LARGE-SYSTEM COMPARISON OF SPACE-TIME TRANSMIT DIVERSITY AND ORTHOGONAL TRANSMIT DIVERSITY FOR DOWNLINK W-CDMA

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ABSTRACT

In this paper, we continue our study of the performance of W-CDMA Transmit Diversity (TD) that we started in [4]. We compare the performance of two open-loop TD techniques for W-CDMA: The Space Time Transmit Diversity (STTD) and the Orthogonal Transmit Diversity (OTD). We use the output SINR provided by a RAKE receiver when combined with each of these diversity techniques. It is impossible to obtain useful conclusions about the performance of the two techniques because the SINR depend in a complex way on many parameters like the spreading codes, the channel, etc. In order to obtain positive results, we follow the classical approach used for the first time in [5] and assume that the spreading codes are random following a certain distribution. In this case, the SINRs can be interpreted as random variables. We consider the limit of the these SINRs when the spreading factor N and the number of users K tend to ∞ with fixed ratio. Under these conditions, the output SINRs can be shown to converge to deterministic limits independent of the "random" spreading codes. We interpret the output SINRs and discuss the different parameters influencing the performance of the two techniques.

1. INTRODUCTION

Third Generation (3G) mobile communication systems such as cdma2000 and W-CDMA are intended to provide higher data rates than current 2G systems. The performance of these systems depend heavily on their ability to combat channel fading. Recently, antenna diversity has proved to be one of the most effective ways to combat channel fading. Multiple antennas at the receiver can be used to provide diversity. The dilemma is that, in the downlink, multiple antennas at the receiver induces an increase in the size of the mobile unit, while significant effort is being done to make wireless mobile devices smaller and cheaper. Alamouti [1] has shown that the diversity provided by using two transmit antennas and one receive antenna is the same as that provided by one transmit antenna and two receive antennas. However, this result is valid for flat fading channels only. The Alamouti scheme allows to provide diversity without the need to include multiple antennas at the receiver side.

There are two popular techniques of (open-loop) Transmit Diversity that have been adopted in W-CDMA and cdma2000 [2, 6]. The first one is the Space Time Transmit Diversity (STTD) based on the Alamouti Space Time Block Code(STBC) that is applied to the transmitted symbols after spreading. The second one is the Orthogonal Transmit Diversity (OTD) where two successive symbols are transmitted simultaneously from two different antennas using two orthogonal spreading codes derived from the spreading code of the single antenna case. By looking at the output SINR in each case, it is very difficult to compare the two techniques and draw insights onto the parameters that affect the performance of each one.

In this paper, we consider the performance of STTD and OTD in the downlink of W-CDMA. We follow the classical approach used for the first time in [5]. We assume that the spreading factor N and the number of users K tend to $+\infty$ at the same rate. The spreading codes are supposed to coincide with Walsh Hadamard codes scrambled by an Independent Identically Distributed (i.i.d) sequence. In this context, the SINRs of the two receiver tend to deterministic limits independent of the scrambling and the spreading codes. We derive the asymptotic SINRs, compare them and draw some conclusions.

Notations: Throughout the paper, we denote by $\mathbf{A}^H$ and $\mathbf{A}^T$ the conjugate and the transpose of $\mathbf{A}$ respectively. $\overline{\mathbf{A}}$ denotes $(\mathbf{A}^H)^T$. $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$.

