T \\ \bar{\mathbf{s}}_2(l) \triangleq [s_2(lN), \cdots, s_2(lN+N-1), \underbrace{0, \cdots, 0}_{L+1}]^T, \end{cases}$$

where L is the maximal channel order among all of sub-channels in the system. After zero-padding, the size of each extended block is $P \triangleq N + L + 1$.

Let $\mathbf{r}_j(l) = [r_j(lP), \cdots, r_j(lP+P-1)]$ be the lth received block at the jth receive antenna. For $n = 0, \cdots, P-1$,

$$r_j(lP+n) = \sum_{i=1}^{2} \sum_{m=0}^{L} h_{ji}(m)\bar{s}_i(lP+n-m) + w_j(lP+n), \quad (1)$$

where $\mathbf{h}_{ji} \triangleq [h_{ji}(0), h_{ji}(1), \cdots, h_{ji}(L)]^T$ denotes the channel impulse response between the ith transmit antenna and the jth receive antenna, and $\mathbf{w}_j(l) \triangleq [w_j(lP), \cdots, w_j(lP+P-1)]^T$ is the additive noise sequence.

Our discussion in the following sections is based on the following assumptions:

(**A1**) The input information sequence is zero mean, mutually independent and i.i.d.. This implies that $E\{s_i(lN+m)s_j(kN+n)\} = \sigma_s^2 \delta_{i-j}\delta_{l-k}\delta_{m-n}$, where σ_s^2 is the signal power.

(**A2**) The noise is additive white Gaussian, independent of the information sequence, with variance σ_w^2.

(**A3**) There are $2K$ blocks in a data frame and the channel is time-invariant within each frame.

(**A4**) All the transmit antennas of a single user are synchronized.

3. SUBSPACE-BASED BLIND CHANNEL ESTIMATION

3.1. Without Prior Channel Order Information

Stack the lth block received from each of the two antennas into a $2P \times 1$ vector $\mathbf{z}(l)$. First consider $\{\mathbf{z}(l)\}_{l=1}^{K}$, we have,

$$\mathbf{z}(l) = \underbrace{\begin{bmatrix} \mathbf{H}_{11} & \mathbf{0}_{1\times N} \\ \mathbf{0}_{1\times N} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{0}_{1\times N} \\ \mathbf{0}_{1\times N} & \mathbf{H}_{22} \end{bmatrix}}_{\triangleq \mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{s}_1(l) \\ \mathbf{s}_2(l) \end{bmatrix}}_{\triangleq \mathbf{x}(l)} + \begin{bmatrix} \mathbf{w}_1(l) \\ \mathbf{w}_2(l) \end{bmatrix}. \quad (2)$$

where

$$\mathbf{H}_{ji} \triangleq \begin{bmatrix} h_{ji}(0) & 0 & \cdots & 0 \\ h_{ji}(1) & h_{ji}(0) & \ddots & \vdots \\ \vdots & h_{ji}(1) & \ddots & 0 \\ h_{ji}(L) & \ddots & \ddots & h_{ji}(0) \\ 0 & h_{ji}(L) & \ddots & h_{ji}(1) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{ji}(L) \end{bmatrix}_{(P-1)\times N} \quad (3)$$

The correlation matrix of $\mathbf{z}(l)$ is

$$\mathbf{R}_z = E\{\mathbf{z}(l)\mathbf{z}(l)^{\mathcal{H}}\} = \sigma_s^2 \mathbf{H}\mathbf{H}^{\mathcal{H}} + \sigma_w^2 \mathbf{I}_{2P}. \quad (4)$$

