JOINTLY OPTIMAL PRECODING/POSTCODING FOR COLORED MIMO SYSTEMS

Duong H. Pham[1], *Hoang D. Tuan*[1], *Ba-Ngu Vo*[2], *and Truong Q. Nguyen*[3]

[1]School of Electrical Engineering and Telecommunications, The University of New South Wales, Sydney, NSW 2052, Australia
[2]Department of Electrical and Electronic Engineering, The University of Melbourne, Melbourne, VIC 3052, Australia
[3]Department of Electrical and Computer Engineering, UCSD, San Diego, CA 92093-0407, USA

ABSTRACT

The problem of designing a jointly optimal linear precoder and decoder for a multiple-input multiple-output (MIMO) channel has received much interest recently. However, most existing works only deal with white input signal. When the input signal is colored, prewhitening and its inverse operation are often applied prior to precoding and after decoding, respectively. Consequently, the precoder and decoder are no longer optimal with respect to the original colored signal. In this paper, we propose a closed-form solution for optimal linear precoder and decoder for colored input signal. Our approach is based on minimizing the symbol mean squared error under an average output power constraint, and is applicable to both MIMO flat fading and frequency selective fading channels. Simulations show the advantage of our solution over prewhitening-based method.

1. INTRODUCTION

Transmission with multiple transmit and receive antennas is a promising method for providing high data rate while not consuming extra valuable transmission bandwidth [1]. To achieve the maximum information rate, appropriate coding, precoding and modulation techniques are required depending on the availability of channel state information. When channel state information is not available at the transmitter, appropriately mapping input symbols in space and time can improve diversity gains, and hence transmission rate [2]. On the other hand, when channel state information is available at both the transmitter and receiver, appropriately allocating power and bits over multiple antennas according to the channel state can improve the system information rate [3, 4]. This allocation process involves designing optimal linear precoder at the transmitter and optimal decoder at the receiver based on channel state information.

The problem of designing jointly optimal linear precoder and decoder has been intensively investigated [3, 5, 4, 6, 7]. However, most solutions are proposed for the special case of white input symbols, which are not applicable in many practical cases where the input symbols are encoded for transmission [8]. Although it is possible, as mentioned in [4, 3], to apply a prewhitening operation prior to precoding at the transmitter and use the corresponding inverse operation at the receiver, optimality is no longer guaranteed. This is because the precoder and decoder design is based on the prewhitened signal, rather than on the actual input signal. To the author's best knowledge, the only the work that considered optimal precoder and coder for colored input signal is [6]. However, this approach requires a special assumption on the channel matrix, which can restrict its applicability in practice.

In this paper, we derive a jointly optimal linear precoder and decoder for the more general case of colored inputs and arbitrary channel matrix, subject to average output power constraints. Unlike previous approaches to joint precoder and decoder design, (including those for white inputs), which required intricate matrix calculus and analysis [3, 4], our approach is based on elementary linear matrix inequality (LMI) arguments and appropriate matrix partitioning that yield a direct solution. LMI is a very powerful tool for most numerical solutions of modern control problems (see e.g. [9, 10] and reference therein). By viewing the joint precoder and decoder design problem for MIMO system as a linear closed-loop control problem, LMI based analysis then allows a closed-form solution to be obtained. We consider both MIMO flat fading and frequency selective fading channels. The proposed design is compared with the case of prewhitening scheme via simulations.

The paper is organized as follows. Section 2 describes the communication model. Section 3 presents the joint design of optimal precoder and decoder. The viability of our results is verified via simulations in Section 4. Concluding remarks are given in Section 5.

Notation: The superscript T denotes the transposition, the superscript H the Hermitian, $\mathbf{I}_N$ the $N \times N$ identity matrix, $\mathbf{0}_{N\times M}$ the $N \times M$ zero matrix. $\mathbf{I}$ and $\mathbf{0}$ denote the identity and zero matrices when their sizes are clear from the context. The $\|.\|$ denotes the Frobenius norm, $\mathrm{E}\{.\}$ the expectation, $\langle . \rangle$ the trace of a matrix, $\jmath = \sqrt{-1}$. The notation $\mathbf{A} > 0$ ($\mathbf{A} \geq 0$, resp.) means that $\mathbf{A}$ is a Hermitian positive definite (positive semidefinite, resp.) matrix. Analogously, $\mathbf{A} < 0$ ($\mathbf{A} \leq 0$, resp.) means that the matrix $\mathbf{A}$ is negative definite (negative semidefinite, resp.). For symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} < \mathbf{B}$ ($\mathbf{A} \leq \mathbf{B}$, resp.) are equivalent to $\mathbf{A} - \mathbf{B} < 0$ ($\mathbf{A} - \mathbf{B} \leq 0$, resp.). Note that $\mathbf{A} \geq \mathbf{B} > 0$ ($\mathbf{A} > \mathbf{B} > 0$, resp.) if and only if $0 < \mathbf{A}^{-1} \leq \mathbf{B}^{-1}$ ($0 < \mathbf{A}^{-1} < \mathbf{B}^{-1}$, resp.) and both of these imply $\langle \mathbf{A} - \mathbf{B} \rangle \geq 0$ ($\langle \mathbf{A} - \mathbf{B} \rangle > 0$, resp.). We denote by $\mathcal{M}_n$ the set of all unitary matrices of dimension $n \times n$.

