NON-REDUNDANT AND REDUNDANT POST CODING IN OFDM SYSTEMS

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) provides a viable solution to communicate over frequency selective fading channels by converting them to an equivalent collection of flat fading channels. In doing so, OFDM systems fail to reap the benefits of diversity available in multipath fading channels. To ameliorate this shortcoming of OFDM, explicit diversity in the form of redundant or non-redundant coding is needed. Our main goal in this paper is to explore different options for low complexity encoder design and compare their performance and complexity. Specifically, in the class of non-redundant codes we discuss the use of signal space diversity codes in OFDM systems. For redundant codes, we introduce a novel low complexity postcoded-OFDM system where coding is employed after performing the IFFT (inverse fast Fourier transform) in the transmitter to reduce system complexity. Simulation results show that postcoded-OFDM with redundant coding outperforms other choices of encoder design considered in this paper.

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) offers several advantages like resilience to multipath fading and intersymbol interference, low complexity and others. It is believed to be a promising technique for future broadband wireless communications [1].

While OFDM systems convert a multipath fading channel into a series of equivalent parallel flat fading channels, they lack the inherent diversity available in multipath channels. Different coded OFDM systems have been reported that employ some form of channel coding or precoding [2, 3] to improve system performance. The error analysis of communication systems over fading channels shows that it is the Hamming distance (defined later in the paper) that governs performance over fading channels [4]. It is important to mention that the Hamming distance of a signal constellation can be increased by non-redundant [5] or redundant coding [3].

Our objective in this paper is to apply non-redundant and redundant coding to uncoded OFDM system and compare the performance and complexity of the resulting systems. We introduce the idea of postcoding that helps reduce the complexity of OFDM transmitter and obtain the best performance.

The paper is organized as follows. Section 2 presents the system details and formulates the problem mathematically. In Section 3, we discuss the error analysis of coded OFDM systems over fading channels and highlight the importance of Hamming distance. Section 4 and 5 discuss the non-redundant and redundant coding schemes, respectively. We present simulation results in Section 6 and conclude the paper in Section 7.

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2. SYSTEM DETAILS AND PROBLEM FORMULATION

Consider an uncoded OFDM system that is implemented by using an N-point IFFT/FFT. The information symbols are mapped to the signal space according to the modulation scheme. The serial stream of modulated data symbols $b(n)$ are grouped in blocks of size N such that the ith block is expressed as $\mathbf{b}(i) := \left[b(iN),\ b(iN+1)\ \cdots b(iN+N-1)\right]$. Let $\mathbf{F}_N$ be the $N \times N$ FFT (fast Fourier transform) matrix with (n,k)th entry as

$$[\mathbf{F}_N]_{n,k} = (1/\sqrt{N})\exp\{-j2\pi(n-1)(k-1)/N\}. \tag{1}$$

Ignoring the block index i, the output of IFFT (inverse fast Fourier transform) block is an OFDM symbol in the form of $N \times 1$ vector and is given by

$$\mathbf{x} = \mathbf{F}_N^{\mathcal{H}}\,\mathbf{b}. \tag{2}$$

The insertion of the cyclic-prefix (CP) at the transmitter and CP-removal at the receiver, renders the channel matrix $\mathbf{H}$ an $N \times N$ circulant matrix $\widetilde{\mathbf{H}}$. The received OFDM symbol can therefore be expressed as:

$$\mathbf{r} = \widetilde{\mathbf{H}}\mathbf{x} + \tilde{\boldsymbol{\eta}} = \widetilde{\mathbf{H}}\mathbf{F}_N^{\mathcal{H}}\,\mathbf{b} + \tilde{\boldsymbol{\eta}}, \tag{3}$$

where $\tilde{\boldsymbol{\eta}}$ represents the $N \times 1$ additive Gaussian noise vector. At the receiver, multiplication with the FFT matrix $\mathbf{F}_N$ diagonalizes the channel matrix $\widetilde{\mathbf{H}}$ such that it contains the N point discrete frequency response of the channel given by [6]:

$$\mathbf{F}_N\widetilde{\mathbf{H}}\mathbf{F}_N^{\mathcal{H}} = \mathbf{H}_D = \mathrm{diag}\left[\mathbf{F}_N\tilde{\mathbf{h}}\right], \tag{4}$$

where $\tilde{\mathbf{h}}$ is $N \times 1$ vector obtained from the concatenation of L_h channel taps, $\{h_l\}_{l=1}^{L_h}$, and $N - L_h$ zeros. Thus, the received OFDM symbols can be simply written as:

$$\mathbf{u} = \mathbf{H}_D\mathbf{b} + \boldsymbol{\eta}. \tag{5}$$

The diagonalization of $\widetilde{\mathbf{H}}$ converts an ISI channel into an ISI free channel and eliminates the need for a complex receiver. Although OFDM systems provide a means to have simple receivers, the system performance deteriorates severely in the presence of channel frequency nulls. This deterioration can be avoided by employing explicit diversity or redundancy (coding) in the OFDM symbols.

Depending on the ease of implementation, the coding process can be called before or after the IFFT block in the transmitter as shown in Fig. 1. We refer to the former as precoded OFDM. In this case, the transmitted OFDM symbols can be written as:

$$\mathbf{y} = \mathbf{F}_{NL}^{\mathcal{H}}\underline{\mathbf{A}}\mathbf{b}. \tag{6}$$

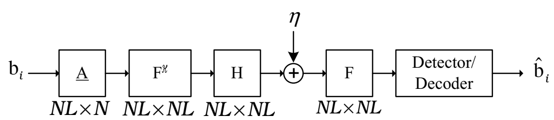
We refer to the latter as postcoded-OFDM (PC-OFDM). In this case, we encode the OFDM symbols after the IFFT as:

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \mathbf{A}\mathbf{F}_N^{\mathcal{H}}\mathbf{b}. \tag{7}$$

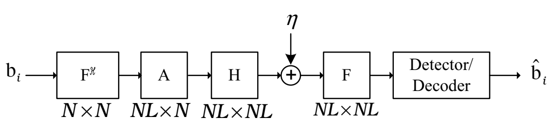
In both cases, we consider complex field coding *i.e.*, $\mathbf{A}$ (or $\underline{\mathbf{A}}$) $\in \mathbb{C}^{K\times N}$ with $K \geq N$, instead of Galois field as it provides more degrees of freedom [3]. It is important to note that any postcoding scheme can be made equivalent to a precoding scheme by selecting

$$\underline{\mathbf{A}} = \mathbf{F}_{NL}\mathbf{A}\mathbf{F}_N^{\mathcal{H}}. \tag{8}$$

Another important factor in the design of encoding matrix is the availability of bandwidth. If the system can tolerate a decrease in bandwidth efficiency, it is always desirable for the sake of system performance to use redundant encoding where $\mathbf{A}$ (or $\underline{\mathbf{A}}$) has a tall structure of $K \times N$ with $K > N$. Similarly, to save bandwidth one can use non-redundant coding by selecting $\mathbf{A}$ with square structure, *i.e.*, $N \times N$. An obvious advantage of postcoding over precoding is the savings in the IFFT module especially for the redundant case. Before discussing these possible choices in detail, we first outline the general criterion used to construct "good" encoding matrices in the next section.



(a) Precoded OFDM system



(b) Postcoded OFDM system

Fig. 1. Precoded vs. Postcoded OFDM systems

3. CODE DESIGN CRITERION FOR FADING CHANNELS

It has been shown in the recent research that the criteria commonly used to design codes for additive white Gaussian noise (AWGN) channels have to be adjusted when dealing with a fading channel (see [4] and references therein). As we shall see soon, the performance of a code over fading channels depends on the minimum Hamming distance and not on the Euclidean distance between codewords. To see how the choice of encoder affects the system performance, consider the precoding scheme of Fig. 1(a) where the received symbol can be expressed as:

$$\mathbf{u} = \mathbf{H}_D\underline{\mathbf{A}}\mathbf{b} + \boldsymbol{\eta}. \tag{9}$$

