CO-ORDINATE INTERLEAVED SPATIAL MULTIPLEXING WITH CHANNEL KNOWLEDGE AT TRANSMITTER AND RECEIVER

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ABSTRACT

Spatial multiplexing (SM) over multiple-input multiple-output wireless channels provides significant capacity gains. In a SM scheme, the eigenmode having the least signal-to-noise ratio (SNR), degrades the overall error rate performance. In this paper, we propose co-ordinate interleaved spatial multiplexing that maximizes the minimum SNR over all eigenmodes. This linearly decodable SM scheme needs the knowledge of the right singular vectors of the channel at the transmitter, and the singular values and left singular vectors at the receiver. We derive the SNR expressions for the proposed scheme and compare its performance with other closed-loop schemes using computer simulations.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless systems, employing multiple transmit and receive antennas at both ends, promise significant improvement in the capacity compared to conventional single-input single-output (SISO) systems [1, 2]. In MIMO spatial multiplexing (SM) systems, the incoming data is divided into multiple sub-streams and each stream is transmitted on a different antenna. If successfully decoded, this increases the capacity nearly linearly with the number of transmit antennas [1, 2]. Initially, SM systems were developed assuming channel state information (CSI) only at the receiver, e.g., Bell labs layered space time (BLAST) scheme [3]. However, it has been shown recently that, having CSI both at transmitter and receiver can significantly improve the error rate performance compared to that of having CSI only at the receiver. CSI can be obtained at the transmitter by utilizing channel symmetry in time division duplex (TDD) systems, and by having an explicit feedback link from receiver to transmitter in non-TDD systems.

With CSI available at both ends, it is well known that channel diagonalization by singular value decomposition (SVD) and water filling achieves capacity [1]. As described in section 3, water filling has significant implementation difficulties, and this has prompted the researchers to design joint transceiver techniques for SM, that can efficiently exploit CSI at both ends with minimum complexity (see [8], and references therein).

In this paper, we propose a novel SM scheme that utilizes the concept of co-ordinate interleaving (CI) [4] along with SVD of the MIMO channel. The proposed scheme, which we call *co-ordinate interleaved spatial multiplexing* (CISM), achieves better diversity order than existing SM schemes that use CSI at both ends and also enables single symbol decodability at the receiver.

2. SYSTEM MODEL

We consider a $N_t \times N_r$ MIMO system with N_t transmit and N_r receive antennas, $N_r \geq N_t$. Let $\mathbf{x} = [x_1\ x_2 \ldots x_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ be the transmit symbol vector and tr$\{E\left[\mathbf{x}\mathbf{x}^H\right]\} = 1$. Further, assume that $x_i \in \mathcal{A}, \forall i$, where $\mathcal{A}$ is a complex signal set with cardinality $|\mathcal{A}| = M$. Note that superscripts T and H denote transpose and conjugate transpose, respectively. E denotes expectation operator and I_N denotes $N \times N$ identity matrix. Discrete time baseband input-output relation of the MIMO channel is,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where, $\mathbf{y} = [y_1\ y_2 \ldots y_{N_r}]^T \in \mathbb{C}^{N_r \times 1}$ is the received signal vector and $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix with h_{ij} representing the channel gain from j^{th} transmit antenna to i^{th} receive antenna. Further, h_{ij} are i.i.d circularly symmetric complex Gaussian with zero mean and unit variance, i.e., $h_{ij} \sim \mathcal{CN}(0, 1)$. Throughout the paper, we assume that $\mathbf{H}$ is a full rank matrix, i.e.. $K = rank(\mathbf{H}) = \min\{N_t, N_r\}$. Also, $\mathbf{n} = [n_1\ n_2 \ldots n_{N_r}]^T$ is the additive noise vector with $n_i \sim \mathcal{CN}(0, \sigma_n^2)$, and $E[\mathbf{n}\mathbf{n}^{\mathbf{H}}] = \sigma_n^2 I_{N_r}$.

With perfect CSI at the receiver, optimum error rate performance can be achieved by maximum likelihood (ML) receiver. ML receiver achieves N_r^{th} order diversity but with complexity $\mathcal{O}(M^{N_t})$ which becomes prohibitive to implement even for moderate values of M and N_t [5] .

The first two authors acknowledge the support extended by the Center of Excellence in Wireless Technology (CEWiT) for this work.