In the absence of noise, the eigendecomposition of $\mathbf{R}_z$ can be represented as

$$\mathbf{R}_z = \begin{bmatrix} \mathbf{U} & \tilde{\mathbf{U}} \end{bmatrix} \begin{bmatrix} \sum & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{U}^{\mathcal{H}} \\ \tilde{\mathbf{U}}^{\mathcal{H}} \end{bmatrix}, \quad (5)$$

where $\sum$ is a diagonal matrix of size $2N \times 2N$ with nonzero diagonal entries, and $\tilde{\mathbf{U}}$ is a $2P \times (2P-2N)$ matrix, whose columns span the null space $\mathcal{N}(\mathbf{R}_z)$. Write $\tilde{\mathbf{U}} = [\tilde{\mathbf{u}}_1, \cdots, \tilde{\mathbf{u}}_{2P-2N}]$. Because $\mathcal{N}(\mathbf{R}_z)$ is orthogonal to the range space $\mathcal{R}(\mathbf{H})$, it follows that

$$\tilde{\mathbf{u}}_k^{\mathcal{H}}\mathbf{H} = \mathbf{0}_{1\times 2N}, \qquad \forall k \in [1, 2P-2N]. \quad (6)$$

Let $\tilde{\mathbf{h}}_1 \triangleq \mathbf{H}(:,1)$ and $\tilde{\mathbf{h}}_2 \triangleq \mathbf{H}(:,N+1)$. And

$$\tilde{\mathbf{H}}_\mu \triangleq \mathbf{V}\tilde{\mathbf{h}}_\mu, \; u = 1, 2, \quad (7)$$

where $\mathbf{V}$ denotes a $2P \times 2P$ Vandermonde matrix with the $(m+1, n+1)$st entry $e^{(-j\frac{\pi}{P}mn)}$.

Note that $\mathbf{H}$ can be factorized as

$$\mathbf{H} = [\mathbf{F}_{2P}^{\mathcal{H}}\mathbf{D}_1\mathbf{F}_{2P}\Theta \;\; \mathbf{F}_{2P}^{\mathcal{H}}\mathbf{D}_2\mathbf{F}_{2P}\Theta], \quad (8)$$

where $\mathbf{D}_\mu \triangleq diag(\tilde{\mathbf{H}}_\mu)$, $\mathbf{F}_{2P}$ denotes a $2P \times 2P$ FFT matrix with the $(m+1, n+1)$st entry $\frac{1}{\sqrt{2P}}e^{(-j\frac{\pi}{P}mn)}$, Θ is the first N columns of an identity matrix with size $2P$. It follows from (6) & (8) that

$$\begin{aligned} \tilde{\mathbf{u}}_k^{\mathcal{H}}\mathbf{H} &= \tilde{\mathbf{u}}_k^{\mathcal{H}}[\mathbf{F}_{2P}^{\mathcal{H}}\mathbf{D}_1\mathbf{F}_{2P}\Theta \;\; \mathbf{F}_{2P}^{\mathcal{H}}\mathbf{D}_2\mathbf{F}_{2P}\Theta] \\ &= \tilde{\mathbf{u}}_k^{\mathcal{H}}\mathbf{F}_{2P}^{\mathcal{H}}[\mathbf{D}_1\mathbf{F}_{2P}\Theta \;\; \mathbf{D}_2\mathbf{F}_{2P}\Theta] \\ &= \mathbf{0}_{1\times 2N} \quad . \end{aligned} \quad (9)$$

Let $\tilde{\mathbf{v}}_k \triangleq \mathbf{F}_{2P}\tilde{\mathbf{u}}_k$ for $k \in [1, 2P-2N]$, we obtain

$$\begin{cases} \tilde{\mathbf{v}}_k^{\mathcal{H}}\mathbf{D}_1\mathbf{F}_{2P}\Theta = \tilde{\mathbf{h}}_1^T\mathbf{V}^T diag(\tilde{\mathbf{v}}_k^*)\mathbf{F}_{2P}\Theta = \mathbf{0}_{1\times N} \\ \tilde{\mathbf{v}}_k^{\mathcal{H}}\mathbf{D}_2\mathbf{F}_{2P}\Theta = \tilde{\mathbf{h}}_2^T\mathbf{V}^T diag(\tilde{\mathbf{v}}_k^*)\mathbf{F}_{2P}\Theta = \mathbf{0}_{1\times N} \end{cases} \quad (10)$$

Let $\mathbf{Q}_k \triangleq \mathbf{V}^T diag(\tilde{\mathbf{v}}_k^*)\mathbf{F}_{2P}\Theta$, $\forall k \in [1, 2P-2N]$, it yields

$$\tilde{\mathbf{h}}_\mu^T \underbrace{[\mathbf{Q}_1, \cdots, \mathbf{Q}_{2P-2N}]}_{\mathbf{Q}} = \mathbf{0}, \; \mu = 1, 2. \quad (11)$$

