PERFORMANCE ANALYSIS OF A DSTTD SYSTEM WITH DECISION-FEEDBACK DETECTION

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ABSTRACT

We investigate closed-form bit error rate (BER) expressions for the double space-time transmit diversity (DSTTD) system with a zero-forcing decision-feedback (ZF-DF) detector. We show that the lower Alamouti's STTD unit can obtain the second order diversity gain. However, the upper one cannot guarantee the fourth order diversity due to the effect of error propagation. For example, it simply gives 3.5 dB signal-to-noise ratio (SNR) advantage over the lower one at the bit error rate (BER) of 10^{-3}. Under the same environment, overall BER performance also suffers from 5.6 dB performance degradation over a maximum likelihood (ML) detector.

1. INTRODUCTION

Recently, it has been shown that multiple transmit/receive antenna techniques can achieve enormous spatial multiplexing gain and diversity gain over rich scattering environment [1], [2]. To exploit this advantage, Bell labs layered space-time (BLAST) architectures which maximize spatial multiplexing gain were introduced in [3], [4]. In order to increase transmit diversity gain, space-time block codes (STBC) were addressed [5], [6].

Next, a hybrid architecture, called double space-time transmit diversity (DSTTD) system, was proposed as a simple way to obtain transmit diversity gain and spatial multiplexing gain simultaneously [7]. It consists of two Alamouti's space-time transmit diversity (STTD) units. Alamouti's STTD itself gives transmit diversity gain and its dual structure improves spatial multiplexing gain. In addition, it requires four transmit antennas and at least two receive antennas, so that it is favorable in practical applications.

To detect transmitted signals in the DSTTD system, a zero-forcing decision-feedback (ZF-DF) detector is often considered at the receiver in that it can accomplish moderate performance with high computational efficiency. For theoretical analysis of the DSTTD system with the ZF-DF detector, we derive closed-form performance expressions in this paper.

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2. TRANSMISSION SYSTEM MODEL

We consider a DSTTD system with four transmit and two receive antennas as shown in Fig. 1. Channel responses are frequency-flat fading and remain constant during a frame transmission. Thus, the stacked received signal vector over two symbol periods can be expressed as:

$$\mathbf{y} = \mathbf{H}_{eff}\mathbf{s} + \mathbf{n} \tag{1}$$

where

$$\begin{aligned} \mathbf{H}_{eff} &= \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 & \mathbf{h}_4 \end{bmatrix} \\ &= \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{12}^* & -h_{11}^* & h_{14}^* & -h_{13}^* \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{22}^* & -h_{21}^* & h_{24}^* & -h_{23}^* \end{bmatrix}. \end{aligned} \tag{2}$$

Also $\mathbf{s} \in \mathbf{C}^{4\times1}$ and $\mathbf{y} \in \mathbf{C}^{4\times1}$ are complex transmitted and received signal vectors, respectively. $\mathbf{n} \in \mathbf{C}^{4\times1}$ is a zero-mean complex additive white Gaussian noise (AWGN) vector with $E[\mathbf{n}\mathbf{n}^H] = \sigma^2\mathbf{I}$. $\mathbf{H}_{eff} \in \mathbf{C}^{4\times4}$ is the complex channel matrix characterizing frequency-flat channel and h_{ij} is channel response from the jth transmit antenna to the ith receive antenna. Each channel coefficient follows independent and identically distributed (i.i.d.) complex Gaussian with zero-mean and unit-variance. Overall transmit power is normalized to have unit-energy.

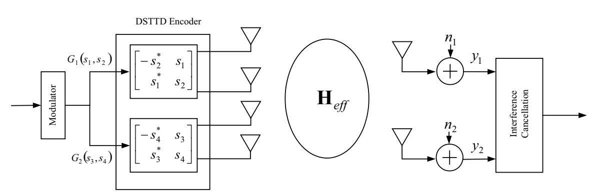

Fig. 1. Block diagram of a DSTTD system.