To assess system performance over uncorrelated fading channels, we adopt the average pairwise error probability (PEP) technique that has been derived in similar context in [3, 7]. By definition, the PEP is the probability of erroneously detecting $\mathbf{b}'$ when $\mathbf{b}$ was transmitted. We consider ML detection and perfect channel knowledge at the receiver. In order to find the PEP (see [3] for details), we need to define a matrix $\mathbf{A}_e := (\mathbf{D}_e\mathbf{V})^{\mathcal{H}}\mathbf{D}_e\mathbf{V}$ where $\mathbf{V}$ is the truncated FFT matrix with $[\mathbf{V}]_{(k,l)} = e^{-j2\pi kl/NL}$ and $\mathbf{D}_e = \underline{\mathbf{A}}(\mathbf{b}-\mathbf{b}')$. Now, for Rayleigh fading channels with uncorrelated paths, the PEP is given by:

$$\Pr(\mathbf{b}\to\mathbf{b}') \leq \left(\frac{1}{4\mathrm{N_o}}\right)^{-\mathrm{G_d}} \left(\prod_{\mathrm{l}=1}^{\mathrm{G_d}} \alpha_{\mathrm{l}}\lambda_{\mathrm{e,l}}\right)^{-1}, \tag{10}$$

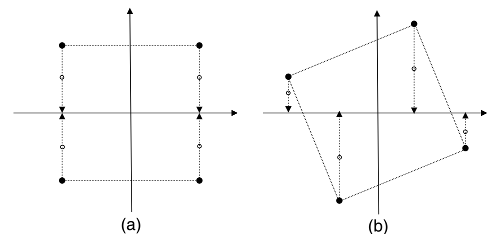


Fig. 2. Effect of signal space diversity on 4-PSK. (a) Without signal space diversity, $\delta_{\min}(\underline{\mathcal{A}})$ = 1. (b) With signal space diversity, $\delta_{\min}(\underline{\mathcal{A}})$ = 2

where $N_o/2$ is the power spectral density of additive white Gaussian noise, $\alpha_l = \mathrm{E}[|h_l|^2]$ is the channel correlation and λ_e are the eigenvalues of $\mathbf{A}_e$. It can be seen from (10) that the PEP depends on the following two factors:

- *Diversity gain (G_d):* Roughly speaking, the diversity gain represents the slope of the PEP curve especially at high SNR. It is related to the rank of $\mathbf{A}_e$ [7].
- *Coding gain (G_c):* The coding gain controls the shift in the PEP curve and depends on the product of eigenvalues $\{\lambda_{e,l}\}_{l=1}^{L_h}$ of $\mathbf{A}_e$ such that $G_c = \left(\prod_{l=1}^{G_d}\lambda_{e,l}\right)^{1/G_d}$

It was shown in [3] that the rank of $\mathbf{A}_e$ is related to the minimum Hamming distance of the codewords. If $\underline{\mathcal{A}}$ is the set of codewords such that $\mathbf{Ab}, \mathbf{Ab}' \in \underline{\mathcal{A}}$ then the Hamming distance $\delta(\mathbf{Ab}, \mathbf{Ab}')$ between these codewords is the number of non-zero entries in $\underline{\mathbf{A}}(\mathbf{b}-\mathbf{b}')$. The minimum Hamming distance of the codeset $\underline{\mathcal{A}}$ is defined as: $\delta_{\min}(\underline{\mathcal{A}}) = \min\{\delta(\mathbf{Ab}, \mathbf{Ab}')|\mathbf{Ab}, \mathbf{Ab}' \in \underline{\mathcal{A}}\}$

The second parameter that controls the shift in the PEP curve is the coding gain. However, it is obvious from (10) that since G_d appears as exponent it can affect the system performance more than G_c.