2. BACKGROUND

The following subsections describe the MIMO flat fading and frequency selective fading channel models and the formulation of the joint precoder decoder design problem.

Flat Fading Channel: In a communication system operating over a N_t input and N_r output flat fading channel, the input sequence of (possibly) encoded symbols is broken into blocks of fixed length L. Each block is then precoded before being sent over the channel. The received block $\mathbf{y} \in \mathbb{C}^{N_r}$ corresponding to the input block $\mathbf{s} \in \mathbb{C}^L$ can be modeled as

$$\mathbf{y} = \mathbf{HFs} + \mathbf{n} \tag{1}$$

where $\mathbf{F} \in \mathbb{C}^{N_t \times L}$ is the precoding matrix, $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ the channel matrix, and $\mathbf{n} \in \mathbb{C}^{N_r}$ denotes additive noise.

Frequency Selective Fading Channel: Now, suppose that our communication system is operating over a N_t input and N_r output frequency selective fading channel with maximum channel order

ν. Similar to the flat fading case, the input sequence of (possibly) encoded symbols is broken into blocks of fixed length L. Different from the flat fading case, each block is precoded by multiplication with the precoder matrix $\mathbf{F} \in \mathbb{C}^{P \times L}$, with $L \geq N_t$, namely $\mathbf{x} = \mathbf{F}\mathbf{s}$. To prevent inter-block interference, the precoded block $\mathbf{x}$ is broken into N_t equal size sub-blocks, $\mathbf{x}_i \in \mathbb{C}^{P/N_t},\ i = 1, \ldots, N_t$. Finally, the ith sub-block is appended with ν zeros and transmitted via the ith antenna. Let $\mathbf{y}_j \in \mathbb{C}^{P/N_t+\nu}$ denote the received block at the jth antenna $j = 1, \ldots, N_r$. Then $\mathbf{y} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_{N_r}^T]^T \in \mathbb{C}^{N_r(P/N_t+\nu)}$ is related to the input block $\mathbf{s}$ by

$$\mathbf{y} = \begin{bmatrix} \mathbf{H}_{11}, \ldots, \mathbf{H}_{N_t 1} \\ \vdots \quad\quad\quad \vdots \\ \mathbf{H}_{1N_r}, \ldots, \mathbf{H}_{N_t N_r} \end{bmatrix} \mathbf{F}\mathbf{s} + \mathbf{n} \tag{2}$$

where $\mathbf{H}_{ij} \in \mathbb{C}^{(P/N_t+\nu) \times (P/N_t)},\ i = 1, \ldots, N_t,\ j = 1 \ldots, N_r$ is the channel matrix of the transmission link between the ith transmit antenna and the jth receive antenna, which is a Toeplitz matrix constructed from the channel vector $\mathbf{h}_{ij} \in \mathbb{C}^{\nu+1}$ such that $[\mathbf{h}_{ij}^T, 0 \ldots, 0]^T$ is the first column and $[h_{ij}(1), 0 \ldots, 0]$ the first row.

Since the system model (2) has the same form as (1), the design solutions for (1) will be applicable to (2). In fact (1) covers a wide class of MIMO channel models, including multicarrier channel, multi-antenna wireless channel, wireline DSL channel, CDMA channel [3]. Therefore, in the subsequent sections, we will consider (1) without loss of generality.

2.1. Assumptions

The following standard assumptions are made throughout:
(A1) The signal is zero-mean, correlated with $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{R}_L > 0$.
(A2) The noise is zero-mean, correlated with $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \mathbf{R}_n > 0$.
(A3) The input signal and noise are independent, i.e. $\mathrm{E}\{\mathbf{n}\mathbf{s}^H\} = \mathbf{0}$.
(A4) $L \leq \min\{N_t, N_r\}$, so $N_t - L$ is the transmission redundancy (note that it is not assumed that $L \leq \mathrm{rank}(\mathbf{H})$ as in [3, 5, 4, 6, 7]).
(A5) The average output power P_T is fixed, i.e. $\langle \mathbf{F}\mathbf{R}_L\mathbf{F}^H \rangle \leq P_T$.
(A4) Channel state is known at both the transmitter and receiver.