3. PERFORMANCE ANALYSIS

The channel matrix $\mathbf{H}_{eff}$ is decomposed as $\mathbf{H}_{eff} = \mathbf{QR}$ by using the QR decomposition. $\mathbf{Q} \in \mathbf{C}^{4\times4}$ and $\mathbf{R} \in \mathbf{C}^{4\times4}$ are the unitary matrix and the upper triangular matrix, respectively. By pre-multiplying (1) with $\mathbf{Q}^H$, the modified received vector is:

$$\widetilde{\mathbf{y}} = \mathbf{R}\mathbf{s} + \mathbf{n}' \tag{3}$$

where $\mathbf{n}'$ is $\mathbf{Q}^H\mathbf{n}$. Since $\mathbf{Q}$ is unitary, the statistics of the noise term $\mathbf{Q}^H\mathbf{n}$ remains unchanged. The ith element in $\widetilde{\mathbf{y}}$ depends only on the ith and higher stream symbols as follows:

$$\widetilde{y}_i = R_{ii}s_i + \sum_{j=i+1}^{4} R_{ij}s_j + n'_i. \quad (4)$$

It makes detection order be from lower stream to higher stream with successive interference cancellation. By assuming that all previous decision are correct, the estimated symbol $\hat{s}_i$ is:

$$\hat{s}_i = Q\left[\frac{\widetilde{y}_i - \sum_{j=i+1}^{4} R_{ij}\hat{s}_j}{R_{ii}}\right] \quad (5)$$

where $\hat{s}_i$ is the estimated symbol of s_i and $Q[\cdot]$ is a decision device. It performs until all streams are detected.

We now proceed to show that it is sufficient to compute only two columns of the unitary $\mathbf{Q}$ matrix and two rows of the upper triangular $\mathbf{R}$ matrix in the QR decomposition. The channel matrix $\mathbf{H}_{eff}$ has two unique characteristics. First, it is quasi-orthogonal matrix since $\mathbf{h}_1$ and $\mathbf{h}_3$ are orthogonal to $\mathbf{h}_2$ and $\mathbf{h}_4$, respectively. It introduces zero-valued elements into the $\mathbf{R}$ matrix. Second, $\mathbf{h}_1$ and $\mathbf{h}_3$ are modified forms of $\mathbf{h}_2$ and $\mathbf{h}_4$, respectively, so several elements can be represented as phase rotation of other elements. The QR decomposition is performed with the modified Gram-Schmidt (MGS) method [8]. In view of these properties, the unitary $\mathbf{Q}$ matrix can be shown as follows:

$$\begin{aligned}\mathbf{Q} &= \begin{bmatrix}\mathbf{q}_1 & \mathbf{q}_2 & \mathbf{q}_3 & \mathbf{q}_4\end{bmatrix} \\ &= \begin{bmatrix} q_{11} & q_{21}^* & q_{13} & q_{23}^* \\ q_{21} & -q_{11}^* & q_{23} & -q_{13}^* \\ q_{31} & q_{41}^* & q_{33} & q_{43}^* \\ q_{41} & -q_{31}^* & q_{43} & -q_{33}^* \end{bmatrix}\end{aligned} \quad (6)$$

where

$$\begin{aligned}& q_{i1} = \mathbf{h}_{1,i}/\|\mathbf{h}_1\|_F \\ & q_{i3} = \frac{\mathbf{h}_{3,i} - \sum_{j=1}^{2}\langle\mathbf{h}_3, \mathbf{q}_j\rangle \mathbf{q}_{j,i}}{\|\mathbf{h}_3 - \sum_{j=1}^{2}\langle\mathbf{h}_3, \mathbf{q}_j\rangle\mathbf{q}_j\|_F} \\ & i \in \{1,2,3,4\}.\end{aligned} \quad (7)$$

