OPTIMAL POWER ALLOCATION FOR FULL-DUPLEX COOPERATIVE MULTIPLE ACCESS

Wessam Mesbah and Timothy N. Davidson

Department of Electrical and Computer Engineering,
McMaster University, Hamilton, Ontario, Canada.

ABSTRACT

Multiple access schemes in which the transmitting nodes are allowed to cooperate have the potential to provide higher quality of service than conventional schemes. In the class of pairwise cooperative multiple access schemes in which channel state information is available at the transmitters, the allocation of transmission power plays a key role in the realization of these quality of service gains. Unfortunately, the natural formulation of the power allocation problem for full-duplex cooperative schemes is not convex, but it is shown herein that this non-convex formulation can be simplified and re-cast in a convex form. In fact, in most scenarios a closed form expression for the optimal power allocation for each point on the boundary of an achievable rate region can be obtained.

1. INTRODUCTION

In conventional multiple access schemes each node attempts to communicate its message directly to the master node; e.g., the base station in a cellular wireless system. While such schemes can be implemented in a straightforward manner, alternative schemes in which nodes are allowed to cooperate have the potential to improve the quality of service that is offered to the transmitting nodes by enlarging the achievable rate region and by reducing the probability of outage; e.g., [1, 2]. The basic principle of cooperative multiple access is for the nodes to mutually relay (components of) their messages to the master node, and hence the design of such schemes involves the development of an appropriate composition of several relay channels [3, 4]. In particular, communication resources must be allocated to the direct transmission and cooperation tasks. The realization of the potential improvement in quality of service provided by cooperation is contingent on this allocation, and in this paper we develop efficient algorithms for optimal power allocation for the class of full-duplex cooperative multiple access schemes.

We will focus on cooperative multiple access schemes in which the transmitting nodes cooperate in pairs and have access to full channel state information. The transmitting nodes will operate in full-duplex mode, and hence will be allowed to simultaneously transmit and receive in the same time-frequency cell. Although this mode places stringent requirements on the communication hardware, it represents an idealized scenario against which more practical schemes can be measured. The nodes will cooperate by completely decoding the cooperative messages transmitted by their partners. We will consider an independent block fading model for the channels between the nodes, and will assume that the coherence time is long. This enables us to neglect the communication resources assigned to the feeding back of channel state information to the transmitters, and also suggests that an appropriate system design objective would be to enlarge the achievable rate region for each channel realization.

An impediment to the development of reliable, efficient power allocation algorithms for full-duplex cooperative multiple access is that the direct formulation of the problem is not convex. By studying the optimality conditions of this problem, we will show that this non-convex formulation can be transformed into a convex one. In particular, we will show that for a given rate requirement for one of the nodes, the optimal power allocation problem can be transformed into a convex problem that has a closed-form solution in most scenarios. In addition to the computational efficiencies that this closed form provides, the ability to directly control the rate of one of the users can be convenient in the case of heterogeneous traffic, especially if one node has a constant rate requirement and the other is dominated by "best effort" traffic. The derivation of our closed-form expressions involved the concurrent discovery of some of the observations in [5] regarding the properties of the optimal solution to the sum-rate optimization problem. Our approach has the advantage that in most scenarios it results in a closed-form solution, as distinct from the reduced-dimension optimization problem formulated in [5].

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada. The work of the second author is also supported in part by the Canada Research Chairs program.

2. FULL-DUPLEX MODEL

A block diagram of the model for full-duplex pairwise cooperative multiple access is provided in Fig 1; see [1, 6]. Let $w_{ij}(n)$ denote the n^{th} message from node i to node j, and let node 0 denote the master node. At the n^{th} (block) channel use, node i transmits the code word

$$X_i = X_{i0} + X_{ij} + U_i, \tag{1}$$

where $X_{i0}(w_{i0}(n), w_{ij}(n-1), w_{ji}(n-1))$ carries the information sent by user i directly to the master node, $X_{ij}(w_{ij}(n), w_{ij}(n-1), w_{ji}(n-1))$ carries the information that is sent by user i to the master node via user j, and $U_i(w_{ij}(n-1), w_{ji}(n-1))$ carries the cooperative information. (Note that all three components of X_i depend on the cooperative messages sent in the previous block.). Let P_i, P_{i0}, P_{ij} and P_{Ui} denote the power allocated to each component in (1). Assuming perfect isolation and echo cancellation, and that each transmitter knows the phase of the channels into which it transmits and has the means to cancel this phase, the received signal at each node can be written as