2. OPEN LOOP TRANSMIT DIVERSITY FOR W-CDMA

2.1. System Model

We consider a CDMA system with K users and a spreading factor N. Let $b_k(m),\ k = 1...K$, denote the transmitted symbol of user k at time instant m. $s(n)$ denotes the scrambling sequence and $d(n)$ the chip sequence. The transmitted chip vector in one symbol period $\mathbf{d}(m) = \left[d(mN), d(mN+1), ..., d(mN+N-1)\right]^T$ is given by:

$$\mathbf{d}(m) = \mathbf{S}(m)\mathbf{C}\mathbf{b}(m) \tag{1}$$

where $\mathbf{S}(m)$ is the $N \times N$ diagonal matrix whose diagonal elements are $s(mN), s(mN+1), ..., s(mN+N-1)$ and $\mathbf{C}$ is a $N \times K$ matrix whose columns are the spreading codes assigned to different users and $\mathbf{b}(m) = [b_1(m), ..., b_K(m)]^T$.
For both STTD and OTD , the sum chip signal (1) is transmitted through two multipath frequency-selective fading channels whose

impulse responses are given by

$$h_i(t) = \sum_{q=0}^{P-1} \lambda_q^i p(t - \tau_q^i) \qquad i = (1,2) \tag{2}$$

where $p(t)$ is the total shaping filter (including the transmitter and the receiver matched filters), λ_q^i and τ_q^i are the complex gain and the delay associated with path q of the channel between transmit antenna $i = (1,2)$ and the receiver, and P is the total number of resolvable paths. For the sake of simplicity we suppose that the number of resolvable paths is the same for both channels.

2.2. Alamouti-based STTD

STTD uses a symbol-level Alamouti STBC. This is equivalent to transmitting the chip vectors defined by equation 1 according to Table. 1.

time Antenna	$m-2$	$m-1$	m	$m+1$
1	$\mathbf{d}(m-2)$	$\mathbf{d}(m-1)$	$\mathbf{d}(m)$	$\mathbf{d}(m+1)$
2	$\mathbf{d}^*(m-1)$	$-\mathbf{d}^*(m-2)$	$\mathbf{d}^*(m+1)$	$-\mathbf{d}^*(m)$

Table 1. The Alamouti STBC for W-CDMA

If we call the chips transmitted from antenna 1 $d_1(n)$ and the chips transmitted from antenna 2 $d_2(n)$ then the chip-rate sampled received signal is given by:

$$x(n) = \sum_{l=1}^{L-1} h_{1,l} d_1(n-l) + \sum_{l=1}^{L-1} h_{2,l} d_2(n-l) + v(n) \tag{3}$$

where $h_{i,l} \triangleq h_i(t)|_{t=lT_c}$, L is the overall channel length (in chip periods) and $v(n)$ is a centered white gaussian noise process with variance σ^2.
It is more convenient to express the model (3) in matrix form. By concatenating the received signal in $2N$ chips we get:

$$\begin{bmatrix} \mathbf{x}(m) \\ \overline{\mathbf{x}}(m+1) \end{bmatrix} = \mathcal{H}_0^{STTD} \begin{bmatrix} \mathbf{d}(m) \\ \overline{\mathbf{d}}(m+1) \end{bmatrix} + \text{ISI} + \begin{bmatrix} \mathbf{v}(m) \\ \overline{\mathbf{v}}(m+1) \end{bmatrix} \tag{4}$$

where $\mathcal{H}_0^{STTD} = \begin{bmatrix} \mathbf{H}_{1,0} & \mathbf{H}_{2,0} \\ -\overline{\mathbf{H}}_{2,0} & \overline{\mathbf{H}}_{1,0} \end{bmatrix}$ and $\mathbf{H}_{i,0}$ is the $N \times N$ classical band limited Toeplitz filtering matrix associated with $h_i(z)$ $(i = 1, 2)$. $\mathbf{x}(m)$ and $\mathbf{v}(m)$ are defined from $x(n)$ and $v(n)$ as $\mathbf{d}(m)$ from $d(n)$.