2.2. Problem formulation

Given the received block $\mathbf{y}$, the problem is to estimate the input block $\mathbf{s}$ that was sent over the channel. More concisely, let $\mathbf{G} \in \mathbb{C}^{L \times N_r}$ denote the decoder. Then the problem is to design $\mathbf{F}$ and $\mathbf{G}$ such that the estimation:

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{y} = \mathbf{G}\mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{G}\mathbf{n} \tag{3}$$

is optimal in some sense. It follows from (1) and (3) that the problem of minimizing the mean squared error $\mathrm{E}\{||\hat{\mathbf{s}} - \mathbf{s}||^2\}$ under the average power constraint (A5) can be posed as

$$\min_{\mathbf{G},\mathbf{F},\langle \mathbf{F}\mathbf{R}_L\mathbf{F}^H \rangle \leq P_T} \langle [\mathbf{G}\mathbf{H}\mathbf{F} - \mathbf{I}_L]\mathbf{R}_L[\mathbf{G}\mathbf{H}\mathbf{F} - \mathbf{I}_L]^H + \mathbf{G}\mathbf{R}_n\mathbf{G}^H \rangle. \tag{4}$$

The works in [7, 5, 3] consider (4) for the special case of white input, i.e. $\mathbf{R}_L = \sigma\mathbf{I}_L$. When the input is colored as considered in this paper, it was suggested in [7, 3] that pre-whitening is applied prior precoding. Let $\mathbf{s}_{pw}$ denote the pre-whitened signal then

$$\mathbf{s}_{pw} = \mathbf{R}_L^{-1/2}\mathbf{s} \ \Rightarrow\ \mathrm{E}\{\mathbf{s}_{pw}\mathbf{s}_{pw}^T\} = \mathbf{I}.$$

Let

$$\begin{aligned} f &= \langle [\mathbf{G}\mathbf{H}\mathbf{F} - \mathbf{I}_L]\mathbf{R}_L[\mathbf{G}\mathbf{H}\mathbf{F} - \mathbf{I}_L]^H + \mathbf{G}\mathbf{R}_n\mathbf{G}^H \rangle \\ &= \langle [\mathbf{G}\mathbf{H}\mathbf{F}_{pw} - \mathbf{I}_L][\mathbf{G}\mathbf{H}\mathbf{F}_{pw} - \mathbf{I}_L]^H + \mathbf{G}\mathbf{R}_n\mathbf{G}^H \rangle \end{aligned}$$

where $\mathbf{F}_{pw} = \mathbf{F}\mathbf{R}_L^{1/2}$. In essence, the problem (4) is replaced by the following problem

$$\min_{\mathbf{G},\mathbf{F}_{pw},\langle \mathbf{F}_{pw}\mathbf{F}_{pw}^H \rangle \leq P_T} f. \tag{5}$$

Let $(\mathbf{F}_{pw}, \mathbf{G}_{pw})$ be the optimal precoder-decoder pair for the whitening optimization problem (5). The suggested prewhitening approach is to take $(\mathbf{F}_{pw}\mathbf{R}_L^{-1/2}, \mathbf{R}_L^{1/2}\mathbf{G}_{pw})$ as the solution for the optimization problem (4). This solution is clearly not optimal.

Regarding rank requirement of the channel matrix, only the special case $L = \mathrm{rank}\{\mathbf{H}\}$ of the optimization problem (4) was considered in [6]. Moreover, for simpler cases of white input, only solutions for $L \leq \mathrm{rank}\{\mathbf{H}\}$ were proposed [3, 5, 7].

3. JOINTLY OPTIMAL PRECODER AND DECODER

We now present a novel approach for solving problem (4) in its full generality. In Subsection 3.1, the non-convex problem (4) is first converted into an equivalent convex optimization problem in the precoder variable. A closed-form solution to the equivalent convex problem is then derived in Subsection 3.2.