$$Y_0 = K_{10}X_1 + K_{20}X_2 + Z_0, \tag{2a}$$
$$Y_1 = K_{21}X_2 + Z_1, \qquad Y_2 = K_{12}X_1 + Z_2, \tag{2b}$$

respectively, where K_{ij} is the magnitude of the channel gain between node i and node j, and Z_i represents the additive zero-mean white circular complex Gaussian noise with variance σ_i^2 at

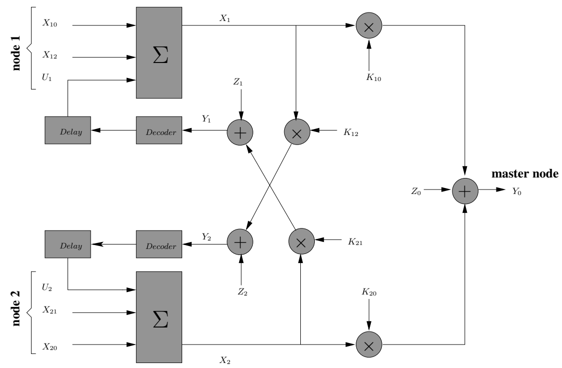

Fig. 1. Full-duplex pairwise cooperative multiple access.

node i. We define the gain-to-noise ratio of each channel to be $\gamma_{ij} = K_{ij}^2/\sigma_j^2$.

The data rate of node i in the above model is $R_i = R_{i0} + R_{ij}$, where R_{i0} is the rate of the messages transmitted directly to the master node, and R_{ij} is the rate of the messages transmitted via node j. Under the assumption that all the channel parameters γ_{ij} are known at both transmitting nodes, an achievable rate region for a given channel realization is the closure of the convex hull of the rate pairs (R_1, R_2) that satisfy the following constraints [1][1]

$$R_{i0} \leqslant \log\left(1 + \gamma_{i0} P_{i0}\right), \tag{3a}$$

$$R_{10} + R_{20} \leqslant \log\left(1 + \gamma_{10} P_{10} + \gamma_{20} P_{20}\right), \tag{3b}$$

$$R_i \leqslant R_{i0} + \log\left(1 + \frac{\gamma_{ij} P_{ij}}{1 + \gamma_{ij} P_{i0}}\right), \tag{3c}$$

$$R_1 + R_2 \leqslant \log\left(1 + \gamma_{10} P_1 + \gamma_{20} P_2 + 2\sqrt{\gamma_{10}\gamma_{20} P_{U1} P_{U2}}\right). \tag{3d}$$

Here, (3a) and (3b) bound the conventional multiple access region (with no cooperation), and (3c) and (3d) capture the impact of cooperation. A natural design objective would be to operate the system in Fig. 1 at rates that approach the boundary of the region specified in (3), subject to constraints on the transmitted power. The power allocation required to do so can be found by maximizing a convex combination of R_1 and R_2 subject to (3) and a bound on the transmitted powers, or by maximizing R_i for a given value of R_j, subject to (3) and the bound on the transmitted powers. Unfortunately, the direct formulation of both these problems is not convex in the transmitted powers, due to the interference components in (3c). The lack of convexity renders the development of a reliable efficient algorithm for the solution of the direct formulation fraught with difficulty. However, in the following subsections we will show that by adopting the latter of the two objectives above, the direct formulation can be transformed into a convex optimization problem that in most scenarios can be analytically solved to obtain closed-form expressions for the optimal power allocation.