4. NON-REDUNDANT CODING IN OFDM

It follows from the above discussion that Hamming distance plays a major role in determining the system performance. While saving the bandwidth, the system performance can still be improved by using non-redundant coding with $\mathbf{A}$ (or $\underline{\mathbf{A}}$) $\in \mathbb{C}^{N\times N}$, *i.e.*, unity code rate. An example of non-redundant coding is *signal space diversity* where the original signal constellation is mapped to a lattice constellation of larger Hamming distance. The name signal space diversity reminds us that it is the choice of signal space that increases the diversity. A simpler way to achieve this is by choosing $\underline{\mathbf{A}}$ as a rotation matrix that can rotate the signal constellation to increase the Hamming distance [4]. The design of rotational matrices for signal space diversity is discussed in [5]. Fig. 2 illustrates the application of signal space diversity where the Hamming distance of 4-PSK is increased from 1 to 2 by selecting $\underline{\mathbf{A}}$ as

$$\underline{\mathbf{A}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & e^{j\pi/8} \\ 1 & -e^{j\pi/8} \end{bmatrix}. \tag{11}$$

In this paper, we apply signal space diversity to an OFDM system by selecting $\underline{\mathbf{A}}$ as a rotational matrix and evaluate system performance. Because of non-redundant coding, we apply this rotational matrix in the form of precoding. Due to the increased Hamming distance, the application of signal space diversity helps improve the performance of the OFDM system without sacrificing the bandwidth efficiency. This assertion will be confirmed through simulations.

5. REDUNDANT CODING IN OFDM

For redundant coding, we are interested in the encoder design such that $\mathbf{A}$ (or $\underline{\mathbf{A}}$) $\in \mathbb{C}^{K\times N}$ with $K > N$. While recent research emphasized redundant precoding [8], we explore the use of redundant postcoding in OFDM systems and refer to this system as postcoded-OFDM (PC-OFDM). In postcoding, the redundant encoding is performed after the IFFT that leads to a reduced complexity IFFT in the transmitter.

5.1. PC-OFDM Encoder Design

The first step in PC-OFDM is to introduce explicit frequency diversity in OFDM symbols that can be fairly easily achieved by upsampling the output of IFFT by L. Since upsampling the signal in time domain creates multiple replicas of the signal in frequency domain, this operation is equivalent of repeating the modulated source symbols prior to IFFT. Since upsampling alone cannot increase the Hamming distance, we therefore multiply the upsampler output with unit magnitude complex number sequence. The block diagram of PC-OFDM system is shown in Fig. 3. This particular design of encoder will render the $NL \times N$ postcoding matrix as:

$$\mathbf{A} = \begin{cases} [\mathbf{A}]_{n,k} = \frac{e^{jn}}{NL} & \text{for } (n,k) = (iL-1, i) \text{ for } i = 1, \cdots, N \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

To gain some insight into PC-OFDM, we find the equivalent precoding matrix of PC-OFDM by using (8). Notice that multiplication operation in Fig. 3(a) corresponds to circular convolution in frequency domain, thus

$$\underline{\mathbf{A}} = \mathbf{C} \oplus \mathbf{L}, \tag{13}$$

where $\mathbf{C}$ and $\mathbf{L}$ are the frequency domain matrices of number sequence $\mathbf{c} = \frac{1}{NL}[e^{j1}, \ldots, e^{jNL}]^T$ and the upsampling operation, respectively. As upsampling by factor L corresponds to L times repetition in frequency domain, the matrix of upsampling operation of order $NL \times N$ is obtained by column-wise concatenation of L identity matrices $\mathbf{I}_N$, *i.e.*

$$\mathbf{L} = \begin{bmatrix} \mathbf{I}_N \\ \vdots \\ \mathbf{I}_N \end{bmatrix}_{NL\times N}.$$

The frequency domain matrix $\mathbf{C}$ can be computed as:

$$\mathbf{C} = \mathbf{F}_{NL}^{\mathcal{H}}\,\mathbf{c}.$$

In matrix form, the circular convolution of (13) can be implemented as the product of circulant matrix formed by $\mathbf{C}^{\mathcal{H}}$ and $\mathbf{L}$. After simplification, the equivalent precoding matrix of PC-OFDM is given by:

$$\underline{\mathbf{A}} = \begin{bmatrix} \mathbf{c}^{\mathcal{H}}\,\mathbf{F}_{NL,1} \\ \vdots \\ \mathbf{c}^{\mathcal{H}}\,\mathbf{F}_{NL,NL} \end{bmatrix} \begin{bmatrix} \mathbf{I}_N \\ \vdots \\ \mathbf{I}_N \end{bmatrix}, \tag{14}$$

where $\mathbf{F}_{NL,i}$ is obtained from the ith (row-wise) circulant shift in $\mathbf{F}_{NL}$. Let us consider an example of PC-OFDM system.

