Accurate BER of Transmitter Antenna Selection/Receiver-MRC over Arbitrarily Correlated Nakagami Fading Channels

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ABSTRACT

Recently, a combined transmitter-selection combining/ receiver–maximal–ratio -combining (TAS/MRC) scheme has been proposed to reduce the complexity of system and retain the diversity advantage. An accurate bit error rate expression is derived for arbitrarily correlated Nakagami fading channels. The Gauss-Laguerre quadrature based numerical method is employed to evaluate the derived expression. For the channels with integer fading parameter, an exact BER expression is obtained. The numerical results illustrate that the derived approximate formula of BER is in excellent agreement with the Monte Carlo results.

1. INTRODUCTION

Although Multiple-input multiple-output (MIMO) signaling has been widely recognized to be capable of improving wireless communications by diversity combining or space-time coding [1][2][3], a price paid in hardware complexity scales with the number of antennas. Antenna selection in the receiver or transmitter can alleviate the cost for multiple antennas and at the same time retain many advantages of MIMO systems.

Recently, a combined transmitter-selection combiner/receiver-maximal-ratio-combining (TAS/MRC) scheme was proposed [4][5]. It selects the transmit antenna that maximizes the total received signal power at the receiver for transmission. The other transmit antennas are kept inactive. In addition to the reduction in the hardware complexity, the scheme alleviates the requirement of the number of radio frequency chains in MIMO systems.

It is very interesting to analyze the performance of TAS/MRC scheme. In [4] and [5], the authors presented the outage probability and BER expression of the scheme in an independent Rayleigh fading channels. Ref. [6] derived the BER expression for the correlated Rayleigh fading channels.

In addition to Rayleigh model, Nakagami distribution is usually adopted to model the fading channel in wireless communications. The acceptance of Nakagami model lies in its physical justification and its capability to account for both severe and weak fading.

In this paper, we will investigate the performance of the TAS/MRC scheme over arbitrary correlated Nakagami fading channels. An accurate bit-error rate expression is derived by using characteristics function method and Gauss-Leguerre quadrature rule [11]. The obtained BER expression is exact when the channels gains are independent and the fading parameter is integer. The numerical results illustrate that the BER expression is in an excellent agreement with the Monte Carlo results.

2. SYSTEM AND CHANNEL MODELS

Consider a multiple-input and multiple-output (MIMO) wireless communication system with L_t transmitter antennas and L_r receiver antennas. The channel between *i*th transmitter antenna and *j*th receiver antenna, denoted as $h_{ji} = \alpha_{ji} e^{j\phi_{ji}}$, is assumed to be quasi-static fading, and its envelope α_{ij} follows Nakagami-m distribution

$$p_{\alpha_{ji}}(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}}\right)^m x^{2m-1} \exp\left(-\frac{mx^2}{\bar{\gamma}}\right) \tag{1}$$

where $m = \left(\bar{\gamma}^2 \big/ E\left[\left(\alpha_{ji}^2 - \bar{\gamma}^2\right)\right]\right), \bar{\gamma} = E\left(\alpha_{ji}^2\right)$, $\Gamma(n) = \int_0^\infty u^{n-1} e^{-u} du$. The above assumption implies that $\gamma_{ji} = \alpha_{ji}^2$ follows the Gamma distribution

$$p_{\gamma_{ji}}(x) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left(-\frac{mx}{\bar{\gamma}}\right) \tag{2}$$

When the distance between the receiver antennas is not large enough, the correlation will be not negligible [7][8]. In this paper, we assume that the correlation exists between the channel gain α_{ij} and α_{ik}. The covariance matrix of $\left[\gamma_{i1},\ \gamma_{i2},\ \cdots,\ \gamma_{iL_r}\right]^T$ is denoted as $\mathbf{R} = \left(R_{jk}\right)_{L_r \times L_r}$.

In this paper, we assume that the channel gain is known or exactly estimated at the receiver. No delay is considered for the feedback of the channel gain from the receiver to the transmitter. At any time, only one out of transmitter antenna that maximizes the total received signal power is selected and activated for transmission.

3. PDF OF THE INSTANTANEOUS SNR OF THE SCHEME

Under the assumption given in the previous section, the $\left[\gamma_{i1}, \ \gamma_{i2}, \ \cdots, \ \gamma_{iL_r}\right]^T$ are correlated Gamma variables with covariance matrix **R**. As shown in [8], this vector can be represented as the weighted sum of a set of independent Gamma random variables, specifically,

$$\begin{bmatrix} \gamma_{i1} \\ \vdots \\ \gamma_{iL_r} \end{bmatrix} = \begin{bmatrix} U_{11} & 0 & \cdots & 0 \\ U_{21} & U_{22} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ U_{L_r 1} & U_{L_r 2} & \cdots & U_{L_r L_r} \end{bmatrix} \begin{bmatrix} w_{i1} \\ \vdots \\ w_{iL_r} \end{bmatrix} = \mathbf{U}\mathbf{w}_i \quad (3)$$