2.3. Orthogonal Transmit Diversity

In orthogonal transmit diversity, two successive symbols are transmitted from different antennas using two orthogonal Walsh-Hadamard codes. More precisely, if symbols $b(m)$ and $b(m+1)$ are to be transmitted using a length N spreading code $\mathbf{c}_1$. Then, in the OTD scheme, the symbol $b(m)$ is transmitted from antenna 1 using the $2N$ spreading code $[\mathbf{c}_1\ \mathbf{c}_1]$ while symbol $b(m+1)$ is transmitted from antenna 2 using the $2N$ spreading code $[\mathbf{c}_1 - \mathbf{c}_1]$. Note that the two $2N$ spreading codes are orthogonal whence the name Orthogonal Transmit Diversity. The chips transmitted from the two antennas in the case of OTD are shown in Table 2

time Antenna	$m-2$	$m-1$	m	$m+1$
1	$\mathbf{d}(m-2)$	$\mathbf{d}(m-2)$	$\mathbf{d}(m)$	$\mathbf{d}(m)$
2	$\mathbf{d}(m-1)$	$-\mathbf{d}(m-1)$	$\mathbf{d}(m+1)$	$-\mathbf{d}(m+1)$

Table 2. The Orthogonal Transmit Diversity Code for W-CDMA

The received signal in two symbol periods is given by:

$$\begin{bmatrix} \mathbf{x}(m) \\ \mathbf{x}(m+1) \end{bmatrix} = \mathcal{H}_0^{OTD} \begin{bmatrix} \mathbf{d}(m) \\ \mathbf{d}(m+1) \end{bmatrix} + \text{ISI} + \begin{bmatrix} \mathbf{v}(m) \\ \mathbf{v}(m+1) \end{bmatrix}, \tag{5}$$

where $\mathcal{H}_0^{OTD} = \begin{bmatrix} \mathbf{H}_{1,0} & \mathbf{H}_{2,0} \\ \mathbf{H}_{1,0} & -\mathbf{H}_{2,0} \end{bmatrix}$.

3. ASYMPTOTIC ANALYSIS

As far as the asymptotic SINR is concerned, the ISI term can be dropped in models (4) and (5) (see for example [3]). Furthermore, we can replace the channel matrices $\mathbf{H}_{0,i}$ by $\mathbf{H}_i$ given by

$$\mathbf{H}_i = \begin{bmatrix} h_{i,0} & 0 & h_{i,L-1} & \cdots & h_{i,1} \\ \vdots & h_{i,0} & & \ddots & \vdots \\ h_{i,L-1} & & & & h_{i,L-1} \\ & \ddots & \ddots & & \\ 0 & & h_{i,L-1} & & h_{i,0} \end{bmatrix}$$

It is more convenient to use the following equivalent model:

$$\mathbf{y} = \mathcal{H}\mathcal{C}\mathcal{B} + \mathcal{V} \tag{6}$$

where the quantities are defined as follows:

- In the case of STTD:

$\mathbf{y} = [x(mN{+}1)\ x^*((m{+}1)N{+}1)...x(mN{+}N)\ x^*((m{+}1)N{+}N)]^T$,

$\mathcal{H}$ is a block Toeplitz matrix of the same structure as $\mathbf{H}_i$ whose 2×2 blocks are equal to $\begin{bmatrix} h_{1,l} & h_{2,l} \\ -(h_{2,l})^* & (h_{1,l})^* \end{bmatrix}$,

$$\mathcal{C} = (\mathbf{S}(m)\mathbf{C}) \otimes \mathbf{A}_{1,1} + (\overline{\mathbf{S}}(m+1)\mathbf{C}) \otimes \mathbf{A}_{\mathbf{2},\mathbf{2}}$$

$\mathbf{A}_{i,j}$ stands for a 2 by 2 matrix whose entry (i, j) is equal to 1 and all other entries are equal to zero,

$\mathcal{B} = [b_1(m)\ b_1^*(m{+}1)\ b_2(m)\ b_2^*(m{+}1)...b_K(m)\ b_K^*(m{+}1)]^T$.

