ZERO-FORCING BEAMFORMING FOR NON-COLLABORATIVE SPACE DIVISION MULTIPLE ACCESS

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ABSTRACT

In this paper we explore non-collaborative beamforming techniques for space division multiple access (SDMA) scenarios where the base station (BS) and subscriber stations (SS) are equipped with multiple antennas. In non-collaborative SDMA, the SS array weights are designed independent of the BS and of one another. We consider zero-forcing beamforming schemes where the BS ensures that the signal stream to one user does not interfere with others. We derive an expression for average SNR achieved per user in the system and show that the average receive SNR per user degraded linearly with increasing number of users in the system. Further, we show that the dominant right singular vector of the channel matrix is the optimal choice of SS beamforming vector. Simulation results are included to verify our theoretical results.

1. INTRODUCTION

Space Division Multiple Access (SDMA) [1, 2] has recently emerged as a popular technique for the next generation communication systems and have been adopted in emerging standards such as IEEE 802.16 [3]. The key idea in SDMA is to enable the Base Station (BS) with multiple antennas to send/receive multiple data streams to/from different subscriber stations (SS) (with or without multiple antennas) on the same time-frequency channel while separating them in the spatial dimension.

In this work, we assume that the SS is equipped with multiple antennas. The MIMO-SDMA system is depicted in Figure 1 where the BS is transmitting a single stream to each SS. The problem we address is the design of the beamforming weights of the BS and the SS. Signal processing algorithms in literature (see [4, 5, 6, 7]) jointly design BS and SS beamformer weights and can be collectively classified under collaborative methods. The joint design requires knowledge of all channel matrices and can be done at the BS. This requires each SS to feedback the channel matrices to the BS followed by a feed foward of the receiver beamformer weights from the BS to each SS (as is proposed in [4, 6]). This causes an great increase in feedback/feedforward overheads. Moreover, standard specifications (for example IEEE 802.16e, 3GPP) etc. may render collaborative methods incapable of deployment in many cases due to these high overhead constraints. On the other hand, under non-collaborative methods, beamformer weights are first designed by the SS independent of each other following which the BS designs it's transmit beamformer weights. The non-collaborative approach eliminates the requirement of a feed forward channel, and further condenses the information that need to be fed back by the BS to the SS. Hence, these methods may provide a viable alternative in most such scenarios.

In this paper, we consider non-collaborative MIMO-SDMA methods. We assume a sub-optimal downlink beamforming strategy commonly referred to as zero-forcing beamforming (ZFBF), in which the BS beamformer weights are designed so that each SSs data stream causes no interference to any other SSs data streams. Relative ease of implementation and reasonably good performance make ZFBF an attractive option for next generation wireless systems. Thus, in this design, the SSs design their receive beamfomers first which are then used by the BS to design the ZFBF vectors. In the current literature, to the best of our knowledge there is no known method for SS beamfomer design which guaranties any optimality. The optimal beamformer for the SS in a single user case is well-known to be given by the singular value decomposition of the instantaneous channel matrix. However, the optimality is no longer clear in the multi-user domain with SDMA. Moreover, it can be easily argued that a statistical metric of performance is more meaningful that an instantaneous one in a non-collaborative setting. Here, we present our analysis of the performance characteristics of non-collaborative zero-forcing SDMA methods and derive an expression for the *average SNR per user*[1] in the system under isotropic channel conditions. We show that performance degrades linearly in the number of users and that the dominant right singular vector is a *statistically optimal* choice for the SS beamformer. We also compare our results with the non-collaborative scheme in [8].

The rest of the paper is organized as follows. The general system model is presented in Section 2. In Section 3 we present the non-collaborative MIMO-SDMA scheme and our analysis. In Section 4 we present the simulation results to establish the validity of our theoretical results. We conclude in Section 5.

Notation : Uppercase and lowercase bold face symbols represent matrices and vectors respectively. For a matrix $\mathbf{H}$, $\mathbf{H}^T$, $\mathbf{H}^H$ and $\mathbf{H}^{-1}$ denote the transpose, the conjugate transpose and the inverse respectively.