3.1. Equivalent convex problem

For notational convenience, we write (4) in a more compact form by using the variable changes $\mathbf{G} \leftarrow \mathbf{G}\mathbf{R}_n^{-1/2}$ and $\mathbf{H} \leftarrow \mathbf{R}_n^{1/2}\mathbf{H}$

$$\min_{\mathbf{G},\mathbf{F},\langle \mathbf{F}\mathbf{R}_L\mathbf{F}^H \rangle \leq P_T} \langle [\mathbf{G}\mathbf{H}\mathbf{F} - \mathbf{I}_L]\mathbf{R}_L[\mathbf{G}\mathbf{H}\mathbf{F} - \mathbf{I}_L]^H + \mathbf{G}\mathbf{G}^H \rangle. \tag{6}$$

This problem appears to be highly non-convex, however, it can be converted into an equivalent convex problem as follows. Let $\mathbf{F}$ be fixed, the optimal solution $\mathbf{G}(\mathbf{F})$ to (6) is given by

$$\mathbf{G}(\mathbf{F}) = \mathbf{R}_L\mathbf{F}^H\mathbf{H}^H[\mathbf{H}\mathbf{F}\mathbf{R}_L\mathbf{F}^H\mathbf{H}^H + \mathbf{I}_{N_r}]^{-1}. \tag{7}$$

Substituting this optimal solution for each fixed $\mathbf{F}$ into the objective of (6) the problem (6) reduces to

$$\min_{\langle \mathbf{F}\mathbf{R}_L\mathbf{F}^H \rangle \leq P_T} \langle [\mathbf{R}_L^{-1} + \mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F}]^{-1} \rangle. \tag{8}$$

3.2. Closed-form solution

Our solution to (8) is based on the following observation, whose proof is omitted due to space limitation.

Proposition 1 *Suppose that Σ and Ψ are given $n \times n$ diagonal matrices with $\{\Sigma(i,i)\}_{i=1,2,\ldots,n}$ arranged in decreasing order. Then*

$$\max_{\mathbf{X} \in \mathcal{M}_n} \langle \Sigma\mathbf{X}\Psi\mathbf{X}^H \rangle = \sum_{i=1}^{n} \Sigma(i,i)\Psi(\tau(i),\tau(i)) \tag{9}$$

is attained at $\mathbf{X} = \mathbf{V}_{\max}$ with $\mathbf{V}_{\max}$ such that

$$\Psi = \mathbf{V}_{\max}^H \mathit{diag}[\Psi(\tau(i),\tau(i))]_{i=1,2,..,n}\mathbf{V}_{\max}$$

where $\tau : \{1,2,..,n\} \to \{1,2,..,n\}$ is a permutation that arranges the sequence $\{\Psi(i,i)\}_{i=1,2,\ldots,n}$ in a decreasing order

$$\{\Psi(\tau(i),\tau(i))\}_{i=1,2,\ldots,n}.$$

Analogously,

$$\min_{\mathbf{X} \in \mathcal{M}_n} \langle \Sigma\mathbf{X}\Psi\mathbf{X}^H \rangle = \sum_{i=1}^{n} \Sigma(i,i)\Psi(\bar{\tau}(i),\bar{\tau}(i)) \tag{10}$$

is attained at $\mathbf{X} = \mathbf{V}_{\min}$ *with* $\mathbf{V}_{\min}$ *such that*

$$\Psi = \mathbf{V}_{\min}^H diag[\Psi(\bar{\tau}(i), \bar{\tau}(i))]_{i=1,2,..,n} \mathbf{V}_{\min}$$

where $\bar{\tau} : \{1, 2, .., n\} \to \{1, 2, .., n\}$ *is a permutation that arranges the sequence* $\{\Psi(i, i)\}_{i=1,2,...,n}$ *in increasing order*

$$\{\Psi(\bar{\tau}(i), \bar{\tau}(i))\}_{i=1,2,...,n}.$$

To solve (8), we first consider the singular value decompositions (SVD) of $\mathbf{R}_L$ and $\mathbf{H}^H\mathbf{H}$, which result in

$$\mathbf{R}_L = \mathbf{U}_L^H \mathbf{\Sigma}_L \mathbf{U}_L \quad \text{and} \quad \mathbf{H}^H\mathbf{H} = \mathbf{U}_H^H \mathbf{\Sigma} \mathbf{U}_H \tag{11}$$

respectively, where $\mathbf{U}_L \in \mathcal{M}_L$, $\mathbf{U}_H \in \mathcal{M}_{N_t}$, and $\mathbf{\Sigma}_L > 0$, $\mathbf{\Sigma} = \text{diag}\begin{bmatrix}\mathbf{\Sigma}_H^2 & \mathbf{0}\end{bmatrix}$, $\Sigma_H > 0$ are diagonal matrices having diagonal elements in decreasing order. The dimensions of $\mathbf{\Sigma}_L$ and $\mathbf{\Sigma}_H$ are $L\times L$ and $H\times H$. Let $\mathbf{F}_{HL} \in \mathbb{C}^{H\times L}$ be such that $\mathbf{U}_H\mathbf{F} = \begin{bmatrix}\mathbf{F}_{HL}\\ *\end{bmatrix}$. Then, it follows that $\mathbf{F}^H\mathbf{H}^H\mathbf{H}\mathbf{F} = \mathbf{F}_{HL}^H\mathbf{\Sigma}_H^2\mathbf{F}_{HL}$, and thus