where $\left[w_{i1}, \ \cdots, \ w_{iL_r}\right]^T$ are independent Gamma RVs with identity covariance matrix, and the low triangular matrix **U** is obtained from the Cholesky decomposition of **R,** i.e., $\mathbf{R} = \mathbf{U}\mathbf{U}^T$. This representation transforms the correlated random variables into the weighted sums of independent variables and retains the branch correlations. From (3), we have

$$\gamma_i = \sum_{j=1}^{L_r} \gamma_{ij} = \sum_{j=1}^{L_r} \left(w_j \sum_{k=1}^{j} U_{jk} \right) \quad (4)$$

Considering $\{w_j\}$ are independent, we can approximate γ_i as a Gamma distributed RV, namely, $\hat{\gamma}_i$.

The m-parameter and the power of $\hat{\gamma}_i$ are given by

$$\tilde{\gamma} = L_r \overline{\gamma} \quad (5)$$

and

$$\tilde{m} = \tilde{\gamma}^2 \left(\sum_{j=1}^{L_r} m_{wj}^{-1} \left(\overline{\gamma}_{wj} \sum_{k=1}^{j} U_{jk} \right)^2 \right)^{-1} \quad (6)$$

where $m_{wj}, \overline{\gamma}_{wj}$ are calculated using the following equations:

$$\begin{aligned} m_{w1} &= m, \\ m_{wj} &= U_{jj}^{-2} \left(\sum_{k=1}^{j-1} U_{jk} \sqrt{m_{wk}} - \overline{\gamma} \right)^2 \\ \gamma_{wj} &= \sqrt{m_{wj}} \end{aligned} \quad (7)$$

With the derived PDF of $\hat{\gamma}_i$, it is straightforward to obtain an approximated PDF of $\gamma_{\max} = \max\left\{\gamma_1, \cdots, \gamma_{L_t}\right\}$, namely,

$$p_{\alpha_{\max}}(x) = L_t \left(\hat{F}(x) \right)^{L_t - 1} \hat{f}(x) \quad (8)$$

where

$$\hat{f}(x) = \left(\frac{\tilde{m}}{\tilde{\gamma}} \right)^m \frac{x^{\tilde{m}-1}}{\Gamma(m)} \exp\left(-\frac{\tilde{m}x}{\tilde{\gamma}} \right) \quad (9)$$

and

$$\begin{aligned} \hat{F}(x) &= \int_0^x \hat{f}(u) du \\ &= \frac{1}{\Gamma(m)} \int_0^{\frac{\tilde{m}}{\tilde{\gamma}}x} u^{\tilde{m}-1} e^{-u} du \end{aligned} \quad (10)$$

is the incomplete Gamma function, and denoted as $\gamma\left(\tilde{m}, \frac{\tilde{m}}{\tilde{\gamma}}x\right)$. Therefore, the PDF of instantaneous SNR of the TAS/MRC scheme would be

$$p_\eta(x) = \frac{1}{\xi} p_{\alpha_{\max}}\left(\frac{\eta}{\xi}\right) \quad (11)$$

where $\eta = \xi\gamma_{\max}$, $\xi = E_b / N_0$, and E_b is the average energy per bit at the transmitter, N_0 is the one-side power spectral density.

4. ACCURATE BER OF THE SCHEME OVER THE FADING CHANNEL WITH NON-INTEGER FADING PARAMETER

For uncoded BPSK modulated communication systems, the conditional bit-error rate is given by $Q\left(\sqrt{2\eta}\right)$. Thus, the average BER of the TAS/MRC scheme can be obtained as

$$BER = E_\eta\left(Q\left(\sqrt{2\eta}\right)\right) = \frac{1}{\pi} \int_0^{\pi/2} \Psi_\eta\left(\frac{1}{\sin^2\theta}\right) d\theta \quad (12)$$

where $\Psi_\eta(\bullet)$ denotes the moment generating function (MGF) of η .

Using (11), we can directly obtain the MGF $\Psi_\eta(\bullet)$ as

$$\Psi_\eta(s) = \frac{L_t}{\Gamma(\tilde{m})} \int_0^\infty e^{-u\left(1 - \frac{\tilde{\gamma}\xi}{\tilde{m}}s\right)} u^{\tilde{m}-1} \left(\gamma\left(m, u\right)\right)^{L_t - 1} du \quad (13)$$

This integral has a closed form solution in which Laurecella's hypergeometric function [10].