2. SYSTEM MODEL

The system model used in this paper is illustrated in Figure 1. We assume BS with N antennas and m different SS ($N \geq m$) with k_i antennas at the i^{th} SS. The MIMO channels from the BS to SS_i is denoted by $\mathbf{H}_i$ ($N \times k_i$) where $i = 1, 2, \ldots, m$. The BS beamforms m different data streams to the m SS on the same time-frequency channel. Though we demonstrate the system model only for the downlink case, the same model applies as is for the uplink case. The modulated data symbol to be transmitted to SS_i is s_i. The antenna

[1] Note that optimization of worst case performance will result in lower performance since it will necessarily degrade the users with the best channels - this has been verified by our simulations

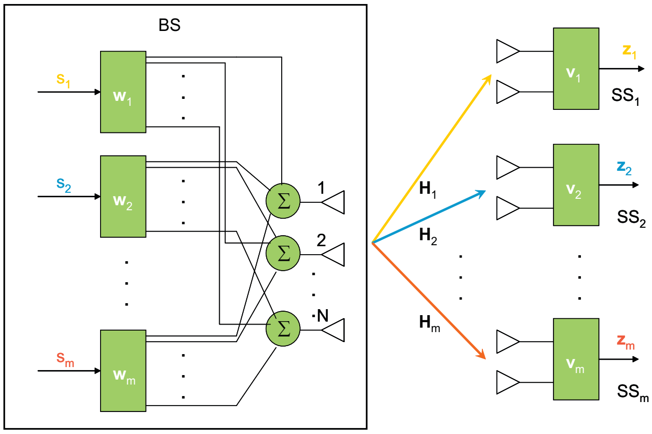

Fig. 1. System model

weights used to transmit symbol s_i is denoted by the beamforming vector $\mathbf{w}_i$.

The transmit vector is given by $\mathbf{t} = \sum_{i=1}^{m} s_i \mathbf{w}_i$. Then the receive signal vector at SS_1 is given by

$$\mathbf{y}_1 = s_1 \mathbf{H}_1^H \mathbf{w}_1 + \sum_{i=2}^{m} s_i \mathbf{H}_1^H \mathbf{w}_i + \boldsymbol{\eta} \tag{1}$$

where $\boldsymbol{\eta}$ is a complex Gaussian white noise vector with variance $\sigma_n^2 \mathbf{I}$. The first term on the RHS of Eq. 1 is the desired receive signal while the summation terms represents the interference terms. The SS estimates s_1 as $\hat{s}_1 = \mathbf{y}_1^H \mathbf{v}_1$ using beamformer $\mathbf{v}_1 (k_1 \times 1)$.

We assume that the m users first design the beamformers $\mathbf{v}_i, i = 1, 2, \ldots, m$ and the knowledge of the $\mathbf{v}_i$ is then used to by the BS to design the transmit beamformers. The optimal zero-forcing beamformers for a given set of $\mathbf{v}_i$ and $\mathbf{H}_i$ can be derived by posing the problem using a linearly constrained minimum variance (LCMV) formulation given by

$$\mathbf{w}_i = \arg \min_{\mathbf{w}^H \mathbf{w}} \mathbf{X}^H \mathbf{w} = \mathbf{e}_i \tag{2}$$

whose solution can be obtained as

$$\mathbf{w}_i = \mathbf{X}[\mathbf{X}^H \mathbf{X}]^{-1} \mathbf{e}_i \tag{3}$$

where $\mathbf{X} = [\mathbf{u}_1 \; \mathbf{u}_2 \; \ldots \; \mathbf{u}_m]$ and $\mathbf{u}_i = \mathbf{H}_i \mathbf{v}_i$ with $i = 1, 2, \ldots, m$, and e_i is a column vector of all zeros except for a 1 at the i^{th} position.

In the above solution the designed gain to each user is normalized to unity. Normalizing the transmit power to user i to be equal to unity would result in differential SNRs to each user - the SNR is then proportional to $1/|\mathbf{w}_i^H \mathbf{w}_i|$. *Important Note:* Observe that, after the BS has designed its weights, the SS can no longer modify its beamformers for that transmission since it will lead to the violation of the conditions of the above solution - so if the SS modifies its weights after the data transmission begins, it will end up receiving unwanted interference from the streams meant for other users.

3. NON-COLLABORATIVE MIMO SDMA

The problem now remains of specifying a means of designing the weight vectors for the SS. In a collaborative setting, the SS and BS weights would be jointly designed using channel information to all SS, arguably at the BS and the SS weights would be fed forward to the different SS. Non-collaborative methods eliminate the need for feed forward of any information while trading off on the SNR gain to each user. In a non-collaborative setting an instantaneous metric of performance is not meaningful since a search for an optimal instantaneous metric requires collaboration. Performance metrics such as sum-capacity, worst case performance etc. is difficult to use in this context since each SS designs its beamformer independent of the channel states of the other SS. Hence, we use average SNR per user as the metric for performance in this paper which is more meaningful in this context. We derive an expression for the average SNR per SS as a function of its choice of beamforming vector.