Successive interference cancellation (SIC) receiver is a low complexity approach that achieves a diversity order of $N_r - N_t + 1$ [5, 6]. It decodes the symbols sequentially by using *successive nulling and cancellation* technique. $\mathbf{H}$ is decomposed as $\mathbf{H} = \mathbf{QR}$ using standard QR decomposition, where $\mathbf{Q}$ is $N_r \times N_t$ unitary matrix and $\mathbf{R}$ is $N_t \times N_t$ upper triangular matrix [7]. Left-multiplying $\mathbf{y}$ with $\mathbf{Q}^H$ yields,

$$\mathbf{z} = \mathbf{Rx} + \mathbf{w} \tag{2}$$

where $\mathbf{z} = \mathbf{Q}^H\mathbf{y}$ and $\mathbf{w} = \mathbf{Q}^H\mathbf{n}$. The estimates $\hat{x}_{N_t}, \ldots, \hat{x}_1$ are obtained by (successive) back-substitution. If $\hat{x}_n$ is in error, then all the subsequent symbol estimates in that block can be in error. Ordered SIC (OSIC) improves SIC by optimally ordering $\mathbf{y}$. OSIC receiver realizes a diversity order that is greater than $N_r - N_t + 1$ but less than N_r [5].

Ignoring error propagation, we can observe that SIC receiver decomposes the MIMO channel into N_t parallel SISO channels [8], $z_i = r_{ii}x_i + w_i, \quad i = 1, 2, \ldots, N_t$, with i^{th} sub-channel SNR given by,

$$SNR_{i,sic} = r_{ii}^2 \frac{E[|x_i|^2]}{\sigma_n^2}, \quad i = 1, 2, \ldots, K \tag{3}$$

It is important to note that overall error rate performance of SIC/OSIC receiver is limited by the error rate of the sub-channel having lowest SNR.

3. CSI AVAILABLE AT BOTH TRANSMITTER AND RECEIVER

With CSI at both both ends of the link, the MIMO channel can be diagonalized using SVD. Let the SVD of $\mathbf{H}$ be given by $\mathbf{H} = \mathbf{UDV}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{D} = diag\left(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_K},\ 0, \ldots, 0\right)$. Here, $\sqrt{\lambda_i}$ is the i^{th} singular value of $\mathbf{H}$ and $\sqrt{\lambda_1} \geq \sqrt{\lambda_2} \geq \ldots \geq \sqrt{\lambda_K}$ [7]. To diagonalize the channel, we transmit $\mathbf{s} = \mathbf{Vx}$ and left-multiply the received vector $\mathbf{y}$ by $\mathbf{U}^H$, to get $\mathbf{z} = \mathbf{Dx} + \mathbf{w}$, where $\mathbf{z} = \mathbf{U^H y}$ and $\mathbf{w} = \mathbf{U}^H\mathbf{n}$. Since $\mathbf{D}$ is a diagonal matrix,

$$z_i = \sqrt{\lambda_i}x_i + w_i, \qquad i = 1, 2, \ldots, K \tag{4}$$

Thus the rank K MIMO channel matrix is decomposed into K non-interfering parallel SISO sub-channels which are also referred to as eigen sub-channels or eigen modes of $\mathbf{H}$. The SNR of the i^{th} sub-channel is given by,

$$SNR_{i,svd} = \lambda_i \frac{E[|x_i|^2]}{\sigma_n^2} \qquad i = 1, 2, \ldots, K \tag{5}$$

which is very similar to the expression for SNR in (3) for the SIC receiver. For a typical MIMO channel $\mathbf{H}$, $\frac{\lambda_{max}}{\lambda_{min}} \gg 1$ which means that λ_i's differ in magnitude by large amounts. As in the SIC receiver, the overall error rate of this SVD based receiver is degraded by the sub-channel having lowest SNR. Information theoretic results shows that the capacity of the MIMO channel can be achieved by water filling on these unequal eigen sub-channels [1]. Water filling may result in widely varying modulation and coding schemes across the sub-channels which requires significant additional information at the receiver so as to enable it to identify and decode the sub-streams correctly. Also, optimum water filling requires non-integer number of bits to be allotted to some of the sub-channels which is not possible with standard signal constellations. These practical difficulties have made it necessary to design new transceiver techniques with CSI at both ends.

