DIVERSITY-MULTIPLEXING TRADEOFF OF OSTBC OVER CORRELATED NAKAGAMI-M FADING CHANNELS

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ABSTRACT

This paper investigates the tradeoff between diversity and multiplexing of orthogonal space-time block coding (OSTBC) for correlated Nakagami-m fading. For a given transmission rate, the outage probability as function of the multiplexing gain and signal-to-noise ratio (SNR) is derived. Following the negative slope of the outage probability versus SNR curve, we obtain the diversity gain at finite SNR and its maximum attainable value to predict the system performance at operating SNR's. In addition, the asymptotic diversity gain at high SNR is given via two approaches, which provides a theoretical limit for practical SNR's. The impact of the complex component correlation among the channel links on the outage performance is also presented.

1. INTRODUCTION

Orthogonal space-time block coding (OSTBC) has been proposed as an innovative transmit diversity scheme for multiple-input multiple-output (MIMO) systems that offers full spatial diversity through simple linear maximum likelihood (ML) decoding [1], [2]. A unified view of the fundamental diversity-multiplexing tradeoff in MIMO systems has been established by Zheng and Tse [3], which is evaluated as the signal-to-noise ratio (SNR) reaches infinity. Assuming independent and identically distributed (i.i.d.) Rayleigh fading, they also compared the tradeoff attainable by the Alamouti scheme [1] with the optimal tradeoff curve achievable using Gaussian random codes. [4] focused on a short structured STBC construction that achieves the optimal tradeoff only for the two-transmit two-receive antenna case. [5] developed a new framework to characterize the nonasymptotic tradeoff of space-time codes at finite SNR over i.i.d. Rayleigh fading channels. The maximum achievable diversity performance of OSTBC at high SNR for correlated Rician fading was tackled [6].

We quantify the performance of OSTBC over spatially correlated Nakagami-m fading channels through the outage probability corresponding to a target transmission rate. Assuming capacity achieving codes in one codeword (packet), the channel outage probability equals the packet error rate. By means of the statistics of the squared Frobenius norm of channel matrix, a series-form outage probability can be expressed in terms of the nonasymptotic multiplexing gain and SNR with excellent convergence. To measure the diversity gain at a particular operating SNR, the negative slope of the outage probability versus SNR curve is computed. As for the asymptotic tradeoff, two approaches are posed to capture a theoretical limit. One is to start from the outage probability available, and the other is to draw the limit of the diversity gain as SNR $\to \infty$. The impact of real-world propagation conditions on the diversity performance of OSTBC was assessed in [6]. Likewise, one of the goals of this paper is to analytically examine the effects of Nakagami-m fading correlation on the outage performance.

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2. SYSTEM MODEL

Consider a wireless system equipped with n_t transmit antennas and n_r receive antennas. We assume frequency-flat, quasi-static and spatially correlated complex channels with the same propagation delay for all the transmitter-receiver antenna pairs. Perfect channel state information is assumed available at the receiver. Transmit diversity over the wireless links using OSTBC with a code rate $r_s \leqslant 1$ is fulfilled. Capacity achieving codes are used in each STBC codeword. During an arbitrary codeword, the channel impulse response between the qth transmit and pth receive antenna is denoted by $h_{pq} = \eta_{pq} e^{j\varphi_{pq}}$, where the phase φ_{pq} is uniformly distributed over $[0, 2\pi)$ and the envelope η_{pq} is Nakagami-m distributed with a probability density function (PDF) given by

$$f_{\eta_{pq}}(\eta) = \frac{2}{\Gamma(m_{pq})} \left(\frac{m_{pq}}{\Omega_{pq}} \right)^{m_{pq}} \eta^{2m_{pq}-1} e^{-\frac{m_{pq}}{\Omega_{pq}}\eta^2} \quad (1)$$

where $\Gamma(\cdot)$ is the gamma function, $m_{pq} \geqslant 0.5$ is the fading parameter describing the fading severity and $\Omega_{pq} = E\left[\eta_{pq}^2\right]$ is the average power of h_{pq}. All channels form a matrix $\mathbf{H} = [h_{pq}] \in \mathbb{C}^{n_r \times n_t}$. Furthermore, it is assumed that each fading channel undergoes the same extent of Nakagami fading and has a unit power, i.e., $m_{pq} = m$ and $\Omega_{pq} = 1$ for $q = 1, 2, \ldots, n_t$ and $p = 1, 2, \ldots, n_r$.