The key assumption that we make is that the channel matrices corresponding to the multi-antenna channels between the BS and each SS are isotropic. Such as assumption is reasonable from a practical viewpoint and is quite common in literature involving wireless channel models (for example, see [9]). For isotropic channels, equivalently the left and right singular vectors of the channel matrices are statistically equally likely in all directions in the complex vector space corresponding to the MIMO channel dimensions. The isotropic assumption has different implications under different scattering environments. For example, under line-of-sight (LoS) scenarios, the isotropic assumption implies that the azimuthal angle of arrival/departure for signal from/to a specific SS to/from the BS is uniform or equally likely in the $(0, 2\pi]$ interval. For the i.i.d. Rayleigh flat fading model, the isotropic assumption implies that the directions of each of the columns of the channel matrix is equally likely in all complex directions within the complex vector space spanned by them.

We emphasize that contrary to previous work in [4, 5, 6], we consider the case when the SS and BS weights are designed in sequence separately at the SS and the BS respectively thus eliminating the need to feed forward the SS weight information as would be required if they are co-designed at the BS. Further, in the collaborative MIMO literature cited above, the complete channel information (MIMO matrices) need to be fed back from each SS to the BS, whereas with non-collaborative methods only the SS weight vectors need be fed back to enable the BS to design the zero-forcing beamformers which substantially reduces the feedback channel bandwidth required.

Important Note: Without loss of generality, we derive our results only for downlink SDMA transmissions - the same analysis applies as is to uplink transmission using SDMA with the same design parameters.

The following lemma is relevant to the proof of the rest of the theorems stated in this paper.

Lemma 3.1 *Let $\mathbf{X} = [\mathbf{u}_1 \; \mathbf{u}_2 \; \ldots \; \mathbf{u}_m]$ be a $N \times m$ matrix with $m < N$, where $\mathbf{u}_i$, $i = 1, 2, \ldots, m$ are mutually independent random unit vectors and let $\mathbf{Y} = \mathbf{X}[\mathbf{X}^H \mathbf{X}]^{-1} \mathbf{X}^H$. If $\mathbf{u}_r \in \mathbb{C}^{N \times 1}$ is an isotropic unit length vector independent of the $\mathbf{u}_i$ $i = 1, 2, \ldots, m$, then $\mathbb{E}[\mathbf{u}_r^H \mathbf{Y} \mathbf{u}_r] = m/N$*

Proof: Since $\mathbf{X}$ has independent columns, it is rank m with probability one. Since, $\mathbf{Y}$ is idempotent with rank m, it has m unit eigen values and $N - m$ zero eigen values. Hence the Eigen Value Decomposition (EVD) of $\mathbf{Y}$ is given by

$$\mathbf{Y} = \mathbf{V} \boldsymbol{\Sigma} \mathbf{V}^H$$

where $\mathbf{V} = [\mathbf{v}_1 \; \mathbf{v}_2 \; \ldots \; \mathbf{v}_N]$ is a unitary matrix and $\boldsymbol{\Sigma}$ is diagonal with m unity elements and $N - m$ zero elements. Without loss of generality assume that the top m diagonal elements of $\boldsymbol{\Sigma}$ are unity.

Since $\mathbf{u}_r$ is isotropic, the distribution of $\mathbf{u}_r$ is invariant to any unitary transformation. $\mathbf{V}$ is a unitary matrix and hence

$$\begin{aligned}\mathbb{E}[\mathbf{u}_r^H \mathbf{Y} \mathbf{u}_r] &= \mathbb{E}[\mathbf{u}_r^H \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}^H \mathbf{u}_r] = \hat{\mathbf{u}}_r^H \boldsymbol{\Sigma} \hat{\mathbf{u}}_r \\ &= \textstyle\sum_{i=1}^m \mathbb{E}[|\hat{u}_{r,i}|^2]\end{aligned} \tag{4}$$