The key observation in the derivation of these closed-form expressions is that the monotonicity of the logarithm implies that for constants a, b, c,and d, the optimization problem

$$\max_{x} \quad \log\left(\tfrac{a+bx}{c+dx}\right) \tag{4a}$$

$$\text{subject to} \quad 0 \leqslant x \leqslant P \tag{4b}$$

[1]All logarithms are to base 2, and all rates are in bits per two dimensions.

is equivalent to a linear fractional program in x. Therefore, it can be transformed [7] into the linear program

$$\max_{z} \quad a/c + (b - ad/c)z \tag{5a}$$

$$\text{subject to} \quad 0 \leqslant z \leqslant P/(c + Pd), \tag{5b}$$

where $z = x/(c + dx)$. This linear program has an analytical solution, and hence if $b - ad/c \neq 0$ the optimal solution to (4) is

$$x_{opt} = \begin{cases} P & \text{if } b - ad/c > 0, \\ 0 & \text{if } b - ad/c < 0. \end{cases} \tag{6}$$

If $b - ad/c = 0$, then any feasible x is optimal.

To formalize the development, we now explicitly state the power allocation problem for the case in which a target rate for node 2 is specified. Given the channel gains K_{ij}, bounds on the transmitted powers, $P_i \leqslant \bar{P}_i$, and a feasible target rate for node 2, $R_{2,\text{tar}}$, solve

$$\max_{P_{i0}, P_{ij}, P_{Ui}} \quad R_1 \tag{7a}$$

$$\text{subject to} \quad 0 \leqslant P_{i0} + P_{ij} + P_{Ui} \leqslant \bar{P}_i, \tag{7b}$$

$$\text{and equation (3) with } R_2 = R_{2,\text{tar}}. \tag{7c}$$

In the following subsections we will provide a solution to (7) that in most cases has a closed form. Since problems of the form in (4) appear in two of the underlying components of (7) and since (6) has two important cases, we will need to consider four cases. In each case, we exploit the fact that since we are attempting to maximize R_1, the upper bound constraint in (7b) for node 1 will be active at optimality.

2.1. Case 1: $\gamma_{10} \leqslant \gamma_{12}$ and $\gamma_{20} \leqslant \gamma_{21}$

In this case the cooperative channel for both nodes has a higher gain-to-noise ratio than the direct channel. It can be shown using (6) that the minimum value of the sum of the transmission powers $P_{20} + P_{21}$ required so that the constraint in (3c) is satisfied for $R_2 = R_{2,\text{tar}}$ is achieved when the sum is allocated to the channel with the higher gain-to-noise ratio. Since $\gamma_{20} \leqslant \gamma_{21}$ then to achieve the required $R_{2,\text{tar}}$ with minimum power we will send all the information of user 2 through the cooperative channel; i.e, $P_{20} = 0$, $R_{20} = 0$ and $R_{2,\text{tar}} = R_{21} = \log\left(1 + \gamma_{21} P_{21}\right)$. Hence $P_{21} = (2^{R_{2,\text{tar}}} - 1)/\gamma_{21}$. Therefore, the constraint in (3c) for node 1 can be written as

$$R_1 \leqslant \log\left(1 + \gamma_{10} P_{10}\right) + \log\left(1 + \frac{\gamma_{12} P_{12}}{1 + \gamma_{12} P_{10}}\right). \tag{8}$$

Using (4) and (6) it can be shown that the choices $P_{10} = 0$ and $P_{12} = \bar{P}_1 - P_{U1}$ maximize the bound in (8) and hence the two constraints on R_1 will be (3d) and $R_1 \leqslant \log\left(1 + \gamma_{12}(P_1 - P_{U1})\right)$. Therefore, we have reduced the problem in (7) to

$$\max_{P_{U1}} \quad \min\left\{\beta_1(P_{U1}), \beta_2(P_{U1})\right\} \tag{9a}$$

$$\text{subject to} \quad 0 \leqslant P_{U1} \leqslant \bar{P}_1, \tag{9b}$$

where $\beta_1(P_{U1}) = \log\left(1 + \gamma_{12}(\bar{P}_1 - P_{U1})\right)$ and $\beta_2(P_{U1}) = \log\left(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2 + 2\sqrt{\gamma_{10}\gamma_{20} P_{U1} P_{U2}}\right) - R_{2,\text{tar}}$. In order to solve (9) analytically, we observe that the argument of the logarithm in $\beta_1(P_{U1})$ is linearly decreasing in P_{U1} while the argument of the logarithm in $\beta_2(P_{U1})$ is concave increasing. Therefore, the solution of (9) is the value of P_{U1} for which the two upper bounds on R_1 intersect; i.e, $\beta_1(P_{U1}) = \beta_2(P_{U1})$. If we define