The orthogonality property of STBC converts a MIMO channel into an equivalent scaled AWGN channel [7], [8]. Hence, the effective instantaneous SNR is expressed in [7] as $\gamma_s = \mu \|\mathbf{H}\|_F^2 / (n_t r_s)$, where μ is measured as the average symbol power transmitted over all n_t antennas to noise power ratio and notation $\|\cdot\|_F$ stands for the Frobenius norm. For a fixed channel realization $\mathbf{H}$, the mutual information of OSTBC is given by [7], [9]

$$I = r_s \log_2 \left(1 + \frac{\mu}{n_t r_s} \|\mathbf{H}\|_F^2\right) \quad \text{bps/Hz}. \tag{2}$$

To calculate the outage performance of such a system, it is necessary to acquire the statistical characteristics of $\|\mathbf{H}\|_F^2$, which have been addressed for independent Nakagami fading with integral fading parameter in [9].

3. PDF OF $\|\mathbf{H}\|_F^2$

Let $\mathbf{u}$ be the vectorized version of $\mathbf{H}$. Then the squared Frobenius norm of $\mathbf{H}$ is written as[1]

$$\|\mathbf{H}\|_F^2 = \sum_{p=1}^{n_r} \sum_{q=1}^{n_t} \eta_{pq}^2 = \mathbf{u}^H \mathbf{u}. \tag{3}$$

It is well known that η_{pq}^2 follows the gamma distribution, i.e., $\eta_{pq}^2 \sim \Upsilon(m, 1/m)$ [2]. Thus, $\|\mathbf{H}\|_F^2$ is the sum of $n_r n_t$ correlated gamma variables. The statistics of $\mathbf{u}$ are characterized by a rank-D correlation matrix $\mathbf{\Lambda} = [\rho_{pq,p'q'}] \in \mathbb{C}^{n_r n_t \times n_r n_t}$ with entries

$$\rho_{pq,p'q'} = \frac{E\left[h_{pq} h_{p'q'}^*\right]}{\sqrt{\Omega_{pq}\Omega_{p'q'}}} = E\left[h_{pq} h_{p'q'}^*\right]. \tag{4}$$

We introduce $\mathbf{\Lambda} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$ by eigenvalue decomposition, where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Sigma}$ is a diagonal matrix whose jth nonzero diagonal element denoted by $\lambda_j, j = 1, 2, \ldots, D$. Subsequently from (3), the $n_r n_t$ correlated gamma variables $\left\{\eta_{pq}^2\right\}$ can be decorrelated into independent variables $\{b_j\}_{j=1}^D$ ($b_j \sim \Upsilon(m, \lambda_j/m)$). Following the PDF of the sum of independent gamma variables [10, Theorem 1], the PDF of $\|\mathbf{H}\|_F^2$ is expressed as

$$f_{\|\mathbf{H}\|_F^2}(y) = \prod_{j=1}^{D} \left(\frac{\lambda_0}{\lambda_j}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\Gamma(Dm+k)} \left(\frac{m}{\lambda_0}\right)^{Dm+k} \times y^{Dm+k-1} e^{-my/\lambda_0} \tag{5}$$

[1]The superscripts ^{H} and * stand for conjugate transpose and conjugate, respectively.

[2]The shorthand notation $X \sim \Upsilon(a, b)$ is used to denote that X is gamma distributed with parameters $a > 0$ and $b > 0$ if the PDF of X is given by

$$f_X(x) = \frac{x^{a-1} e^{-x/b}}{\Gamma(a) b^a}, \quad x \geqslant 0.$$

where $\lambda_0 = \arg\min_j \{\lambda_j\}$ and the coefficients $\{\delta_k\}$ can be obtained recursively by

$$\begin{cases} \delta_0 = 1 \\ \delta_k = \dfrac{m}{k} \displaystyle\sum_{i=1}^{k} \left[\sum_{j=1}^{D} \left(1 - \frac{\lambda_0}{\lambda_j}\right)^i\right] \delta_{k-i}. \\ \qquad k = 1, 2, \ldots \end{cases} \tag{6}$$

It should be noted that the above PDF has a very similar form as the PDF of the sum of correlated gamma variables in [10]. Unlike the correlation between the instantaneous powers of two received signals used for the latter, the complex component correlation considered in this paper is the correlation between two field components, in which various parameters of interest such as the mean angle of arrival (AOA) of the signal, angular spread and array configurations are all taken into account. In addition, it has been shown in [10] that the single gamma series is numerically stable and quickly convergent so that the truncation to the first K terms can be used to meet a specified accuracy.