- In the case of OTD:

$\mathbf{y} = [x(mN{+}1)\ x((m{+}1)N{+}1)...x(mN{+}N)\ x((m{+}1)N{+}N)]^T$,

$\mathcal{H}$ is a block Toeplitz matrix of the same structure as $\mathbf{H}_i$ whose 2×2 blocks are equal to $\begin{bmatrix} h_{1,l} & h_{2,l} \\ h_{1,l} & -h_{2,l} \end{bmatrix}$,

$$\mathcal{C} = (\mathbf{S}(m)\mathbf{C}) \otimes \mathbf{A}_{1,1} + (\mathbf{S}(m+1)\mathbf{C}) \otimes \mathbf{A}_{\mathbf{2},\mathbf{2}}$$

$\mathcal{B} = [b_1(m)\ b_1(m+1)\ b_2(m)\ b_2(m+1)...b_K(m)\ b_K(m+1)]^T$

For both cases $\mathcal{V}$ has the same structure as $\mathbf{y}$. $\mathcal{C}$ can be interpreted as the overall code matrix. Note we have omitted the time index as it is irrelevant.

We now give the output SINR associated to a RAKE receiver when combined with STTD or OTD. The RAKE receiver is a matched filter matched to the signature of the user of interest. Suppose that we want to retrieve $b_1(m)$, that is the symbol transmitted by user 1 at time instant m from antenna 1. Let $\mathcal{C} = [\mathbf{w}_1 \quad \mathbf{U}]$, where $\mathbf{w}_1$ is the overall code of the user of interest and $\mathbf{U}$ represents the matrix of interferers codes.
The soft estimate of $b_1(m)$ is given by:

$$\tilde{b}_1(m) = \mathbf{w}_1^H \mathcal{H}^H \mathbf{y} \tag{7}$$

The SINR, that we index by the spreading factor, corresponding to this receiver is given by :

$$\beta^{(N)} = \frac{|\mathbf{w}_1^H \mathcal{H}^H \mathcal{H} \mathbf{w}_1|^2}{\mathbf{w}_1^H \mathcal{H}^H (\mathcal{H}\mathbf{U}\mathbf{U}^H \mathcal{H}^H + \sigma^2 \mathbf{I}) \mathcal{H} \mathbf{w}_1} \tag{8}$$

We will call $\beta_{OTD}^{(N)}$ the SINR corresponding to OTD, while $\beta_{STTD}^{(N)}$ stands for the SINR corresponding to STTD. $\mathcal{H}$ is defined above for each kind of transmit diversity.

If we suppose that the scrambling sequence is an i.i.d QPSk symbol (a very realistic assumption), then $\beta_{OTD}^{(N)}$ and $\beta_{STTD}^{(N)}$ can be interpreted as random variables. When N and K tend to ∞ while $\frac{K}{N} \to \alpha$, these random variables can be shown to converge to deterministic limits that we will derive in the sequel.

3.1. Space Time Transmit Diversity

Let us define:

$$|h_1(e^{2i\pi f})|^2 + |h_2(e^{2i\pi f})|^2 = \sum_k R_{hh}(k) e^{-2i\pi f} \tag{9}$$

$$X(e^{2i\pi f}) = \overline{h_1}(e^{-2i\pi f}) h_2(e^{2i\pi f}) - h_2(e^{-2i\pi f}) \overline{h_1}(e^{2i\pi f}) \tag{10}$$

Theorem 1 *Under the assumption that the scrambling sequence is i.i.d with variance* 1, $\lim_{N\to\infty, \frac{K}{N}\to\alpha} \beta_{STTD}^{(N)} \to \beta_{STTD}$ *given by:*

$$\beta_{STTD} = \frac{|R_{hh}(0)|^2}{\alpha \sum_{k\neq 0} |R_{hh}(k)|^2 + \alpha \int_0^1 |X(e^{2i\pi f})|^2 df + \sigma^2 R_{hh}(0)} \tag{11}$$

the convergence stands for the convergence in probability.