$$\langle \mathbf{F}\mathbf{R}_L\mathbf{F}^H\rangle = \langle \mathbf{U}_H\mathbf{F}\mathbf{R}_L\mathbf{F}^H\mathbf{U}_H^H\rangle = \tag{12}$$

$$\langle \begin{bmatrix}\mathbf{F}_{HL}\\ *\end{bmatrix} \mathbf{R}_L \begin{bmatrix}\mathbf{F}_{HL}^H & *\end{bmatrix}\rangle \leq \langle \mathbf{F}_{HL}\mathbf{R}_L\mathbf{F}_{HL}^H\rangle. \tag{13}$$

Therefore, without loss of generality we can assume that

$$\mathbf{U}_H\mathbf{F} = \begin{bmatrix}\mathbf{F}_{HL}\\ \mathbf{0}_{(N_t-H)\times L}\end{bmatrix}, \text{i.e.} \mathbf{F} = \mathbf{U}_H^H \begin{bmatrix}\mathbf{F}_{HL}\\ \mathbf{0}_{(N_t-H)\times L}\end{bmatrix}. \tag{14}$$

Thus (8) can be rewritten as

$$\min_{\langle \mathbf{F}_{HL}\mathbf{U}_L^H\mathbf{\Sigma}_L\mathbf{U}_L\mathbf{F}_{HL}\rangle \leq P_T} \langle [\mathbf{U}_L^H\mathbf{\Sigma}_L^{-1}\mathbf{U}_L + \mathbf{F}_{HL}^H\mathbf{\Sigma}_H^2\mathbf{F}_{HL}]^{-1}\rangle. \tag{15}$$

We consider the case $H \geq L$ first. It follows from [11, Th.7.4.5, p. 414] that the following SVD can be performed:

$$\mathbf{\Sigma}_H\mathbf{F}_{HL}\mathbf{U}_L^H\mathbf{\Sigma}_L^{1/2} = \mathbf{U}\begin{bmatrix}\sqrt{\mathbf{D}_x}\\ \mathbf{0}_{(H-L)\times L}\end{bmatrix}\mathbf{V} \tag{16}$$

where $\mathbf{U} \in \mathcal{M}_H, \mathbf{V} \in \mathcal{M}_L$, and $\mathbf{D}_x$ is a $L \times L$ diagonal matrix with $\mathbf{D}_x(i, i) = x_{ii} \geq 0$. Then

$$\begin{gathered}\mathbf{U}_L\mathbf{F}_{HL}^H\mathbf{\Sigma}_H^2\mathbf{F}_{HL}\mathbf{U}_L^H = \mathbf{\Sigma}_L^{-1/2}\mathbf{V}^H\mathbf{D}_x\mathbf{V}\mathbf{\Sigma}_L^{-1/2},\\ \mathbf{F}_{HL}\mathbf{U}_L^H\mathbf{\Sigma}_L\mathbf{U}_L\mathbf{F}_{HL} = \mathbf{\Sigma}_H^{-1}\mathbf{U}\,\text{diag}[\mathbf{D}_x\ \mathbf{0}_{H-L}]\mathbf{U}^H\mathbf{\Sigma}_H^{-1}\end{gathered}$$

and the problem (15) now becomes

$$\min_{\mathbf{D}_x\geq 0, \mathbf{U}\in\mathcal{M}_H, \mathbf{V}\in\mathcal{M}_H,\ \theta(\mathbf{D}_x,\mathbf{U})\leq P_T} \langle \mathbf{\Sigma}_L\mathbf{V}^H[\mathbf{I}_L + \mathbf{D}_x]^{-1}\mathbf{V}\rangle \tag{17}$$

where $\theta(\mathbf{D}_x, \mathbf{U}) = \langle\text{diag}[\mathbf{D}_x\ \mathbf{0}_{H-L}]\mathbf{U}^H\mathbf{\Sigma}_H^{-2}\mathbf{U}\rangle$.
By Proposition 1 we have

$$\min_{\mathbf{V}\in\mathcal{M}_L} \langle \mathbf{\Sigma}_L\mathbf{V}^H[\mathbf{I}_L + \mathbf{D}_x]^{-1}\mathbf{V}\rangle = \sum_{i=1}^{L} \frac{\mathbf{\Sigma}(i,i)}{1 + \mathbf{D}_x(x(i), x(i))} \tag{18}$$

where $x(i)$ is a map from $\{1, ..., L\}$ to itself making decreasing order for $\mathbf{D}_x(i, i)$ (and thus increasing order for $1/(1 + D_x(i, i))$). It can be shown that the constraint $\theta(\mathbf{D}_x, \mathbf{U})$ in (17) can be written as:

$$\min_{\mathbf{U}\in\mathcal{M}_H} \theta(\mathbf{D}_x, \mathbf{U}) = \sum_{i=1}^{L} \Sigma_H^{-2}(i, i)\mathbf{D}_x(i, i) \leq P_T. \tag{19}$$