4. DIVERSITY-MULTIPLEXING TRADEOFF

In this section, according to the outage probability as function of the multiplexing gain and SNR, we quantify the diversity-multiplexing tradeoff of OSTBC in the finite SNR and high SNR regions.

4.1. Finite SNR

Given a transmission rate R (bps/Hz), the nonasymptotic multiplexing gain r is defined as $r = R/\log_2(1+\tau)$ [5], where τ specializes the average SNR per receive antenna. Since each channel has a unit power, τ is equal to μ. For nonergodic fading channels, there exists a nonzero probability that the rate R cannot be supported by the channel realization $\mathbf{H}$. So the outage probability of OSTBC can be computed from (2) and (5) to be

$$\begin{aligned} P_{\text{out}}(r, \mu) &= \Pr\{I < r \log_2(1+\mu)\} \\ &= \Pr\left\{\|\mathbf{H}\|_F^2 < \frac{n_t r_s}{\mu}\left[(1+\mu)^{r/r_s} - 1\right]\right\} \\ &= \prod_{j=1}^{D} \left(\frac{\lambda_0}{\lambda_j}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\Gamma(Dm+k)} \\ &\quad \times \gamma\left(Dm+k, \frac{c}{\mu}\left[(1+\mu)^{r/r_s} - 1\right]\right) \end{aligned} \tag{7}$$

where $c = m n_t r_s / \lambda_0$ and $\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$ is the incomplete gamma function. In the case of i.i.d. Nakagami-

m fading, (7) reduces to

$$P_{\text{out}}(r,\mu) = \frac{\gamma\left(n_r n_t m, m n_t r_s \left[(1+\mu)^{r/r_s} - 1\right]/\mu\right)}{\Gamma(n_r n_t m)}. \tag{8}$$

Moreover for i.i.d. Rayleigh fading, i.e., $m = 1$, we see that (8) can be simplified to [5, eq.(6)].

As described in [5], the diversity gain $d(r,\mu)$ of a system with multiplexing gain r at SNR μ determines the negative slope of the outage probability versus SNR curve in a log-log scale as follows:

$$d(r,\mu) = -\frac{\mu}{P_{\text{out}}(r,\mu)} \cdot \frac{\partial P_{\text{out}}(r,\mu)}{\partial \mu}. \tag{9}$$

Substituting (7) into (9) yields

$$\begin{aligned} d(r,\mu) = {} & \frac{\frac{c}{\mu}\left[(1+\mu)^{r/r_s} - 1 - \frac{r\mu}{r_s}(1+\mu)^{r/r_s-1}\right]}{\sum\limits_{k=0}^{\infty} \frac{\delta_k}{\Gamma(Dm+k)} \gamma\left(Dm+k, \frac{c}{\mu}\left[(1+\mu)^{r/r_s} - 1\right]\right)} \\ & \times e^{-\frac{c}{\mu}\left[(1+\mu)^{r/r_s}-1\right]} \sum_{k=0}^{\infty} \frac{\delta_k}{\Gamma(Dm+k)} \\ & \times \left(\frac{c}{\mu}\left[(1+\mu)^{r/r_s} - 1\right]\right)^{Dm+k-1}. \end{aligned} \tag{10}$$

Letting $r \to 0$ and after some manipulations, we have the maximum achievable diversity gain at finite SNR as

$$d_m(\mu) = \lim_{r\to 0} d(r,\mu) = Dm\left(1 - \frac{\mu}{(1+\mu)\ln(1+\mu)}\right). \tag{11}$$

To aim at system design, this characterizes the performance limit at a particular operating SNR.