where $\hat{\mathbf{u}}_r = \mathbf{V}^H \mathbf{u}_r$ and $\hat{\mathbf{u}}_r = [\hat{u}_{r,1}\ \hat{u}_{r,2}\ \dots\ \hat{u}_{r,N}]^T$. Now, it is known that any isotropic vector $\hat{\mathbf{u}}$ can be written as $\hat{\mathbf{u}} = \mathbf{g}/|\mathbf{g}^H\mathbf{g}| = \hat{\mathbf{g}}$ where $\mathbf{g}$ is a vector of i.i.d $\mathbb{CN}(0,1)$ random variables and $\hat{\mathbf{g}}$ is its normalized version. Note that since $\mathbb{E}[\hat{\mathbf{g}}^H\hat{\mathbf{g}}] = \mathbb{E}[\sum_{i=1}^N |\hat{g}_i|^2] = N\mathbb{E}[|\hat{g}_1|^2] = 1$, we get $\mathbb{E}[|\hat{g}_1|^2] = 1/N$. Using this identity in (4), proves the lemma. ∎

The next theorem presents the main result of this paper - a closed form expression for the average receive SNR for a given SS for a given selection of receive weights $\mathbf{v}_i$.

Theorem 3.1 *Assume a MIMO-SDMA system with N transmit antennas at the BS and m SSs, with the i-th SS having k_i receive antennas. The i-th SS using a receive beamformer denoted as $\mathbf{v}_i$ and the BS enforces a zero-forcing solution using an LCMV formulation. Then, under the assumption that the MIMO channel $\mathbf{H}_i$ between the BS and the i-th SS is isotropic $\forall i$, the average receive SNR for any SS is given by*

$$\mathbb{E}[\rho] = \mathbb{E}[\mathbf{v}^H\mathbf{H}^H\mathbf{H}\mathbf{v}] \left\{1 - \frac{m-1}{N}\right\} \left\{\frac{\sigma_s^2}{\sigma_n^2}\right\}.$$

Proof: Without loss of generality we prove the theorem for user 1, with SNR for user 1 denoted by ρ_1.

We rewrite $\mathbf{X}$ as

$$\mathbf{X} = [\lambda_1\tilde{\mathbf{u}}_1\ \lambda_2\tilde{\mathbf{u}}_2\ \dots \lambda_m\tilde{\mathbf{u}}_m] \tag{5}$$

where $\tilde{\mathbf{u}}_i = \mathbf{u}_i/\lambda_i$ and $\lambda_i = \sqrt{\mathbf{u}_i^H\mathbf{u}_i}$. For transmit power σ_s^2, the receive SNR for user 1 is $\rho_1 = (1/||\mathbf{w}_1||^2)(\sigma_s^2/\sigma_n^2)$. Since the linear constraints on $\mathbf{w}_1$ corresponding to columns $2, 3, \dots m$ of $\mathbf{X}$ are zero constraints, the solution is invariant to the scaling of these columns, i.e., $\mathbf{X}$ can be replaced with $\tilde{\mathbf{X}} = [\lambda_1\tilde{\mathbf{u}}_1\ \tilde{\mathbf{u}}_2\ \tilde{\mathbf{u}}_3\ \dots \tilde{\mathbf{u}}_m]$ without affecting the beamformer solutions.

Then, for the optimal beamformer solution using LCMV, it is easy to show that

$$||\mathbf{w}_1||^2 = \mathbf{e}_1^T[\tilde{\mathbf{X}}^H\tilde{\mathbf{X}}]^{-1}\mathbf{e}_1 = [\tilde{\mathbf{X}}^H\tilde{\mathbf{X}}]_{11}^{-1} \tag{6}$$

Therefore, $\rho_1 = \{[\tilde{\mathbf{X}}^H\tilde{\mathbf{X}}]_{11}^{-1}\}^{-1}(\sigma_s^2/\sigma_n^2)$ where $[\tilde{\mathbf{X}}^H\tilde{\mathbf{X}}]_{11}^{-1}$ is the first row and first column element of $[\tilde{\mathbf{X}}^H\tilde{\mathbf{X}}]^{-1}$.

$[\tilde{\mathbf{X}}^H\tilde{\mathbf{X}}]$ which can be written as

$$\tilde{\mathbf{X}}^H\tilde{\mathbf{X}} = \begin{bmatrix} |\lambda_1|^2 & \mathbf{c}^H \\ \mathbf{c} & \mathbf{W} \end{bmatrix} \tag{7}$$

where

$$\mathbf{c} = \lambda_1 \left[\tilde{\mathbf{u}}_2^H\tilde{\mathbf{u}}_1\ \tilde{\mathbf{u}}_3^H\tilde{\mathbf{u}}_1\ \dots\ \tilde{\mathbf{u}}_m^H\tilde{\mathbf{u}}_1\right]^T$$