Substituting the closed form solution of MGF into (12), we obtain the BER expression as follows

$$\begin{aligned} BER = \frac{L_t \Gamma\left(\tilde{m}L_t\right)}{\tilde{m}^{L_t - 1}\Gamma(\tilde{m})} \frac{1}{\pi} \int_0^{\pi/2} & \frac{1}{\left(L_t - \frac{\tilde{\gamma}\xi}{\tilde{m}} \frac{1}{\sin^2\theta}\right)^{\tilde{m}L_t}} \\ & \times F_A\left(\tilde{m}L, \mathbf{1}_{L_t - 1}, \left(1 + \tilde{m}\right)\mathbf{1}_{L_t - 1}, \left(\frac{1}{L_t - \frac{\tilde{\gamma}\xi}{\tilde{m}} \frac{1}{\sin^2\theta}} \right) \mathbf{1}_{L_t - 1} \right) d\theta \end{aligned} \quad (14)$$

The expression in (14) is very general and applicable to any correlated Nakagami fading channels with integer or noninteger fading parameters.

Unfortunately, there is no closed-form, at least up to the knowledge of the authors, for this integral. The Gauss-Leguerre quadrature (GLQ) rule is employed to evaluate it. The obtained accurate numerical approximation of the BER is given by

$$BER \approx \frac{1}{2P}\sum_{k=1}^{P}\Psi_{\eta}\left(-\frac{1}{1-\cos\left(\frac{2k-1}{2P}\pi\right)}\right)$$

$$\approx \frac{1}{2P}\frac{L_t}{\Gamma(\tilde{m})}\sum_{k=1}^{P}\left(\frac{\tilde{m}\left(1-\cos\left(\frac{2k-1}{2P}\pi\right)\right)}{\tilde{m}\left(1-\cos\left(\frac{2k-1}{2P}\pi\right)\right)+\tilde{\gamma}\xi}\right)^{\tilde{m}} \quad (15)$$

$$\times\sum_{j=1}^{J} w(j)\left[\gamma\left(\tilde{m},\frac{\tilde{m}\left(1-\cos\left(\frac{2k-1}{2P}\pi\right)\right)}{\tilde{m}\left(1-\cos\left(\frac{2k-1}{2P}\pi\right)\right)+\tilde{\gamma}\xi}x(j)\right)\right]^{L_t-1}$$

where P is the number of points in the numerical integration, and J is the number of points in the evaluation of $\Psi_{\eta}(\bullet)$.

5. EXACT BER OF THE SCHEME OVER THE FADING CHANNEL WITH INTEGER FADING PARAMETER

In this section, we consider the case where the channel gains are independent and the fading parameters are integers. We assume that all branches have the same fading parameter and power.

Under the above assumption, the CDF function $\hat{F}(x)$ can be written in a closed form

$$\hat{F}(x) = 1 - e^{-\frac{\tilde{m}}{\tilde{\gamma}}x}\sum_{k=0}^{\tilde{m}-1}\frac{1}{k!}\left(\frac{\tilde{m}}{\tilde{\gamma}}x\right)^{k} \quad (16)$$

where $\tilde{m} = L_r m$ is an integer, $\tilde{\gamma} = L_r\bar{\gamma}$. And the PDF of $\gamma_{\max} = \max\{\gamma_1, \cdots, \gamma_{L_t}\}$ is given by

$$p_{\alpha_{\max}}(x) = \frac{L_t}{\Gamma(\tilde{m})}\left(1 - e^{-\frac{\tilde{m}}{\tilde{\gamma}}x}\sum_{k=0}^{\tilde{m}-1}\frac{1}{k!}\left(\frac{\tilde{m}}{\tilde{\gamma}}x\right)\right)^{L_t-1}\left(\frac{\tilde{m}}{\tilde{\gamma}}\right)^{\tilde{m}} x^{\tilde{m}-1} e^{-\frac{\tilde{m}}{\tilde{\gamma}}x}$$

Thus, for an uncoded BPSK modulated TAS/MRC system, the average bit-error rate can be expressed as

$$BER = \int_0^{\infty} Q\left(\sqrt{2\frac{E_b}{N_0}u}\right)p_{\gamma_{\max}}(u)\,du$$

$$= \frac{L_t}{\Gamma(\tilde{m})}\int_0^{\infty} Q\left(\sqrt{2\frac{E_b}{N_0}\frac{\tilde{\gamma}}{\tilde{m}}u}\right)\left(1 - e^{-u}\sum_{k=0}^{\tilde{m}-1}\frac{1}{k!}u^{k}\right)^{L_t-1} u^{\tilde{m}-1} e^{-u}\,du$$