$$A = \frac{\gamma_{12}}{2\sqrt{\gamma_{10}\gamma_{20}}}\left(1+\gamma_{21}P_{21}\right), \tag{10}$$
$$B = \left[\left(1+\gamma_{21}P_{21}\right)\left(1+\gamma_{12}\bar{P}_1\right) - \left(1+\gamma_{10}\bar{P}_1+\gamma_{20}\bar{P}_2\right)\right]/(2\sqrt{\gamma_{10}\gamma_{20}}),$$

where, as above, $P_{21} = (2^{R_{2,\mathrm{tar}}} - 1)/\gamma_{21}$, then the optimal power allocation for node 1 can be written as

$$P_{U1} = \frac{2AB + P_{U2} - \sqrt{(2AB+P_{U2})^2 - 4A^2B^2}}{2A^2}, \tag{11a}$$
$$P_{10} = 0, \qquad P_{12} = \bar{P}_1 - P_{U1}, \tag{11b}$$

where $P_{U2} = \bar{P}_2 - P_{21}$. Since $P_{10} = P_{20} = 0$, both nodes will use cooperative transmission only.

2.2. Case 2: $\gamma_{10} > \gamma_{12}$ and $\gamma_{20} \leqslant \gamma_{21}$

In this case, the direct channel for node 1 has a higher gain-to-noise ratio than its cooperative channel, but for node 2 the opposite is true. Using a similar arguments to case 1, the minimum value of $P_{20}+P_{21}$ required for $R_{2,\mathrm{tar}}$ to be achievable occurs when $P_{20} = 0$. Thus the power distribution for the second user will be $P_{21} = (2^{R_{2,\mathrm{tar}}} - 1)/\gamma_{21}$, $P_{20} = 0$, $P_{U2} = \bar{P}_2 - P_{21}$. Therefore, the constraint in (3c) for node 1 is the same as in (8). However, in this case it can be shown that the choices $P_{12} = 0$ and $P_{10} = \bar{P}_1 - P_{U1}$ maximize the constraint in (8). The two constraints on R_1 will be (3d) and $R_1 \leqslant \log\left(1+\gamma_{10}(\bar{P}_1 - P_{U1})\right)$. That is, we have reduced the problem in (7) to

$$\max_{P_{U1}} \quad \min\left\{\beta_3(P_{U1}), \beta_2(P_{U1})\right\} \tag{12a}$$
$$\text{subject to} \qquad 0 \leqslant P_{U1} \leqslant \bar{P}_1, \tag{12b}$$

where $\beta_3(P_{U1}) = \log\left(1+\gamma_{10}(\bar{P}_1 - P_{U1})\right)$ and $\beta_2(P_{U1})$ is as defined in Section 2.1. By analogy to Case 1, the solution to this problem is the intersection point between the two terms inside the minimum function. If we define $A = \sqrt{\frac{\gamma_{10}}{4\gamma_{20}}}\left(1+\gamma_{21}P_{21}\right)$ and $B = \left(\gamma_{21}P_{21}(1+\gamma_{10}P_1) - \gamma_{20}P_2\right)/(2\sqrt{\gamma_{10}\gamma_{20}})$, the value of P_{U1} in that solution has the same form as (11a), and the other powers for node 1 are $P_{10} = P_1 - P_{U1}$ and $P_{12} = 0$. From this result it is apparent that node 1 will use direct transmission only, while node 2 will use only cooperative transmission.

2.3. Case 3: $\gamma_{10} \leqslant \gamma_{12}$ and $\gamma_{20} > \gamma_{21}$

In this case the cooperative channel of node 1 has a higher gain-to-noise ratio than the direct channel, while this property is reversed for node 2. This case is symmetric to Case 2, which means that it is optimal to set $P_{10} = 0$, $P_{21} = 0$, $P_{20} = (2^{R_{2,\mathrm{tar}}} - 1)/\gamma_{20}$ and $P_{U2} = \bar{P}_2 - P_{20}$. The problem in (7) can then be written in the same form as (9) except that $R_{2,\mathrm{tar}} = \log\left(1+\gamma_{20}P_{20}\right)$, and hence if we define

$$A = \gamma_{12}\left(1+\gamma_{20}P_{20}\right)/(2\sqrt{\gamma_{10}\gamma_{20}}), \tag{13}$$
$$B = \left[\left(1+\gamma_{20}P_{20}\right)\left(1+\gamma_{12}\bar{P}_1\right) - \left(1+\gamma_{10}\bar{P}_1+\gamma_{20}\bar{P}_2\right)\right]/(2\sqrt{\gamma_{10}\gamma_{20}}),$$

the optimal power allocation for node 1 will have the same form as (11). Therefore, node 1 will use cooperative transmission only, while node 2 will only use direct transmission.