3.2. Orthogonal Transmit Diversity

Let us define:

$$|h_1(e^{2i\pi f})|^2 = \sum_k R_h(k) e^{-2i\pi f} \tag{12}$$

Theorem 2 *Under the assumption that the scrambling sequence is i.i.d with variance* 1, $\lim_{N\to\infty, \frac{K}{N}\to\alpha} \beta_{OTD}^{(N)} \to \beta_{OTD}$ *given by:*

$$\beta_{OTD} = \frac{|R_h(0)|^2}{\alpha \sum_{k\neq 0} |R_h(k)|^2 + \frac{\sigma^2}{2} R_h(0)} \tag{13}$$

3.3. Discussion of the two theorems

The two theorems are proved using large-random matrix analysis as in [3, 5]. The proofs are omitted due to the lack of space. The expression of β_{STTD} contains the desired term in the numerator and three undesired terms in the denominator. The third term stems from the effect of noise and will not be discussed. The first undesired term $\alpha\big(\sum_{k\neq 0} |R_{hh}(k)|^2\big)$ is the classical Multi Access Interference (MAI) due to the non-perfect nature of each channel separately. The second undesired term $\alpha \int_0^1 |X(e^{2i\pi f})|^2 df$ is more interesting and can be interpreted as the Cross-Channel Interference (CCI) due to the simultaneous use of two multipath channels (see equation 10). In the case of single-path channels, the first and the second term of the denominator vanish. On the other hand, when there is no transmit diversity (i.e. $h_2(z) = 0$), part of the first term ($\alpha \sum_{k\neq 0} |R_{hh}(k)|^2$) will still be present (see equation (9)), while the second term will vanish. The remark that the CCI vanishes for single path channels was behind the original Alamouti STBC proposed for single-user flat-fading channels [1]. For multipath channels, however, the CCI can be very high, and the STTD may deteriorate the performances when used with a RAKE receiver. The MAI and CCI terms are both weighted by the load factor α. This explains the fact that the SINR is higher for lightly loaded systems and vice versa.

Concerning β_{OTD}, we first mention that it depends only on one channel (equation (12)). This means that *asymptotically* the second channel does not have any effect (neither positive, nor negative) on the first channel and vice versa. This is a disadvantage with respect to STTD since there is no (symbol-level) diversity provided. On the other hand, we note that there is no second term (CCI) in the denominator of β_{OTD}. In the case of multipath channel, this is an advantage in favor of OTD since STTD suffers from the interference created due to the use of two non-ideal channels. We note also that the SINR provided by OTD is the same as that provided by the no Transmit Diversity case where the power is doubled (whence the term $\frac{\sigma^2}{2} R_h(0)$). This means that there is no symbol-level diversity (before decoding) [6]. the diversity obtained is seen only after including deinterleaving and Viterbi decoding.

As a general conclusion, we note that OTD provides no (symbol-level) diversity since the denominator depends on one channel only but does not suffer from the CCI. The STTD, on the other hand, provides diversity as the denominator of β_{STTD} depends on both channels but suffers from CCI that is created due to the simultaneous use of these channels. For ideal and moderate multipath channels, we expect STTD to provide a better performance while for severe multipath channels OTD is expected to perform better.

4. SIMULATION RESULTS

We verify that our asymptotic analysis allows to predict the performance of W-CDMA with finite spreading factors. We have implemented the physical layer of the downlink of the UMTS-FDD, and we have compared the measured (empirical) raw (without coding nor interleaving) Bit Error Rate (BER) obtained for $N = 32$ and $K = 16$ with its asymptotic evaluation given by $Q(\sqrt{\beta_{STTD}})$ and $Q(\sqrt{\beta_{OTD}})$. The results are presented in Figure 1. The propagation channel is a three path channel with the following profile [1 0 0.2 0 0.1] (every two symbols a different realization of the channel is generated). It is noteworthy that the receiver we implemented is based on the correct model (3), thus showing that the approximation (6) (dropping the ISI term) is justified in this context. Figure 1 shows that our asymptotic evaluations allow to predict rather accurately the BER performance for $N = 32$ (The fit is even better for higher values of N).