Example 1: Consider the design of PC-OFDM encoder for $N = 2$ and $L = 2$. The postcoding and equivalent precoding matrices are:

$$\mathbf{A} = \frac{1}{4}\begin{bmatrix} e^{j1} & 0 \\ 0 & 0 \\ 0 & e^{j3} \\ 0 & 0 \end{bmatrix}, \underline{\mathbf{A}} = \frac{1}{4}\begin{bmatrix} e^{j1}+e^{j3} & e^{j1}-e^{j3} \\ e^{j1}-e^{j3} & e^{j1}+e^{j3} \\ e^{j1}+e^{j3} & e^{j1}-e^{j3} \\ e^{j1}-e^{j3} & e^{j1}+e^{j3} \end{bmatrix}.$$

This bears a close resemblance with rotation matrix of signal space diversity codes (cf. (11)). Thus in a sense, the PC-OFDM system does perform signal constellation rotation through multiplication with unit amplitude phasors and improves the system performance over fading channels. Furthermore, PC-OFDM also increases the Hamming distance of the codeset as stated in the following proposition:

Proposition 1. *The PC-OFDM system achieves the maximum available diversity gain.*

Proof: We omit the proof due to limited space.

5.2. PC-OFDM Decoder

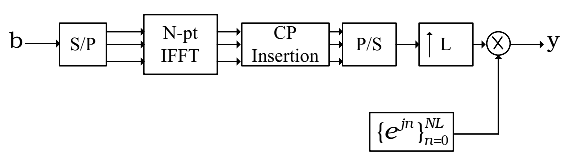
We find that the concept of equivalent precoding matrix of PC-OFDM facilitates the decoder design as well. With the equivalent precoding matrix $\underline{\mathbf{A}}$ as defined in (8) and the assumption that the receiver has channel information, the ML decoder is given by

$$\hat{\mathbf{b}} = \min_{\mathbf{b}_i} ||\mathbf{u} - \mathbf{H}_D \underline{\mathbf{A}} \mathbf{b}_i||.$$

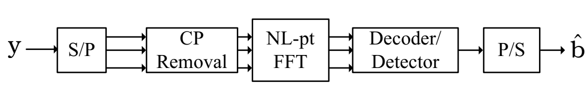
ML detection algorithm is computationally extensive but provides the best performance. Other choices of suboptimum detectors include linear detectors like zero forcing and minimum mean square error detectors [3].

5.3. Complexity Comparison with Precoded OFDM Systems

It is obvious from Fig. 1 that PC-OFDM systems result in less complexity IFFT modules. For instance, a PC-OFDM transmitter with N source symbols requires an N-point IFFT module with computational complexity of $\mathcal{O}(N\log N)$ per N data symbols or simply $\mathcal{O}(\log N)$ per data symbol. In contrast, a redundant precoded



(a) PC-OFDM transmitter



(b) PC-OFDM receiver

Fig. 3. PC-OFDM transceiver block diagram

OFDM transmitter [3] with $NL \times N$ (where $L \in \mathbb{R}$ and $L \geq 1$) encoding has a computational complexity of $\mathcal{O}(L \log NL)$ per data symbol. This reduced complexity of PC-OFDM transmitter makes it suitable for wireless personal area networks. However, receivers of both systems employ equal complexity FFT modules for same code rate $(1/L)$. It is important to mention that the IFFT operation in PC-OFDM transmitter is performed at information symbol data rate, however, in precoded OFDM this operation is performed after encoding and at higher sampling rate. Thus, PC-OFDM saves power by computing the IFFT operation at lower rate.