3.1. Geometric Mean Decomposition (GMD) Transceiver

With the observation that the overall error rate of SIC receiver is limited by the lowest SNR sub-channel, which in-turn depends on $\min_i\{r_{ii}\},\ i = 1, 2, \ldots, N_t$, Jiang *et al.* [8] have proposed a precoding scheme that maximizes the $\min_i\{r_{ii}\}$. The precoding scheme is based on geometric mean decomposition (GMD) of $\mathbf{H}$ developed by the same authors [8]. GMD of any matrix $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ with rank K decomposes $\mathbf{H}$ into $\check{\mathbf{Q}}\check{\mathbf{R}}\mathbf{P}^H$, where $\check{\mathbf{Q}}$ and $\mathbf{P}$ are unitary and $\check{\mathbf{R}}$ is a $K \times K$ upper triangular matrix with its diagonal elements equal to the *geometric mean* of the non-zero singular values of $\mathbf{H}$; i.e.,

$$\check{r}_{ii} = \left(\prod_{j=1}^{K} \sqrt{\lambda_j}\right)^{1/K}, \qquad i = 1,\ 2,\ \ldots,\ K \tag{6}$$

With the knowledge of $\mathbf{D}$ and $\mathbf{V}$ at the transmitter, $\mathbf{P}$ is calculated through GMD algorithm. Transmitting the precoded vector $\mathbf{s} = \mathbf{Px}$ and left multiplying the received signal $\mathbf{y}$ with $\check{\mathbf{Q}}^H$ results in $\mathbf{z} = \check{\mathbf{R}}\mathbf{x} + \mathbf{w}$, where $\mathbf{z} = \check{\mathbf{Q}}^H\mathbf{y}$ and $\mathbf{w} = \check{\mathbf{Q}}^H\mathbf{n}$. $\hat{x}_{N_t}, \ldots, \hat{x}_1$ are obtained by (successive) back-substitution. Ignoring error propagation, we can obtain $SNR_{i,gmd}$, SNR of the i^{th} sub-channel in GMD receiver, as

$$SNR_{i,gmd} = \check{r}_{ii}^2 \frac{E[|x_i|^2]}{\sigma_n^2} \qquad i = 1, 2, \ldots, K \tag{7}$$

4. CO-ORDINATE INTERLEAVED SPATIAL MULTIPLEXING (CISM)

We note that error rate performance of the SVD based receiver can be improved by maximizing $\min_i\{SNR_{i,svd}\}, i = 1, 2, \ldots, K$. We use co-ordinate interleaving to achieve this.

4.1. Co-ordinate Interleaving

The concept of co-ordinate interleaving is to interleave real and imaginary parts of the complex symbols to be transmitted over independent fading channels so that they encounter different fading gains. At the receiver, they are de-interleaved before decoding. Detailed discussion of CI has been given in [4], and the references therein, which shows that the performance of CI technique depends on co-ordinate product distance (CPD) of the signal set $\mathcal{A}$. Co-ordinate product distance of $\mathcal{A}$ is defined as,

$$CPD(\mathcal{A}) = \min_{u \neq v \in \mathcal{A}} |\text{Re}\{u\} - \text{Re}\{v\}||\text{Im}\{u\} - \text{Re}\{v\}| \quad (8)$$

where Re$\{x\}$ and Im$\{x\}$ denote real and imaginary part of x, respectively. CI achieves full diversity iff $CPD(\mathcal{A}) > 0$. This means that no two signal points in $\mathcal{A}$ can lie on a horizontal or vertical line in the complex plane. This requires standard constellations like M-QAM and M-PSK to be rotated to make their $CPD > 0$. Each constellation has an optimum angle of rotation θ_{opt} that maximizes its CPD and for square lattice constellations $\theta_{opt} = 31.7175^0$ [4]. In the following discussion, we assume that the signal set $\mathcal{A}$ that we use has maximum CPD.

4.2. CISM Algorithm

With CSI at both ends, we apply co-ordinate interleaving across the symbols transmitted on different eigenmodes. Let $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ be the symbol vector to be transmitted, with $x_i \in \mathcal{A}$, and let SVD$(\mathbf{H}) = \mathbf{UDV}^H$.

- Interleave real and imaginary parts of x_i to get $\tilde{x}_i$, where,

$$\tilde{x}_i = \text{Re}\{x_i\} + j\text{Im}\{x_{N_t-(i-1)}\}, \ i = 1, \ldots, N_t \quad (9)$$