Also each element in the upper triangular $\mathbf{R}$ matrix is found as:

$$\begin{aligned} R_{11} &= R_{22} = \|\mathbf{h}_1\|_F, R_{12} = \langle\mathbf{h}_2, \mathbf{q}_1\rangle = 0 \\ R_{13} &= \langle\mathbf{h}_3, \mathbf{q}_1\rangle, R_{14} = \langle\mathbf{h}_4, \mathbf{q}_1\rangle \\ R_{23} &= -\langle\mathbf{h}_4, \mathbf{q}_1\rangle^*, R_{24} = \langle\mathbf{h}_3, \mathbf{q}_1\rangle^* \\ R_{33} &= \|\mathbf{h}_3 - \sum_{j=1}^{2}\langle\mathbf{h}_3\mathbf{q}_j\rangle\mathbf{q}_j\|_F, R_{34} = \langle\mathbf{h}_4, \mathbf{q}_3\rangle = 0 \\ R_{44} &= R_{33}. \end{aligned} \quad (8)$$

Hence, the upper triangular $\mathbf{R}$ matrix is simplified as:

$$\mathbf{R} = \begin{bmatrix} R_{11} & 0 & R_{13} & R_{14} \\ 0 & R_{11} & -R_{14}^* & R_{13}^* \\ 0 & 0 & R_{33} & 0 \\ 0 & 0 & 0 & R_{33} \end{bmatrix}. \quad (9)$$

To derive performance, it is necessary to find the statistics of the upper triangular $\mathbf{R}$ matrix. First, $|R_{11}|^2$ has a chi-square distribution with eight degrees of freedom because it is equal to $\|\mathbf{h}_1\|_F^2$. Now, we represent $\mathbf{h}_3$ with the QR decomposition to find the probability density distribution of $|R_{33}|^2$ as:

$$\mathbf{h}_3 = R_{13}\mathbf{q}_1 - R_{14}^*\mathbf{q}_2 + R_{33}\mathbf{q}_3 \quad (10)$$

where $R_{13}\mathbf{q}_1$ and $R_{14}^*\mathbf{q}_2$ is the projection of $\mathbf{h}_3$ onto the subspace spanned by $\mathbf{q}_1$ and $\mathbf{q}_2$. Let $\mathbf{Q}_1 = [\mathbf{q}_1, \mathbf{q}_2]$ contain an orthonormal basis for the interference subspace. Since $\mathbf{g} := [R_{13}, -R_{14}^*]^T = \mathbf{Q}_1^H\mathbf{h}_3$ is a linear transformation of a jointly Gaussian vector, it is jointly Gaussian, and is thus characterized by its covariance matrix:

$$E[\mathbf{g}\mathbf{g}^H] = \mathbf{Q}_1^H E[\mathbf{h}_3\mathbf{h}_3^H]\mathbf{Q}_1 = \mathbf{Q}_1^H\mathbf{I}_4\mathbf{Q}_1 = \mathbf{I}_2. \quad (11)$$

Then, $|R_{13}|^2$ and $|R_{14}|^2$ have a chi-square distribution with two degrees of freedom, respectively. Due to the fact that $\|\mathbf{h}_3\|_F^2$ follows a chi-square distribution with eight degrees of freedom, $|R_{33}|^2$ has a chi-square distribution with four degrees of freedom. Consequently, we summarize properties of the upper triangular $\mathbf{R}$ matrix.

- $|R_{11}|^2$ and $|R_{33}|^2$ follow a chi-square distribution with eight and four degrees of freedom, respectively.
- Off-diagonal elements $|R_{13}|^2$ and $|R_{14}|^2$, have a chi-square distribution of two degrees of freedom, respectively.

With using above results, we can derive closed-form BER expressions. As each stream in the same Alamouti's STTD unit has equal performance, it is enough to obtain performance of the stream 1 and the stream 3 (i.e., $\overline{P}_1(e) = \overline{P}_2(e)$ and $\overline{P}_3(e) = \overline{P}_4(e)$, where $\overline{P}_i(e)$ is the average BER performance of the stream i). We assume that binary phase-shift keying (BPSK) is employed in each stream. First, the conditional bit error probability of the stream 3 is:

$$\begin{aligned} P_3(e|R_{33}) &= Q\left(\sqrt{\frac{2|R_{33}|^2E_b}{4N_0}}\right) \\ &= \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{|R_{33}|^2E_b}{4N_0\sin^2\theta}\right)d\theta \\ &= \frac{1}{\pi}\prod_{j=1}^{2}\int_0^{\pi/2}\exp\left(-\frac{X_jE_b}{4N_0\sin^2\theta}\right)d\theta \end{aligned} \quad (12)$$