and $\mathbf{W} = \hat{\mathbf{X}}^H\hat{\mathbf{X}}$ with $\hat{\mathbf{X}} = [\tilde{\mathbf{u}}_2\ \tilde{\mathbf{u}}_3\ \dots\ \tilde{\mathbf{u}}_m]$. Define, $T = |\lambda_1|^2 - \mathbf{c}^H\mathbf{W}^{-1}\mathbf{c}$. Then using the Schur complements form [10], we have

$$[\tilde{\mathbf{X}}^H\tilde{\mathbf{X}}]^{-1} = \begin{bmatrix} T^{-1} & -T^{-1}\mathbf{c}^H\mathbf{W}^{-1} \\ -\mathbf{W}^{-1}\mathbf{c}T^{-1} & \mathbf{W}^{-1} + \mathbf{W}^{-1}\mathbf{c}T^{-1}\mathbf{c}^H\mathbf{W}^{-1} \end{bmatrix} \tag{8}$$

Observing that T is a scalar, we have $||\mathbf{w}_1||^2 = T^{-1}$ and $\rho_1 = T(\sigma_s^2/\sigma_n^2)$. Therefore,

$$\begin{aligned}\rho_1 &= (|\lambda_1|^2 - \mathbf{c}^H\mathbf{W}^{-1}\mathbf{c})\tfrac{\sigma_s^2}{\sigma_n^2} \\ &= |\lambda_1|^2(1 - \tilde{\mathbf{c}}^H\mathbf{W}^{-1}\tilde{\mathbf{c}})\tfrac{\sigma_s^2}{\sigma_n^2}\end{aligned} \tag{9}$$

where

$$\tilde{\mathbf{c}} = \left[\tilde{\mathbf{u}}_2^H\tilde{\mathbf{u}}_1\ \tilde{\mathbf{u}}_3^H\tilde{\mathbf{u}}_1\ \dots\ \tilde{\mathbf{u}}_m^H\tilde{\mathbf{u}}_1\right]$$

$\mathbf{W}$ does not contain any term related to $\mathbf{u}_1$ and hence is statistically independent of λ_1. The $\tilde{\mathbf{c}}$ contain projection terms of $\mathbf{u}_1$ normalized to unit-length and hence depends only on the angle between the vectors $\tilde{\mathbf{u}}_i$, $i = 2, 3, \dots, m$ and $\mathbf{u}_1$, but not on λ_1, the length of $\mathbf{u}_1$. Hence, it follows that λ_1 and $\tilde{\mathbf{c}}$ are statistically independent.

Therefore, taking expectation on both sides of Eq. 9 we have

$$\begin{aligned}\mathbb{E}[\rho_1] &= \mathbb{E}[|\lambda_1|^2(1 - \tilde{\mathbf{c}}^H\mathbf{W}^{-1}\tilde{\mathbf{c}})]\tfrac{\sigma_s^2}{\sigma_n^2} \\ &= \mathbb{E}[|\lambda_1|^2](1 - \mathbb{E}[\tilde{\mathbf{c}}^H\mathbf{W}^{-1}\tilde{\mathbf{c}}])\tfrac{\sigma_s^2}{\sigma_n^2}\end{aligned} \tag{10}$$

Observing that $\tilde{\mathbf{c}} = \hat{\mathbf{X}}^H\tilde{\mathbf{u}}_1$ we have $\tilde{\mathbf{c}}^H\mathbf{W}^{-1}\tilde{\mathbf{c}} = \tilde{\mathbf{u}}_1^H\mathbf{Y}\tilde{\mathbf{u}}_1$ where $\mathbf{Y} = \hat{\mathbf{X}}[\hat{\mathbf{X}}^H\hat{\mathbf{X}}]^{-1}\hat{\mathbf{X}}^H$. Hence it follows from Lemma 3.1 that

$$\mathbb{E}[\tilde{\mathbf{c}}^H\mathbf{W}^{-1}\tilde{\mathbf{c}}] = (m-1)/N \tag{11}$$

Substituting in Eq. 10, we have

$$\mathbb{E}[\rho_1] = \mathbb{E}[|\lambda_1|^2]\left(1 - \frac{m-1}{N}\right)\left(\frac{\sigma_s^2}{\sigma_n^2}\right) \tag{12}$$

∎

An immediate corollary of Thm. 3.1 follows as

Corollary 3.1 $|\lambda_1|^2$ *and hence* $\mathbb{E}[|\lambda_1|^2]$ *is maximized by choosing* $\mathbf{v}_i$ *as the dominant right singular vector of* $\mathbf{H}_i$*. Hence, the optimal beamformer which maximizes the average receive SNR for any SS is given by the dominant right singular vector of the instantaneous channel matrices.*

Corollary 3.2 *The average SNR per user degrades linearly with the number of users in the system.*

It can be shown that the choice of the right singular vector by each user *does not* necessarily maximize the instantaneous SNR for each user. However, Corollary 3.1 implies that on an average the choice is optimal independent of the choices of the other users and their channels.