It has a closed form as given by

$$BER = \frac{L_t}{\Gamma(\tilde{m})}\sum_{k=0}^{L_t-1}\left\{\frac{(-1)^k\binom{L_t-1}{k}}{\left[2(k+1)\right]^{\tilde{m}}}\sum_{t=0}^{k(\tilde{m}-1)}\left[a_t(\tilde{m},k)\right.\right.$$

$$\times(\tilde{m}+t-1)!\left(1-\sqrt{\frac{\nu}{\nu+k+1}}\right)^{\tilde{m}+t}$$

$$\left.\left.\times\sum_{j=0}^{\tilde{m}+t-1}2^{-j}\binom{\tilde{m}+t-1+j}{j}\left(1+\sqrt{\frac{\nu}{\nu+k+1}}\right)^{j}\right]\right\} \quad (17)$$

where $\nu = \frac{E_b}{N_0}\frac{\tilde{\gamma}}{\tilde{m}}$ and $a_t(\tilde{m},k)$ is the coefficient of x^{2t} in the expansion of $\left(\sum_{i=0}^{\tilde{m}-1}\frac{1}{i!}\left(\frac{x^2}{2(k+1)}\right)^{i}\right)^{k}$.

When m=1, (17) reduces to (25) in [5]. Thus, our results take the result for Reyleigh fading channel as its special case.

6. NUMERICAL SIMULATIONS

In this section, we will present numerical results for the MIMO system with 3 receiver antennas over correlated Nakagami flat fading channels.

The covariance matrix is given by

$$R = \begin{bmatrix} 1 & 0.4 & 0.3 \\ 0.4 & 1 & 0.7 \\ 0.3 & 0.7 & 1 \end{bmatrix}$$

It is in a normalized form. The matrix Γ is determined from **R** and scaled by a factor SNR/m as given by

$$\Gamma = \frac{SNR}{m}\sqrt{R}$$

where $\sqrt{\mathbf{R}}$ denotes the matrix with its entries equal to the square root of the corresponding entries of **R**.

We first investigate the accuracy of the approximate formula (15). Fig. 1 shows the BER calculated by using (15), and by Monte Carlo experiments. It can be seen that the formula is always very accurate in the considered SNR region.

In Fig.2, we compare the performance of TAS/MRC with that of MRC. For MRC scheme, there is only one transmitter antenna, and the received signals at three branches are combined according to the MRC principle. The slopes of the curves illustrate that TAS/MRC has a larger diversity gain than MRC scheme in all three different fading parameter m.

7. CONCLUSION

In this paper, we investigate the performance of the transmitter antenna selection/receiver-MRC scheme over spatially correlated Nakagami fading channels. An accurate bit-error rate expression is derived by using characteristics function based approach and Gauss-Leguerre quadrature rule. The expression is exact when the channel gains are independent and the fading parameter is integer.

The obtained approximate formula is very general and applicable to any correlated Nakagami fading channels. The numerical results illustrate that it is in an excellent agreement with the Monte Carlo experiments.

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8. REFERENCES

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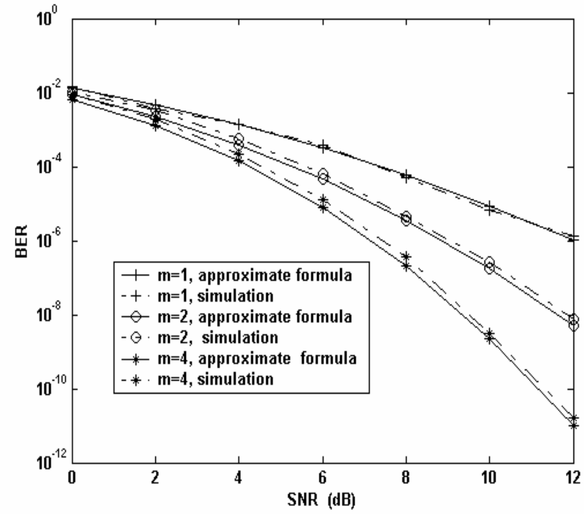

Fig.1 Comparison of the BER of TAS/MRC

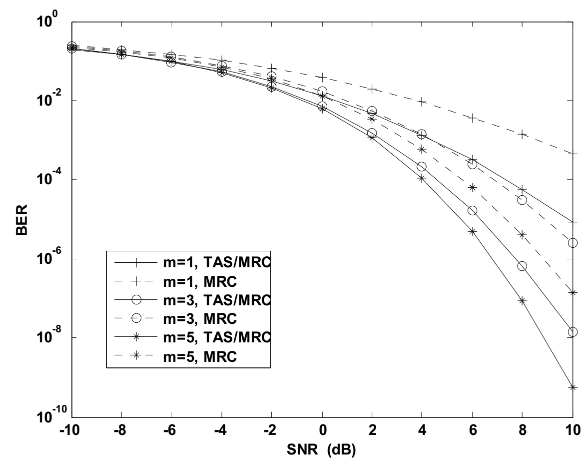

Fig.2 Comparison between TAS/MRC and MRC with single transmitter antenna.