Without loss of generality, we assume that the initial delay is zero. Under (**A4**), the non-zero initial delay assumption is equivalent to $h_{ji}(0) \neq 0, \forall i, j \in [1, 2]$ and implies that $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{h}}_2$ are linearly independent, therefore $\{\tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_2\}$ forms a basis that spans null space $\mathcal{N}(\mathbf{Q})$. Determine two eigenvectors $\{\mathbf{e}_1, \mathbf{e}_2\}$ corresponding to two $\mathbf{0}$ eigenvalues of $\mathbf{Q}$. Since both $\mathbf{e}_1$ and $\mathbf{e}_2$ exist in a two-dimensional space whose basis is $\{\tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_2\}$, it yields the following two equations:

$$\begin{cases} \alpha_1\tilde{\mathbf{h}}_1 + \alpha_2\tilde{\mathbf{h}}_2 = \mathbf{e}_1 \\ \alpha_3\tilde{\mathbf{h}}_1 + \alpha_4\tilde{\mathbf{h}}_2 = \mathbf{e}_2 \end{cases} \quad (12)$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are unknown nonzero constants.

Under assumptions (**A1**) $\sim$ (**A4**), $\tilde{\mathbf{h}}_2$ can be blindly estimated up to a complex scalar:

$$\hat{\mathbf{h}}_2 \quad = \quad \mathbf{e}_1 - \frac{e_1(0)}{e_2(0)}\mathbf{e}_2. \quad (13)$$

Then two individual sub-channels can be extracted from $\hat{\mathbf{h}}_2$.

$$\begin{aligned} \hat{\mathbf{h}}_{12} &= \{\hat{h}_2(1), \cdots, \hat{h}_2(L+1)\}, & (14) \\ \hat{\mathbf{h}}_{22} &= \{\hat{h}_2(P+1), \cdots, \hat{h}_2(P+L+1)\}. & (15) \end{aligned}$$

In the presence of white noise with variance σ_w^2, the SVD of $\mathbf{R}_z$ has the following form:

$$\mathbf{R}_z = \begin{bmatrix} \mathbf{U}_s & \tilde{\mathbf{U}}_w \end{bmatrix} \begin{bmatrix} \sum_s & 0 \\ 0 & \sum_w \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^{\mathcal{H}} \\ \tilde{\mathbf{U}}_w^{\mathcal{H}} \end{bmatrix}, \quad (16)$$

where $\sum\limits_s = diag(\sigma_1^2, \cdots, \sigma_{2N}^2)$, $\sum\limits_w = diag(\sigma_w^2, \cdots, \sigma_w^2)$ with $\sigma_1^2 \geq \cdots \geq \sigma_{2N} > \sigma_w^2$. The only difference is that we need to use $\tilde{\mathbf{U}}_w$ to generate $\mathbf{Q}$ in (11) instead of using $\tilde{\mathbf{U}}$.

To estimate all the sub-channels related to the 1st antenna, similar procedures can be applied to $\{\mathbf{z}(l)\}_{l=K+1}^{2K}$, which can be written as:

$$\mathbf{z}(l) = \begin{bmatrix} \mathbf{0}_{1\times N} & \mathbf{H}_{21} \\ \mathbf{H}_{11} & \mathbf{0}_{1\times N} \\ \mathbf{0}_{1\times N} & \mathbf{H}_{22} \\ \mathbf{H}_{12} & \mathbf{0}_{1\times N} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1(l) \\ \mathbf{s}_2(l) \end{bmatrix} + \begin{bmatrix} \mathbf{w}_1(l) \\ \mathbf{w}_2(l) \end{bmatrix}. \quad (17)$$