4.2. High SNR

The high SNR case ($\mu \gg 1$) allows us to approximate the outage probability (7) as

$$\begin{aligned} P_{\text{out}}(r,\mu) \approx {} & \prod_{j=1}^{D}\left(\frac{\lambda_0}{\lambda_j}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\Gamma(Dm+k)} \\ & \times \gamma\left(Dm+k, c\mu^{r/r_s-1}\right). \end{aligned} \tag{12}$$

By keeping the first term (corresponding to $k = 0$) in the above infinite series and remaining the first term in the power series expansion resulted from the incomplete gamma function, (12) can be finally approximated to

$$P_{\text{out}}(r,\mu) \approx \prod_{j=1}^{D}\left(\frac{m n_t r_s}{\lambda_j}\right)^m \frac{\mu^{-Dm(1-r/r_s)}}{Dm\Gamma(Dm)} \tag{13}$$

which states that the tradeoff curve $d(r)$ is upper-bounded by $d_{\text{out}}(r) = Dm(1 - r/r_s)$, $0 \leqslant r \leqslant r_s$. Another approach to obtain the same bound is to evaluate the limit as $\mu \to \infty$ of the diversity gain (10), namely

$$d_{\text{out}}(r) = \lim_{\mu\to\infty} d(r,\mu) = Dm(1 - r/r_s). \tag{14}$$

In this way, the maximum achievable diversity gain $d_{\max}$ for correlated Nakagami-m fading is $d_{\max} = d_{\text{out}}(0) = Dm$, which also matches the limit of $d_m(\mu)$ as $\mu \to \infty$. This gain reduces to $n_r n_t$ [3], [5] and D [6] for i.i.d. Rayleigh fading and correlated Rayleigh fading, respectively.

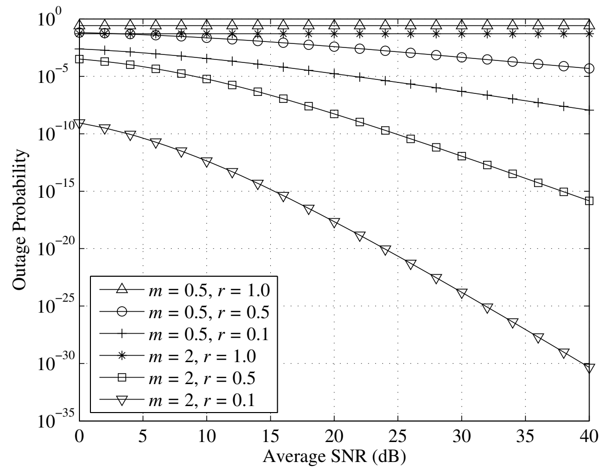

Fig. 1. Outage probability versus the average SNR per receive antenna over i.i.d. fading channels for different fading parameters m and multiplexing gains r.

5. NUMERICAL RESULTS

This section presents some numerical results to illustrate the outage probability and diversity-multiplexing tradeoff of OSTBC over Nakagami-m fading channels. Consider a 2×2 downlink MIMO system using the Alamouti code ($r_s = 1$), where the antenna elements at base station (BS) and mobile terminal (MT) are respectively spaced by d_{BS} and d_{MT}. Furthermore, the MT is assumed to be surrounded by isotropic scattering, while the BS is unobstructed by local scatterers. In such a scenario, based on a closed-form expression for the spatial cross-correlation function [8, eq.(47)], the coefficients $\rho_{pq,p'q'}$ depend on d_{BS}, d_{MT}, mean AOA at the BS α_{BS} and AOA spread Δ.

Fig. 1 exhibits the outage probability versus the average SNR per receive antenna over i.i.d fading channels for different fading parameters m and multiplexing gains r. As expected, the outage performance improves with increased fading parameter m. Given a moderate multiplexing gain $r = 0.5$, $m = 2$, $d_{\text{MT}} = 0.3\lambda_c$ (λ_c is the wavelength) and $\Delta = \pi/32$, the effects of the array configuration and the operating environment (associated with the parameters d_{BS} and α_{BS}) on the outage performance are shown in Fig. 2. Although the series in (7) has the satisfied convergence, we find