Based on (18) and (19), the problem (17) is reduced to

$$\min_{0\leq \mathbf{D}_x(i+1,i+1)\leq \mathbf{D}_x(i,i)} g_l(\mathbf{D}_x) :\ g_h(\mathbf{D}_x) \leq P_T \tag{20}$$

where $g_l(\mathbf{D}_x) := \sum_{i=1}^{L} \mathbf{\Sigma}_L(i, i)(1 + \mathbf{D}_x(i, i))^{-1}$ and
$g_h(\mathbf{D}_x) = \sum_{i=1}^{L} \mathbf{\Sigma}_H^{-2}(i, i)\mathbf{D}_x(i, i)$.
It can be seen that the optimal solution of this problem is the same as that of its relaxed problem: $\min_{\mathbf{D}_x(i,i)\geq 0} g_l(\mathbf{D}_x) : g_h(\mathbf{D}_x) \leq P_T$ and is obtained through the Karush-Kuhn-Tucker (KKT) condition of convex programming (see e.g. [12]) as

$$\mathbf{D}_x(i, i) = (\mu^{-1/2}\gamma(i) - 1)^+ \tag{21}$$

where $\gamma(i) = \mathbf{\Sigma}_L^{1/2}(i, i)\mathbf{\Sigma}_H(i, i)$, $(z)^+ := \max\{0, z\}$, and μ is chosen such that

$$P_T = g_h(\mathbf{D}_x) = \sum_{i=1}^{L}(\mathbf{\Sigma}_H^{-2}(i, i)(\mu^{-1/2}\gamma(i) - 1))^+.$$

It is obvious from the property of $\mathbf{D}_x(i, i)$ that $x(i) = i$ and hence $\mathbf{U} = \mathbf{I}_H$ and $\mathbf{V} = \mathbf{I}_L$ are the optimal solution of (18) and (19). Then from (16) and (21) the optimal solution $\mathbf{F}_{HL}$ is given by

$$\begin{aligned}\mathbf{F}_{HL} &= \mathbf{\Sigma}_H^{-1}\begin{bmatrix}\text{diag}[((\mu^{-1/2}\gamma(i) - 1)^+)^{1/2}]_{i=1,...,L}\\ \mathbf{0}_{(H-L)\times L}\end{bmatrix}\mathbf{\Sigma}_L^{-1/2}\mathbf{U}_L\\ &= \begin{bmatrix}\text{diag}[((\frac{\mu^{-1/2}}{\gamma(i)} - \frac{1}{\gamma(i)^2})^+)^{1/2}]_{i=1,..,L}\\ \mathbf{0}_{(H-L)\times L}\end{bmatrix}\mathbf{U}_L.\end{aligned} \tag{22}$$

Therefore, the optimal solution of (8) is given by

$$\mathbf{F} = \hat{\mathbf{U}}_H^H\ \text{diag}[((\mu^{-1/2}\gamma(i)^{-1} - \gamma(i)^{-2})^+)^{1/2}]_{i=1,..,L}\mathbf{U}_L \tag{23}$$

$$\mathbf{U}_H = \begin{bmatrix}\hat{\mathbf{U}}_H\\ *\end{bmatrix}, \hat{\mathbf{U}}_H \in \mathbb{C}^{L\times N_t}. \tag{24}$$

We now consider the case $L \geq H$. Instead of (16) the following SVD is performed [11, Th. 7.4.5, p.414]:

$$\mathbf{\Sigma}_H\mathbf{F}_{HL}\mathbf{U}_L^H\mathbf{\Sigma}_L^{1/2} = \mathbf{U}\begin{bmatrix}\sqrt{\mathbf{D}_x} & \mathbf{0}_{H\times(L-H)}\end{bmatrix}\mathbf{V} \tag{25}$$

with $\mathbf{D}_x = \text{diag}[\mathbf{D}_x(i, i)]_{i=1,2,...,H} \geq 0$, $\mathbf{U} \in \mathcal{M}_H(\mathbf{R})$, and $\mathbf{V} \in \mathcal{M}_L(\mathbf{R})$. Then, instead of (17), the equivalence of (15) is given by