3.2. With Prior Channel Order Information

Once (12) is obtained, it is possible to estimate $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{h}}_2$ jointly if $h_{12}(L) \neq 0$ or $h_{22}(L) \neq 0$. Specifically,

$$\hat{\mathbf{h}}_2 = \mathbf{e}_1 - \frac{e_1(0)}{e_2(0)}\mathbf{e}_2. \tag{18}$$

$$\begin{aligned} \hat{\mathbf{h}}_1 &= \mathbf{e}_1 - \frac{e_1(L+1)}{e_2(L+1)}\mathbf{e}_2, \qquad if\ h_{12}(L) \neq 0, \\ \hat{\mathbf{h}}_1 &= \mathbf{e}_1 - \frac{e_1(2P-N)}{e_2(2P-N)}\mathbf{e}_2, \qquad if\ h_{22}(L) \neq 0. \end{aligned} \tag{19}$$

The major advantage of joint channel estimation is that now we only need K blocks to achieve the same estimation accuracy as described in Section 3.1, which means the complexity is significantly reduced.

The assumption that $h_{12}(L) \neq 0$ or $h_{22}(L) \neq 0$ can be guaranteed if the transmission sequential is designed according to the following criterion: assuming that the ith transmit antenna has the knowledge of the maximal channel order among all the sub-channels related to the ith transmit antenna, denoted by L_i, the branch that transmits data blocks firstly has the smaller L_i, and the other branch needs to delay a symbol period before transmission.

4. EXTENSION TO MIMO SYSTEMS WITH $N_T > 2$

Assume that N_t transmit antennas and N_r receive antennas are employed in a MIMO system. The transmission scheme is shown in Fig. 2.

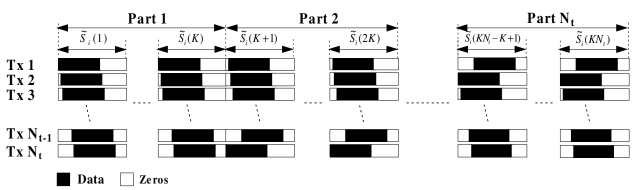


Fig. 2. The proposed transmit delay scheme for general MIMO systems

For the *Part 1*, $\forall i \in [1, N_t]$, $\forall l \in [1, K]$,

$$\tilde{\mathbf{s}}_i(l) \triangleq [\underbrace{0, \cdots, 0}_{i-1}, s_i(lN), \cdots, s_i(lN+N-1), \underbrace{0, \cdots, 0}_{L+N_t-i}]^T,$$

where $\{s_i(lN), s_i(lN+1), \cdots, s_i(lN+N-1)\}$ denotes the lth data block emitted from the ith transmit antenna. For the following parts, each branch delays one more symbol period relative to its previous part. For example, the 1st branch delays one symbol period at the *Part 2*, two symbol periods at the *Part 3*, and so on. If the delay exceeds $(N_t - 1)$, take the value modulo N_t.

If the channel order information $\{L_i\}_{i=1}^{N_t}$ is known *a priori*, sort $\{L_i\}_{i=1}^{N_t}$ in an increasing order to obtain indexes $\{\kappa(i)\}_{i=1}^{N_t}$ such that $\{L_{\kappa(1)} \leq L_{\kappa(2)} \leq \cdots \leq L_{\kappa(N_t)}\}$. The transmission scheme can be simplified to a particular part of the Fig. 2, which satisfies that the $\kappa(i)$th's antenna delays $(i-1)$ symbol periods at the beginning.

Once receiving these blocks, the same procedures as described in Section 3 can be independently applied for each part to estimate one composite channel. Due to space limitation, the details are omitted here.

5. SIMULATIONS

Channel estimation is measured in terms of the averaged normalized mean square error (NMSE) defined as

$$NMSE \triangleq \frac{1}{N}\sum_{\mu=1}^{N} \|\hat{\mathbf{h}}_\mu - \mathbf{h}_\mu\|^2 / \|\mathbf{h}_\mu\|^2, \tag{20}$$

where $\hat{\mathbf{h}}_\mu$ and $\mathbf{h}_\mu$ denote the μth estimated channel and the μth true channel, N is the total number of channels in a MIMO system. The phase ambiguities are assumed to be known here. The channel impulse response between each transmitter-receiver pair is generated randomly and independently. The channel is assumed to be static within each frame, which is composed of K data blocks. For each block, BPSK signals are transmitted and a zero-forcing equalizer is applied for signal detection. More specifically,

$$\hat{\mathbf{x}}(l) = \hat{\mathbf{H}}^\dagger \mathbf{z}(l), \tag{21}$$

where $\hat{\mathbf{H}}$ can be formed by substituting $\hat{\mathbf{h}}_{ji}$ for $\mathbf{h}_{ji}$ in (2), $\dagger$ denotes pseudo-inverse. SNR is defined as the ratio between the average transmission power per bit and noise power E_b/N_0. We set the system parameters as $L = 5$, $N = 23$, $P = N + L + 1 = 29$.