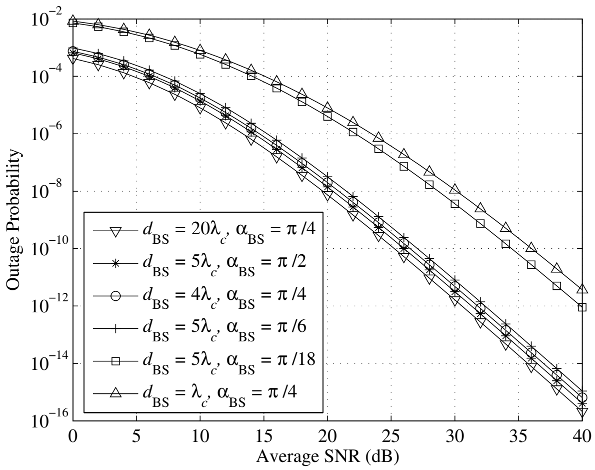

Fig. 2. Outage probability versus the average SNR per receive antenna over correlated fading channels with various values of d_{BS} and α_{BS}.

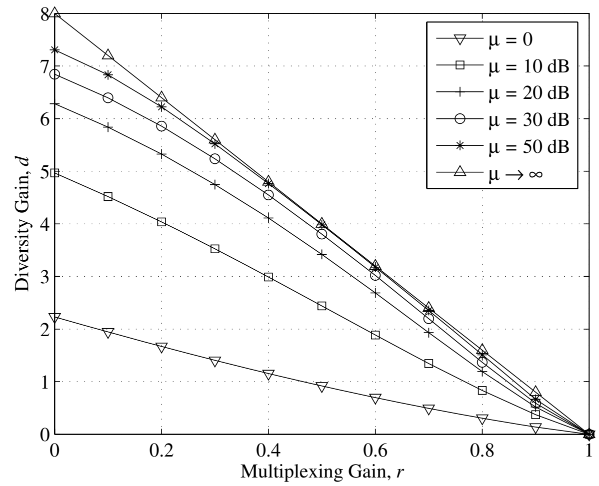

Fig. 3. Diversity-multiplexing tradeoff curves for different values of μ over correlated fading channels.

by simulations that it converges very slowly when the array elements are in a highly correlated situation. Therefore, in order to ensure convergence for six different correlated situations, the series has a common truncation order $K = 100$ for all cases. It is seen that a performance loss of about 0.5 dB is incurred at $d_{\text{BS}} = 20\lambda_c$ compared to the performance shown in Fig. 1, but a drastic degradation of 11 dB is incurred at $d_{\text{BS}} = \lambda_c$. For a fixed $d_{\text{BS}} = 5\lambda_c$, the correlation with a $\pi/6 < \alpha_{\text{BS}} < \pi/2$ results in a close performance, while a 7 dB loss in performance at $\alpha_{\text{BS}} = \pi/18$ occurs in comparison with $\alpha_{\text{BS}} = \pi/6$.

Fig. 3 depicts the diversity-multiplexing tradeoff curves for different SNR's. It has been assumed $m = 2$, $d_{\text{BS}} = 5\lambda_c$, $d_{\text{MT}} = 0.3\lambda_c$, $\alpha_{\text{BS}} = \pi/4$ and $\Delta = \pi/32$. Both series in (10) are truncated after the first ten terms. As can be seen in the figure, the curve at $\mu = 50$ dB in the moderate multiplexing gain region of $0.3 < r < 0.7$ greatly approaches the bound ($\mu \to \infty$), but is still below this theoretical limit for practical SNR's. Due to the full rank property of the correlation matrix $\mathbf{\Lambda}$, the maximum achievable diversity gain at high SNR is 8, while a maximum diversity gain of only around 5 can be achieved at $\mu = 10$ dB.

6. CONCLUSION

Exploiting the channel outage probability as a performance measure, we have analyzed the diversity-multiplexing tradeoff attainable by OSTBC for spatially correlated Nakagami-m fading with arbitrary real-valued m. The nonasymptotic tradeoff at finite SNR and the asymptotic tradeoff at high SNR were presented, respectively. Numerical results indicate that for some strongly correlated situations represented by a certain range of the physical parameters, the outage and diversity performance can be seriously deteriorated.

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