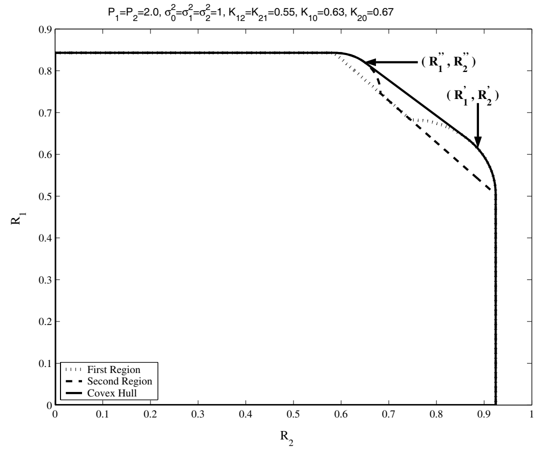

Fig. 2. Convex hull of the two achievable rate regions in Case 4.

2.4. Case 4: $\gamma_{10} > \gamma_{12}$ and $\gamma_{20} > \gamma_{21}$

In this case, the channel gain-to-noise ratios of the direct transmission channels are larger than those of the cooperative channels. Using similar arguments to the previous cases, the minimum value of the sum of the transmission powers $P_{20} + P_{21}$ required so that the constraint in (3c) is satisfied for $R_2 = R_{2,\mathrm{tar}}$ is achieved when $P_{21} = 0$ and $P_{20} = (2^{R_{2,\mathrm{tar}}} - 1)/\gamma_{20}$; i.e., all the information of user 2 will be sent directly to the master node. In order to maximize R_1, the remaining power of node 2 will be used to cooperate with node 1;i.e, $P_{U2} = \bar{P}_2 - P_{20}$. Since $R_{20} \neq 0$, the constraint in (3c) can be written as

$$R_1 \leqslant \log\left(1+\frac{\gamma_{10}P_{10}}{1+\gamma_{20}P_{20}}\right) + \log\left(1+\frac{\gamma_{12}P_{12}}{1+\gamma_{12}P_{10}}\right). \tag{14}$$

The remaining design variables in (7) are P_{10}, P_{12}, and P_{U1}. For a given value of P_{U1}, the bound on the right hand side of (14) can be written in the form of the objective in (4). Using (6) it can be shown that if $R_{2,\mathrm{tar}} > \log\bigl(\gamma_{10}/\gamma_{12}\bigr)$, then $P_{10} = 0$. On the other hand, if $R_{2,\mathrm{tar}} \leqslant \log\bigl(\gamma_{10}/\gamma_{12}\bigr)$, then $P_{12} = 0$. In the first case R_1 now has two bounding constraints, (3d) and $R_1 \leqslant \log\bigl(1+\gamma_{12}P_{12}\bigr)$. Since in this case $P_{12} = \bar{P}_1 - P_{U1}$, the problem of maximizing R_1 will be the same as that in Case 3, and hence the solution will have the form of (11) with A and B being defined as in (13). From this solution it is clear that node 1 will use cooperative transmission only while node 2 will use only direct transmission.

If $R_{2,\mathrm{tar}} \leqslant \log\left(\gamma_{10}/\gamma_{12}\right)$, then $P_{12} = 0$. Since $P_{12} = P_{21} = 0$, there will be no cooperative message at either node and hence there is no need for the transmission of U_1 and U_2. Therefore, $P_{U1} = P_{U2} = 0$ and $P_{10} = \bar{P}_1$. In this case, both nodes will use direct transmission only; i.e., the cooperative scheme will reduce to a conventional multiple access scheme.