- Transmit $\mathbf{s} = \mathbf{V}\tilde{\mathbf{x}}$, where $\tilde{\mathbf{x}} = [\tilde{x}_1, \ \tilde{x}_2, \ldots, \tilde{x}_{N_t}]^T$
- Receive $\tilde{\mathbf{y}} = \mathbf{Hs} + \mathbf{n} = \mathbf{UD}\tilde{\mathbf{x}} + \mathbf{n}$.
- Left-multiply $\tilde{\mathbf{y}}$ with $\mathbf{U}^H$ to get $\tilde{\mathbf{z}} = \mathbf{D}\tilde{\mathbf{x}} + \mathbf{w}$ where $\tilde{\mathbf{z}} = \mathbf{U}^H\tilde{\mathbf{y}}$ and $\mathbf{w} = \mathbf{U}^H\mathbf{n}$. This results in,

$$\tilde{z}_i = \sqrt{\lambda_i}\tilde{x}_i + w_i, \ i = 1, \ldots, N_t \quad (10)$$

- De-interleave $\tilde{z}_i$:

$$z_i = \text{Re}\{\tilde{z}_i\} + j\text{Im}\{\tilde{z}_{N_t-(i-1)}\}, \ i = 1, \ldots, N_t \quad (11)$$

  Substituting (10) and (9) in (11),

$$z_i = \sqrt{\lambda_i}\text{Re}\{x_i\} + j\sqrt{\lambda_{N_t-(i-1)}}\text{Im}\{x_i\} + \text{Re}\{w_i\} + j\,\text{Im}\{w_{N_t-(i-1)}\} \quad (12)$$

- Estimate x_i :

$$\hat{x}_i = \arg\min_{x_i \in \mathcal{A}} \left| z_i - \left(\sqrt{\lambda_i}\text{Re}\{x_i\} + j\sqrt{\lambda_{N_t-(i-1)}}\text{Im}\{x_i\}\right)\right|^2 \quad (13)$$

As can be seen from (10), the received signals gets decoupled and hence can be decoded by single-symbol ML decoding rule given by (13).

In-order to assess the performance of the proposed scheme, we calculate SNR of each sub-stream. Let $z_{i,s} = \sqrt{\lambda_i}\text{Re}\{x_i\} + j\sqrt{\lambda_{N_t-(i-1)}}\text{Im}\{x_i\}$ denotes the signal component of z_i and $z_{i,n} = \text{Re}\{w_i\} + j\,\text{Im}\{w_{N_t-(i-1)}\}$ denotes the noise component. Average signal power in z_i is

$$E[|z_{i,s}|^2] = \lambda_i E[(\text{Re}\{x_i\})^2] + \lambda_{N_t-(i-1)} E[(\text{Im}\{x_i\})^2] \quad (14)$$

Assuming symmetric signal constellation, $E[(\text{Re}\{x_i\})^2] = E[(\text{Im}\{x_i\})^2] = E[|x_i|^2]/2$. This implies,

$$E[|z_{i,s}|^2] = \frac{\lambda_i + \lambda_{N_t-(i-1)} E[|x_i|^2]}{2} \quad (15)$$

and the noise power in z_i is

$$E[|z_{i,n}|^2] = E[(\text{Re}\{w_i\})^2] + E[\left(\text{Im}\{w_{N_t-(i-1)}\}\right)^2] = \frac{\sigma_n^2}{2} + \frac{\sigma_n^2}{2} = \sigma_n^2 \quad (16)$$

Finally, from (15) and (16), we obtain,

$$SNR_{i,cism} = \frac{E[|z_{i,s}|^2]}{E[|z_{i,n}|^2]} = \frac{\lambda_i + \lambda_{N_t-(i-1)}}{2} \frac{E[|x_i|^2]}{\sigma_n^2} \quad (17)$$

Note that CISM needs only to know $\mathbf{V}$ at the transmitter. We can further reduce the amount of feedback by using efficient quantization/encoding schemes for unitary matrices [9].

5. SIMULATION RESULTS

In this section, we evaluate the error rate performance of the proposed scheme through simulations, using the system model given by (1). $\mathbf{H}$ is assumed to be constant over a block length of N_t symbols and varies independently from block to block. We use 4-QAM rotated by $\theta_{opt} = 31.7175^0$ as the signaling constellation. SNR's for SVD, GMD and CISM schemes are computed using (5), (7) and (17), respectively, ignoring the $\frac{E[|x_i|^2]}{\sigma_n^2}$ term which is common in all the three expressions.