$$\min_{\mathbf{D}_x\geq 0, \mathbf{U}\in\mathcal{M}_H, \mathbf{V}\in\mathcal{M}_H,\ \eta(\mathbf{D}_x,\mathbf{V})\leq P_T} g_v(\mathbf{D}_x, \mathbf{V}) \tag{26}$$

where $g_v(\mathbf{D}_x, \mathbf{V}) = \langle\mathbf{\Sigma}_L\mathbf{V}^H(\mathbf{I}_L + \text{diag}[\mathbf{D}_x\ \mathbf{0}_{L-H}])^{-1}\mathbf{V}\rangle$ and $\eta(\mathbf{D}_x, \mathbf{V}) = \langle\mathbf{D}_x\mathbf{U}^H\mathbf{\Sigma}_H^{-2}\mathbf{U}\rangle$. The optimal solution is obtained as

$$\mathbf{F}_{HL} = \begin{bmatrix}\text{diag}[((\frac{\mu^{-1/2}}{\gamma(i)} - \frac{1}{\gamma(i)^2})^+)^{1/2}]_{i=1,..,H} & \mathbf{0}_{H\times(L-H)}\end{bmatrix}\mathbf{U}_L \tag{27}$$

instead of (22). Accordingly, the optimal solution of (8) now is

$$\mathbf{F} = \hat{\mathbf{U}}_H^H\text{diag}[((\frac{\mu^{-1/2}}{\gamma(i)} - \frac{1}{\gamma(i)^2})^+)^{1/2}]_{i=1,..,L}\hat{\mathbf{U}}_L \tag{28}$$

where

$$\mathbf{U}_H = \begin{bmatrix}\hat{\mathbf{U}}_H\\ *\end{bmatrix}, \hat{\mathbf{U}}_H \in \mathbb{C}^{H\times N_t}, \mathbf{U}_L = \begin{bmatrix}\hat{\mathbf{U}}_L\\ *\end{bmatrix}, \hat{\mathbf{U}}_L \in \mathbb{C}^{H\times L} \tag{29}$$

instead of (23), (24).
The following theorem recaps our main results:

Theorem 1 *With SVDs (11), the optimal precoder* $\mathbf{F}$ *of the problem (6) is given either by the formulas (23), (24) when rank*$(H) \geq L$ *or by (28), (29) otherwise. In both cases, the optimal decoder* $\mathbf{G}$ *of the problem (6) is defined by the optimal precoder* $\mathbf{F}$ *by (7).*

4. SIMULATIONS AND COMPARISONS

This section presents simulation results to illustrate the performance of our solution and compares it with that of the pre-whitening scheme where the signal is whitened before being precoded. Due to lack of space, only simulations for frequency selective fading channels are presented. The channel taps are uncorrelated complex Gaussian random variables, i.e. Rayleigh fading channel. Each realization of the channel is assumed known at the transmitter and receiver; and the linear precoder $\mathbf{F}$ and decoder $\mathbf{G}$ were optimized for each channel realization. The signal vectors $\mathbf{s}$ used in the simulations were drawn from the quadrature phase shift keying (QPSK) constellation $\{\pm 1 \pm \jmath\}$, correlated with covariance $\mathbf{R}_L$. The additive noise vectors $\mathbf{n}$ are zero-mean, correlated complex Gaussian random variables with covariance $\mathbf{R}_n$. The total transmit power across all transmit antennas was normalized to unity, i.e. $\langle \mathbf{F}\mathbf{R}_L\mathbf{F}^H \rangle = 1$. The signal to noise ratio (SNR) is defined as $\mathrm{SNR} = \langle \mathbf{F}\mathbf{R}_L\mathbf{F}^H \rangle / \langle \mathbf{R}_n \rangle = 1/\langle \mathbf{R}_n \rangle$, which does not include possible gain/attenuation of the channel realization. In our simulations, the channel matrix $\mathbf{H}$ was normalized so that $\langle \mathbf{H}\mathbf{H}^H \rangle = 1$. Both the normalized least squares error (NLSE) defined as $\mathrm{NLSE} = \|\hat{\mathbf{s}} - \mathbf{s}\|^2 / \|\mathbf{s}\|^2$, and the bit error rate (BER) were used as the figure of merit for system performance.