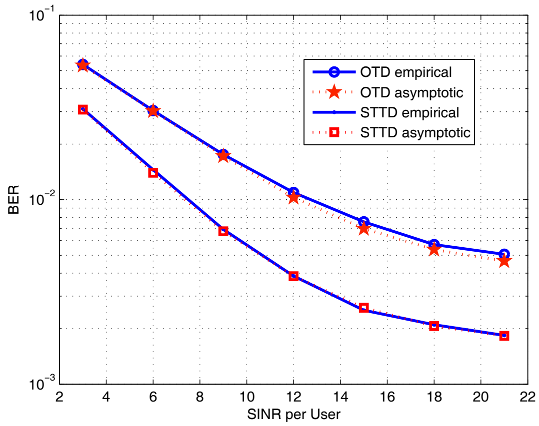

Fig. 1. Comparison of empirical and theoretical BER

In order to have a better understanding of the performance of both schemes, we consider a system with $\alpha = 0.5$ and a SNR of 10 dB. We study the BER performance in the asymptotic regime as a function of the channels. Each propagation channel is taken to be a two path channel. The two paths are spaced by one chip. We vary the (average) amplitude of the second path with respect to the first starting from a single path (when the second path amplitude is equal to zero) to two equal power paths (when the second path amplitude is equal to 1). We note that when the amplitude of the

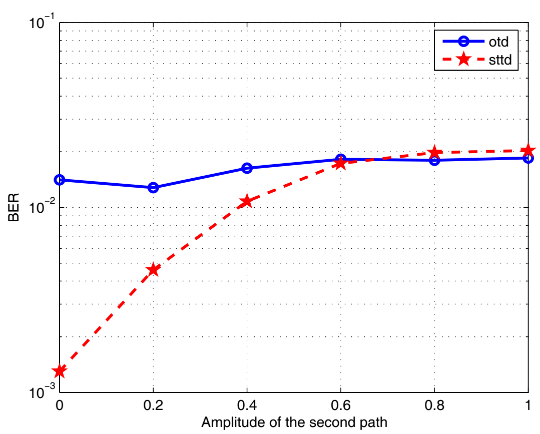

Fig. 2. Effect of the channel on STTD & OTD Performance.

second path is small with respect to the first path, STTD performs much better than OTD. This is due to the diversity provided by the term in the numerator of β_{STTD}. The STTD does not suffer from Cross Channel Interference (CCI) since it is very weak (or even null for a single path channel). On the other hand, for a two (equal) path channel, OTD is slightly better than STTD because OTD does not suffer from CCI. The STTD provides diversity (numerator) but suffers from two much CCI (denominator). The effect of the two phenomena makes it worse than OTD which does not provide (symbol-level) diversity but does not suffer from CCI.

5. CONCLUSION.

In this paper, we have addressed the performance of Space Time Transmit Diversity and Orthogonal Transmit Diversity in the downlink of W-CDMA over frequency-selective fading channels. We have derived asymptotic expressions of the SINR provided by a RAKE receiver when coupled with each technique. The asymptotic expressions are derived by assuming an i.i.d scrambling code and by letting the number of users K and the spreading factor N tend to infinity with fixed ratio. We concluded that Orthogonal Transmit Diversity does not provide any symbol-level diversity. This means that we only observe an improvement over a single-transmit antenna if we use deinterleaving and Viterbi decoding. On the other hand, Space Time Transmit Diversity provides symbol level diversity but suffers for simultaneous use of two multipath channels. Simulation results show that our asymptotic expressions allow to predict the performance of UMTS-FDD for $N = 32$. We have noticed that for moderate channels, STTD gives a better performance while for severe frequency selective channels OTD performs better.

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