Fig. 1 compares the symbol error rate (SER) performance of CISM, open-loop ML and GMD transceiver for a 2x2 SM system. Slope of the SER curve of ML receiver corresponds to second order diversity. Performance of the GMD transceiver, which is close to that of ML receiver, also shows a diversity order of two. As $\lambda_1 > \lambda_2$, second sub-channel in SVD transceiver has lowest SNR i.e., $SNR_{min,svd} = \lambda_2$. From (7) and (17) we can compute that, $SNR_{i,gmd} = \sqrt{\lambda_1\lambda_2}$, $i = 1, 2$, and $SNR_{1,cism} = SNR_{2,cism} = \frac{\lambda_1+\lambda_2}{2}$. It is easy to show that $\lambda_2 \ll \sqrt{\lambda_1\lambda_2} < \frac{\lambda_1+\lambda_2}{2}$. Hence CISM performs significantly better than the other two schemes. Since $\lambda_1 + \lambda_2 = \text{tr}\{H^H H\} = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2$ [10], CISM for 2×2 system achieves fourth order diversity.

Next we compare SER performance of the above mentioned schemes for a 3×3 system. Note that co-ordinate interleaving on 3×3 system interleaves x_1 with x_3 and transmits x_2 without any change. From Fig. 2, we can observe that CISM outperforms other schemes and slope of the curve,

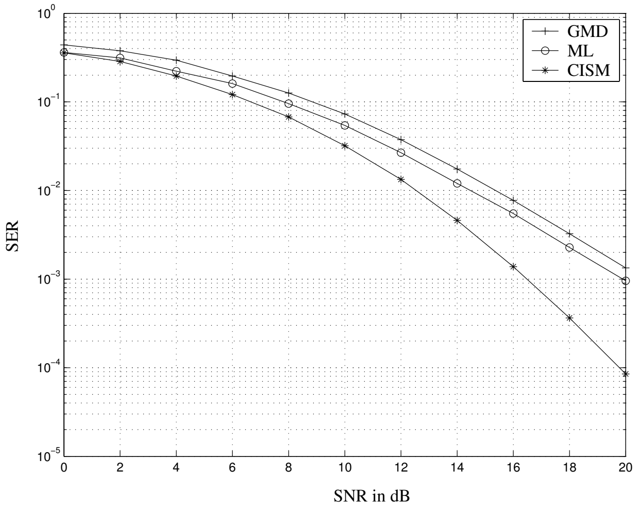

Fig. 1. SER comparison for 2×2 system with rotated 4-QAM

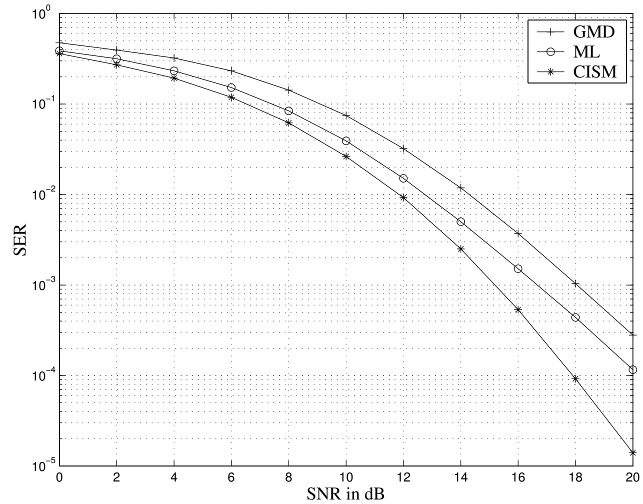

Fig. 2. SER comparison for 3×3 system with rotated 4-QAM

when compared to slope of the ML performance, indicates that CISM achieves diversity order greater than three.

An increase in $K(= rank(\mathbf{H}))$ will generally see a decrease in the least singular value $\sqrt{\lambda_K}$. Hence, $SNR_{i,gmd}, \forall i$, which is proportional to the geometric mean of $\sqrt{\lambda_i}$'s (see (6), (7)), decreases for MIMO channels of larger dimension. Through simulations, by averaging over 10^5 channel realizations, we have observed that $SNR_{min,cism}$ is greater than $SNR_{min,gmd}$ for all K, implying that CISM outperforms GMD transceiver over MIMO channels for any $N_t \times N_r$.

6. CONCLUSIONS

We have proposed a new spatial multiplexing scheme that applies co-ordinate interleaving across the symbols transmitted on different eigenmodes of the MIMO channel. The proposed co-ordinate interleaved spatial multiplexing scheme requires the knowledge of the right singular vectors of the channel at the transmitter while it needs to know singular values and left singular vectors at the receiver. It was shown by deriving SNR expressions that co-ordinate interleaving reduces the SNR imbalance across different eigenmodes significantly. In CISM, the minimum SNR over all streams is always higher than that of other closed-loop schemes, resulting in an improved error rate performance. Received symbol streams in CISM gets decoupled, enabling single-symbol ML decoding. Currently we are investigating the usefulness of this technique for correlated and/or Rician channels.

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