5.1. Robustness to Common Zeros among Sub-channels

The following four sub-channels in a 2×2 system share a common zero at $(1, 0)$, as shown in Fig. 3. From Table 1, the proposed subspace-based blind channel estimation algorithm achieves high estimation accuracy at SNR $\geq$ 10dB, which demonstrates the robustness of our subspace-based method.

$$\begin{aligned}
\mathbf{h}_{11} = [\quad & 0.54 \quad -0.13+0.53i \quad -0.47-0.03i \\ & 0.11-0.18i \quad -0.18+0.00i \quad 0.13-0.32i \quad]^T \\
\mathbf{h}_{12} = [\quad & 0.36 \quad -0.43-0.54i \quad -0.19+0.08i \\ & 0.15+0.26i \quad 0.40+0.07i \quad -0.30+0.13i \quad]^T \\
\mathbf{h}_{21} = [\quad & 0.65 \quad -0.07-0.35i \quad 0.05-0.15i \\ & -0.16-0.09i \quad -0.10+0.12i \quad -0.39+0.46i \quad]^T \\
\mathbf{h}_{22} = [\quad & 0.29 \quad -0.19+0.23i \quad -0.69+0.08i \\ & -0.07-0.31i \quad 0.21-0.06i \quad 0.45+0.06i \quad]^T
\end{aligned}$$

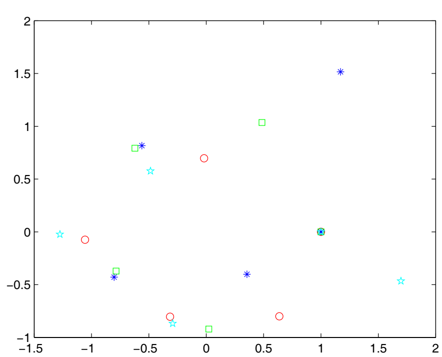


Fig. 3. Plot of zero locations of four sub-channels in the 2×2 system.

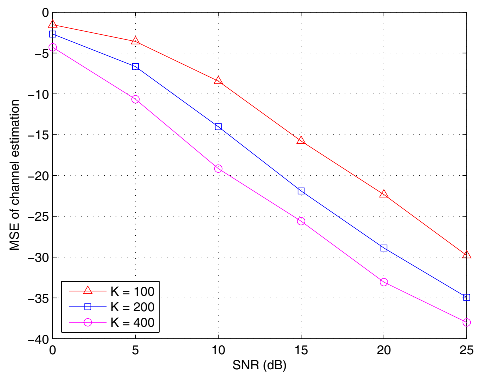
5.2. Effectiveness for MIMO Systems

Systems with different block sizes, $K = 100,\ 200,\ 400$, are tested. All the simulation results are averaged over 500 Monte Carlo runs.

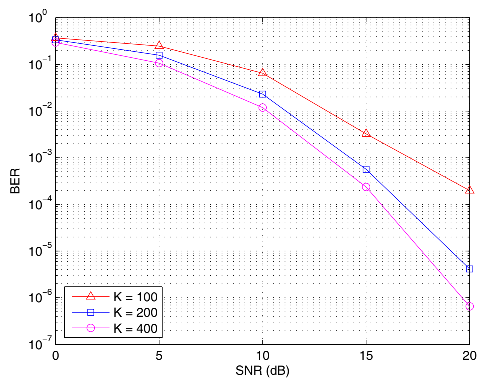
Table 1. Averaged NMSE of channel estimation for all the sub-channels which share a common zero, K = 100.