where $Q(x) = \frac{1}{\pi}\int_0^{\pi/2}\exp(-\frac{x^2}{2\sin^2\theta})d\theta$ and X_j follows a chi-square distribution with two degrees of freedom.
Averaging (12) with respect to $|R_{33}|^2$ yields:

$$\begin{aligned} \overline{P}_3(e) &= \mathcal{E}\left\{\frac{1}{\pi}\prod_{j=1}^{2}\int_0^{\pi/2}\exp\left(-\frac{X_jE_b}{4N_0\sin^2\theta}\right)d\theta\right\} \\ &= \frac{1}{\pi}\int_0^{\pi/2}\mathcal{E}\left\{\prod_{j=1}^{2}\exp\left(-\frac{X_jE_b}{4N_0\sin^2\theta}\right)\right\}d\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2}\left\{\frac{\sin^2\theta}{\sin^2\theta + E_b/4N_0}\right\}^2 d\theta \\ &= \left(\frac{1-\mu(\gamma)}{2}\right)^2\sum_{k=0}^{1}\binom{k+1}{k}\left(\frac{1+\mu(\gamma)}{2}\right)^k \end{aligned} \quad (13)$$

$$\begin{aligned}\overline{P_1}(e) &= \overline{P_3}(e)\overline{P_4}(e)\overline{P_1}(e|s_3 \neq \hat{s}_3, s_4 \neq \hat{s}_4) + 2\overline{P_3}(e)\overline{P_4}(c)\overline{P_1}(e|s_3 \neq \hat{s}_3, s_4 = \hat{s}_4) + \overline{P_3}(c)\overline{P_4}(c)\overline{P_1}(e|s_3 = \hat{s}_3, s_4 = \hat{s}_4) \\ &= \overline{P_3}(e)\overline{P_4}(e)D(4, \frac{E_b}{8E_b + N_0}) + 2\overline{P_3}(e)\overline{P_4}(c)D(4, \frac{E_b}{4E_b + N_0}) + \overline{P_3}(c)\overline{P_4}(c)D(4, \frac{E_b}{N_0}) \\ &= D^2(2, \frac{E_b}{N_0})D(4, \frac{E_b}{8E_b + N_0}) + 2D(2, \frac{E_b}{N_0})(1 - D(2, \frac{E_b}{N_0}))D(4, \frac{E_b}{4E_b + N_0}) + (1 - D(2, \frac{E_b}{N_0}))^2 D(4, \frac{E_b}{N_0}). \quad (16)\end{aligned}$$

where $\mu(\gamma) = \sqrt{\gamma/(\gamma+1)}$, $\gamma = E_b/4N_0$, and

$$\frac{1}{\pi}\int_0^{\pi/2}\left\{\frac{\sin^2\theta}{\sin^2\theta+\gamma}\right\}^m d\theta = \left(\frac{1-\mu(\gamma)}{2}\right)^m \sum_{k=0}^{m-1}\binom{m-1+k}{k}\left(\frac{1+\mu(\gamma)}{2}\right)^k . \quad (14)$$

For convenience, we introduce $D(m, E_b/N_0)$ as:

$$D(m, \frac{E_b}{N_0}) = \left(\frac{1-\mu(\gamma)}{2}\right)^m \sum_{k=0}^{m-1}\binom{m-1+k}{k}\left(\frac{1+\mu(\gamma)}{2}\right)^k . \quad (15)$$

Since we have already derived $\overline{P_3}(e)$, we can calculate $\overline{P}_1(e)$ as (16). Due to $D^2(2, \frac{E_b}{N_0}) \ll 1$ and $(1 - D(2, \frac{E_b}{N_0})) \approx 1$ in the high signal-to-noise ratio (SNR) region, it is simplified as:

$$\overline{P_1}(e) \approx 2D(2, \frac{E_b}{N_0})D(4, \frac{E_b}{4E_b + N_0}) + D(4, \frac{E_b}{N_0}). \quad (17)$$