In addition to the savings in FFT modules, the symmetrical structure of coding matrices in PC-OFDM can be exploited to further reduce the system complexity. As evident from the sparse nature of $\mathbf{A}$ in (12), the postcoding in the transmitter is a low cost operation and requires only $\mathcal{O}(N)$ complex multiplications. Similarly, the decoding matrix needed at the receiver is block repetitive matrix (cf. 14), *i.e.*,

$$\underline{\mathbf{A}} = [\underbrace{\underline{\mathbf{A}}_1^T \quad \cdots \quad \underline{\mathbf{A}}_1^T}_{\text{repeats } L \text{ times}}],$$

where $\underline{\mathbf{A}}_1^T$ is the $N \times N$ block of $\underline{\mathbf{A}}$. Based on these observations, we compared in Table 1 the number of complex multiplications and additions needed in PC-OFDM and precoded OFDM systems [3].

Table 1. Number of complex multiplications and additions

	Transmitter	Receiver
PC-OFDM System	$\mathcal{O}(N)$	$\mathcal{O}(2N^2)$
Pre-coded OFDM	$\mathcal{O}(2N^2L)$	$\mathcal{O}(2N^2L)$

6. SIMULATION RESULTS

We perform simulations to compare the bit error rate (BER) of coded and uncoded OFDM systems as shown in Fig. 4. The information symbols are QPSK modulated to yield $\mathcal{B} = \{\pm 1 \pm j\}$. The simulations are performed over Rayleigh fading channel with five taps that are generated according to the Jakes model. For coded OFDM, we consider different choices of encoder matrix $\mathbf{A}$ (or $\underline{\mathbf{A}}$) discussed in this paper. For non-redundant coding, we consider $N = 2$ and the rotation matrix as given in (11). The results in Fig. 4 show that OFDM with signal space diversity (non-redundant OFDM) can achieve the same BER as that of uncoded OFDM but with 2.5dB less signal to noise ratio (SNR). For redundant coding, we consider the novel PC-OFDM with $N = 2$ and $L = 2$ that results in code rate of $1/2$ and gives the best performance as shown in Fig 4. We also obtained the BER performance of pulsed-OFDM [9] with $N = L = 2$ and the results are shown in Fig. 4. The slope of the curve shows that pulsed-OFDM could not achieve the full diversity order available in the system. To compare with precoded OFDM systems, we employed the real (referred as precoded OFDM-a) and complex (referred as precoded OFDM-b) precoders proposed in [3] and showed their BER results in Fig. 4. As seen from Fig. 4, the BER performance depends on the choice of the precoder.

7. CONCLUSIONS

We presented a detailed account of redundant and non-redundant coding in OFDM systems. To increase the Hamming distance of the signal space, we introduced a non-redundant coding in the form of signal space diversity in OFDM systems and showed that it provides

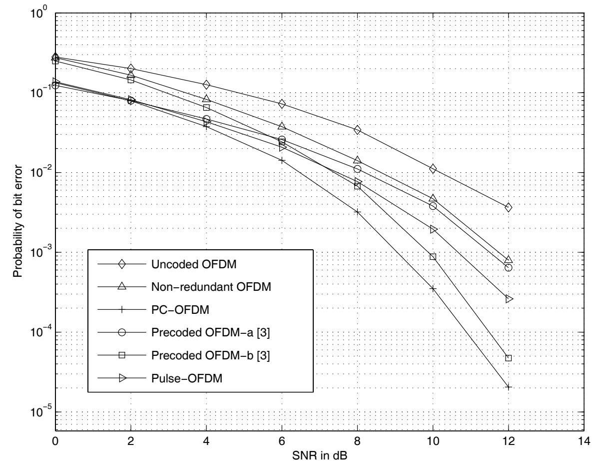


Fig. 4. BER of OFDM system with redundant and non-redundant coding

a gain of 2.5dB in SNR through simulations. The novel PC-OFDM system, discussed in this paper, introduces redundant postcoding in OFDM and is a low complexity system that improves the performance by reducing bandwidth efficiency.

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