4. SIMULATIONS

Simulations results are presented to demonstrate the validity of our theoretical results. System simulations were carried out in MATLAB [2]. We implemented a single cell downlink system with one BS having a fixed number of antennas and different numbers of users having varying number of receive antennas. We simulated results for two different channel models - (i) Rayleigh flat fading channel model, and, (ii) LoS channel model with angles of arrival/departure generated from a uniform distribution in $[0, 2\pi]$. The users in the system employ the dominant right singular vector as their choice of receive beamformers.

The simulation results for the Rayleigh flat-fading are shown in Fig. 2. The number of BS antennas is set to 5; hence a maximum of 5

[2] MATLAB is a registered trademark of The MathWorks Inc.

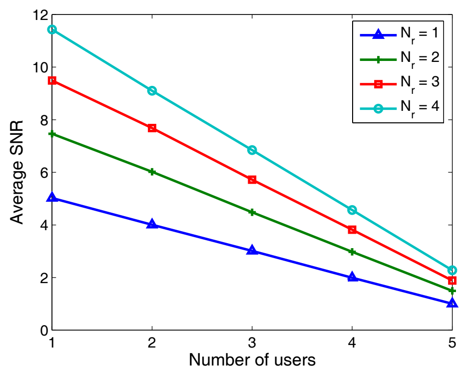

Fig. 2. Average SNR with optimal SS beamformer for varying number of users with Rayleigh channels

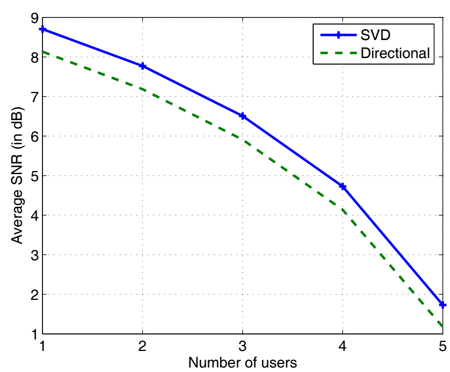

Fig. 3. Comparison of SVD-based scheme with scheme in [8]

users can be supported in the system. The average SNR for a single user in the system is plotted. The $\mathbb{E}[\lambda^2]$ depends on the channel model used and is derived in literature for the Rayleigh case. It is easy to verify that the simulation results conform to the derivation in Thm. 3.1. The LOS case assumes a uniform linear array at half wavelength spacing. The simulated results for LOS exhibit exactly the same pattern as those for the Rayleigh case and are omitted for brevity.

In our next set of simulations we compare the non-collaborative MIMO SDMA scheme in [8] to the optimal method presented in this paper. The work in [8] is based on actual field trials where the SS first acquires the dominant direction (of arrival) of the transmitter (BS) and selects its beamforming vector to point in that direction. We simulated the same approach assuming a uniform linear array at the SS. The SS selects a beamformer aligned towards the most dominant angle of arrival from the BS. Simulation results for a Rayleigh flat fading environment is shown in Fig. 3. The comparison shows that the singular vector scheme performs consistently better than the scheme in [8]. Note that the SVD based method and that in [8] would perform identically in a LOS scenario since the singular vector and the weight vector representing prominent direction of arrival are the same.

5. CONCLUSION

This paper presents new results for non-collaborative MIMO SDMA systems where the BS and SS have multiple antennas. Such systems, while being relevant to many of the scenarios where collaborative schemes are difficult to implement, are not well studied in literature. We present new results for non-collaborative SDMA wherein the SS weights are designed independently of the BS and of each other followed by the design of the BS weights. We derive an expression for the average SNR per user in the non-collaborative MIMO SDMA multi-user system. We prove that the SNR performance degrades linearly in the number of users and that the dominant right singular vector of the channel matrix is the optimal choice of the SS beamformer independent of the choice of the other SS in the system to maximize the average receive SNR. Further results are presented in [11] which considers cases when (possibly non-identical) external interference is present at each SS.

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