Based on (13) and (17), we can calculate overall performance by finding arithmetic mean of the performance of all streams as:

$$\begin{aligned}\overline{P}(e) &= \frac{1}{4}\sum_{j=1}^{4}\overline{P_j}(e) \\ &\approx \frac{1}{2}D(2, \frac{E_b}{N_0}) + \frac{1}{2}D(4, \frac{E_b}{N_0}) \\ &\quad + D(2, \frac{E_b}{N_0})D(4, \frac{E_b}{4E_b + N_0}). \quad (18)\end{aligned}$$

4. SIMULATION RESULTS

In order to validate previous analysis, Monte Carlo simulations have been performed. We assume that channel coefficients are perfectly known at the receiver.

The BER performance of each stream is illustrated in Fig. 2. We can observe that simulation results agree with analytical results. Note that we will just describe performance of the stream 1 and the stream 3 because each stream in the same Alamouti's STTD unit shows identical performance. With interference nulling process by the QR decomposition, the stream 3 provides the second order diversity. In case of the stream 1, however, it cannot attain the fourth order diversity due to imperfect interference cancellation. Accordingly, it merely realizes the second order diversity with SNR gain. For example, the stream 1 accomplishes 3.5 dB performance improvement over the stream 3 at the BER of 10^{-3}.

The performance of both a maximum likelihood (ML) detector and the ZF-DF detector is shown in Fig. 3. It demonstrates that the ML detector produces the fourth order diversity gain, but the ZF-DF detector yields the second order diversity gain. Additionally, the ZF-DF detector cannot fully exploit SNR gain of the stream 1 and the stream 2 because overall performance is dominantly influenced by the worst performance: the stream 3 and the stream 4. To be concrete, the ZF-DF detector undergoes 5.6 dB performance penalty at the BER of 10^{-3}, compared with the ML detector.

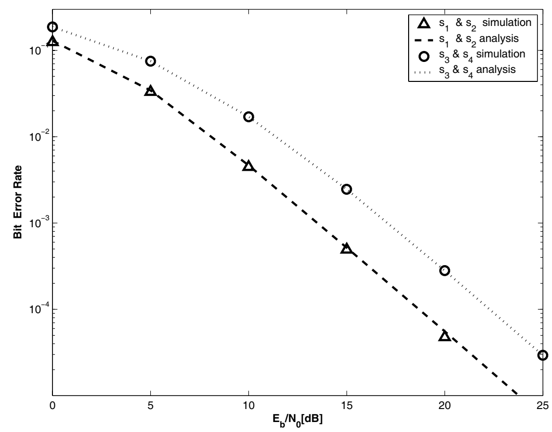

Fig. 2. BER performance of the ZF-DF detector.

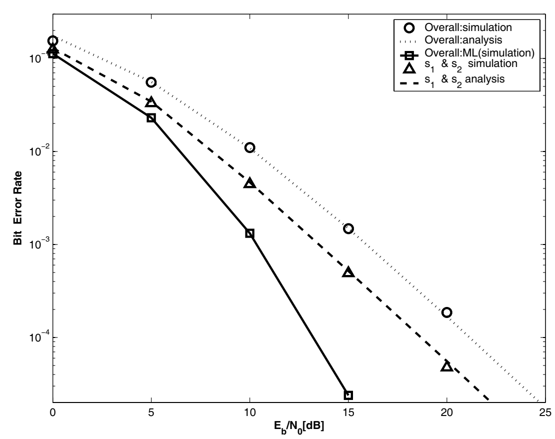

Fig. 3. Overall BER performance of the ZF-DF detector and the ML detector.

5. SUMMARY AND CONCLUSIONS

We have derived closed-form performance expressions for the DSTTD system with the ZF-DF detector. It has turned out that the proposed analysis coincides with simulation results. Both interference nulling process and error propagation contribute to limiting overall performance of the ZF-DF detector by the second order diversity gain. To illustrate this, it is 5.6 dB worse than the ML detector at the BER of 10^{-3}.

6. REFERENCES

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