A system of $N_t = 2$ transmit and $N_r = 2$ receive antennas was considered. The channel order was $\nu = 3$. Each block of input signal $\mathbf{s} \in \mathbb{C}^{10}$ was precoded as $\mathbf{x} = \mathbf{F}\mathbf{s} \in \mathbb{C}^{14}$. To avoid interblock interference, the following transmission scheme was used. Let $\mathbf{x}_1$ and $\mathbf{x}_2$ denote the ν-zero appended versions of the first half and the last half of $\mathbf{x}$ respectively: $\mathbf{x}_1 = [x(1), \ldots, x(7), 0, \ldots, 0]^T \in \mathbb{C}^{10}$ and $\mathbf{x}_2 = [x(8), \ldots, x(14), 0, \ldots, 0]^T \in \mathbb{C}^{10}$. The elements of $\mathbf{x}_1$ were sent to the first antenna and those of $\mathbf{x}_2$ to the second antenna. The NLSCE and the BER in Figures 1 and 2 respectively demonstrate that our method outperforms the pre-whitening scheme.

5. CONCLUSION

We have presented a solution for designing jointly optimal linear precoder and decoder for the general case of input signal. The distinct advantage of our solution is that it is applicable for both cases of white and colored input signals. The design is based on the minimum mean squared error criterion under the average output power constraint. Simulation results verified the theoretical results.

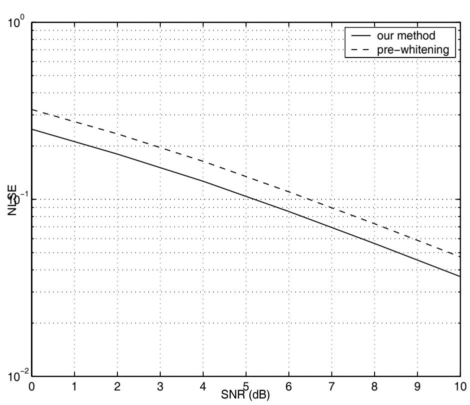

Fig. 1. NLSE: our method vs. prewhitening scheme

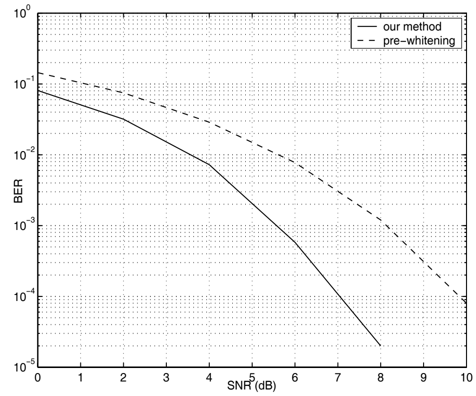

Fig. 2. BER: our method vs. prewhitening scheme

6. REFERENCES

[1] T. L. Marzetta and B. H. Hochwald, "Capacity of a mobile multiple antenna communication link in rayleigh flat fading," *IEEE Transactions on Information Theory*, , no. 45, pp. 139–157, 1999.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, , no. 16, pp. 1451–1458, 1998.

[3] D.P. Palomar, J.M. Cioffi, and M.A. Lagunas, "Joint tx-rx beamforming design for multicarier mimo channels: a unified framework for convex optimization," *IEEE Transactions on Signal Processing*, , no. 51, pp. 2381–2401, 2003.

[4] A. Scaglione, S. Barbarossa, and G.B. Giannakis, "Filterbank transceivers optimizing information rate in block transmissions over dispersive channels," *EEE Transactions on Information Theory*, , no. 45, pp. 1019–1032, 1999.

[5] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for mimo channles using the weighted mmse criterion," *IEEE Transactions on Communications*, , no. 49, pp. 2198–2206, 2001.

[6] A. Scaglione, G.B. Giannakis, and S. Barbarossa, "Redudant filterbank precoders and equalizers. part I: unification and optimal design," *IEEE Transactions on Signal Processing*, , no. 47, pp. 1988–2006, 1999.

[7] A. Scaglione, P. Stoica, S. Barbarossa, G.B. Giannakis, and H. Sampath, "Optimal design for space-time linear precoder and decoders," *IEEE Transactions on Signal Processing*, , no. 50, pp. 1051–1064, 2002.

[8] D. Gesbert, M. Shafi, D. Shiu, P.J. Smith, and A. Naguib, "From theory to prctice: an overview of mimo space-time coded wireless systems," *IEEE Journal on Selected Areas in Communications*, , no. 21, pp. 281–301, 2003.

[9] P. Apkarian and H.D. Tuan, "Parameterized LMIs in control theory," *SIAM Journal of Control and Optimization*, , no. 38, pp. 1241–1264, 2000.

[10] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in systems and control theory*, SIAM, Philadelphia, 1994.

[11] R.A. Horn and C.R. Johnson, *Matrix analysis*, Cambridge University Press, 1985.

[12] D. Luenberger, *Linear and nonlinear programming*, Springer, 2nd Edition, 2003.