SNR (dB)	0	5	10	15	20	25
NMSE (dB)	-5.0	-5.8	-15.6	-21.1	-31.1	-32.4

In Fig. 4 and Fig. 5, the NMSE of the channel estimation and BER performance are both improved as K increases. The reason is that the time averaged R_z approaches the theoretical covariance matrix when K goes up so that the estimation accuracy is further improved.



(a) Normalized channel MSE



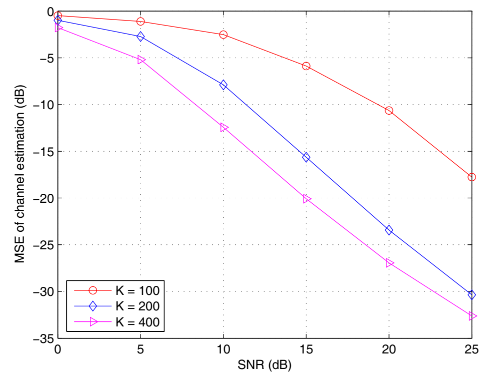
(b) Bit error rate

Fig. 4. Performance of blind channel estimation and blind signal detection for a 2×2 system.

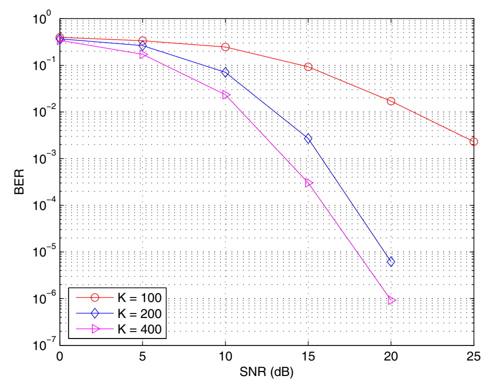
In the simulation, the overall data rate is $\frac{N}{P} = 0.7931$, while the data rate of the corresponding Alamouti scheme in [5] with the same block size is $\frac{N}{2(N+2L)} = 0.3276$.

6. CONCLUSION

In this paper, a structured transmit delay scheme was proposed. Based on the transmitter design, a subspace-based blind channel estimation algorithm was developed for MIMO systems. With the proposed approach, blind MIMO channel estimation can be achieved with no pre-equalization and with no limitations on channel zero locations.



(a) Normalized channel MSE



(b) Bit error rate

Fig. 5. Performance of blind channel estimation and blind signal detection for a 3×3 system.

7. REFERENCES

[1] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. 25th Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Nov 1991, vol. 2, pp. 856–860.

[2] K. Abed-Meraim, P. Loubation, , and E. Moulines, "A subspace algorithm for certain blind identification problems," *IEEE Trans. Inform. Theory*, vol. 43, pp. 499–512, Mar. 1997.

[3] Z. Ding, "Matrix outer-product decomposition method for blind multiple channel identification," *IEEE Trans. Signal Processing*, vol. 45, pp. 3053–3061, Dec. 1997.

[4] J. Shen and Z. Ding, "Direct blind MMSE channel equalization based on second order statistics," *IEEE Trans. Signal Processing*, vol. 48, pp. 1015–0122, Apr. 2000.

[5] S. Zhou, B. Muquet, and G.B. Giannakis, "Subspace-based (semi-) blind channel estimation for block precoded space-time OFDM," *IEEE Trans. Signal Processing*, vol. 50, pp. 1215–1228, May 2002.

[6] N. Ammar and Z. Ding, "Frequency selective channel estimation in time-reversed space-time coding," in *Proc. IEEE Wireless Communications and Networking Conference*, Atlanta, CA, Mar 2004, vol. 3, pp. 1838–1843.

[7] Jinho Choi, "Equalization and semi-blind channel estimation for space-time block coded signals over a frequency-selective fading channel," *IEEE Trans. Signal Processing*, vol. 52, pp. 774–785, Mar. 2004.

[8] D. Gore, S. Sandhu, and A. Paulraj, "Delay diversity codes for frequency selective channels," in *Proc. IEEE Intl. Conf. Commun.*, New York, 28 Apr-2 May 2002, vol. 3, pp. 1949–1953.