In Cases 1–3 above, the solution to (7) generates the achievable rate region directly. However, in the present case the achievable rate region is the convex hull of the rates achieved by solving (7) and those generated by the solution of the symmetric image of (7) in which R_2 is maximized subject to a target rate for node 1. As one might suspect, the solution to that problem is symmetric with that of (7). Fig. 2 shows the rate regions achieved by the solution to (7) and its symmetric image. The figure also shows the convex hull of those two rate regions. The inner pentagon in this figure is the (non-cooperative) multiple access region, and hence the cooperative gain in Case 4 is clear. (Much larger cooperative gains are realized in other cases; e.g., Fig. 3.) Points on the interval $(R_1^{'}, R_2^{'})$ to $(R_1^{''}, R_2^{''})$ in Fig. 2 are not achieved by the solution of (7) or its symmetric inverse, but can be achieved using standard time sharing techniques in which the system operates at the point $(R_1^{'}, R_2^{'})$ for a fraction ρ of the block length, and at the point $(R_1^{''}, R_2^{''})$ for the remainder of the block. Although we do not have a closed form expression for the points $(R_1^{'}, R_2^{'})$ and $(R_1^{''}, R_2^{''})$ at this time, they can be determined from the solution of an auxiliary convex optimization problem. (The formal statement of that problem has been omitted due to space constraints.)

3. SIMULATION RESULTS

In order to assess the benefits of cooperation, we have provided in Fig. 3 the average achievable rate regions in different scenarios. While these figure is similar to that in Fig. 2 of Part I of [1], the key point is that it has been computed using the closed form solution developed herein. In the scenarios considered, the channels were independent block fading channels with long coherent times. The channel gains were Rayleigh distributed, the Gaussian noise variances were normalized to 1, and the transmission powers of the cooperating nodes were set to be equal $\bar{P}_1 = \bar{P}_2 = 2$. (Recall that each node has full channel state information.) Fig. 3 shows the symmetric channel case where the channel between each node and the master node is Rayleigh fading with the same mean value $E(K_{10}) = E(K_{20}) = 0.63$. Different curves are plotted for different values of the mean value of the inter-user channel $E(K_{12})$. (For each realization $K_{12} = K_{21}$.) The average achievable rate region was obtained by taking the direct sum of the achievable rate regions for each channel realization and then dividing by the number of realizations. These plots clearly demonstrate the advantages of cooperative multiple-access, especially when the gain of the cooperative channels is (often) significantly larger than the gain of the direct channels.

In addition to the average achievable rate region, it is interesting to observe the optimal power allocations. Fig. 4 shows the allocation of the different power components for both node 1 and node 2 for one channel realization in which $K_{10} = K_{20} = 0.4$ and $K_{12} = K_{21} = 0.7$. (These gains satisfy the conditions of Case 1 in our closed-form solution.) The figure plots the optimal power components that maximize the rate R_1 for each value of the rate R_2. We note from the figure that there is one power component for each user that is zero for all values of R_2; i.e., in this case $P_{10} = P_{20} = 0$. We also note that the curves for P_{12} and P_{21} intersect at the same value for R_2 as the curves for P_{U1} and P_{U2}. This intersection point represents the equal rate point at which $R_1 = R_2$. The figure also illustrates that as R_2 increases, node 2 allocates more power to P_{21} to increase the data rate sent to node 1. As R_2 increases, node 1 has to reduce its data rate, and this is reflected in the decreasing amount of power that is allocated to P_{12}.

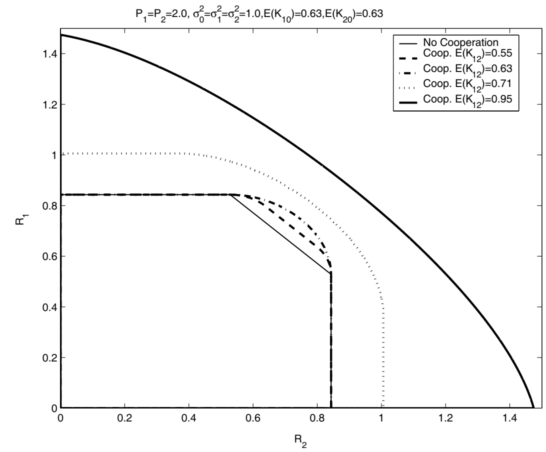

Fig. 3. Achievable rate region in a case of symmetric direct channels.

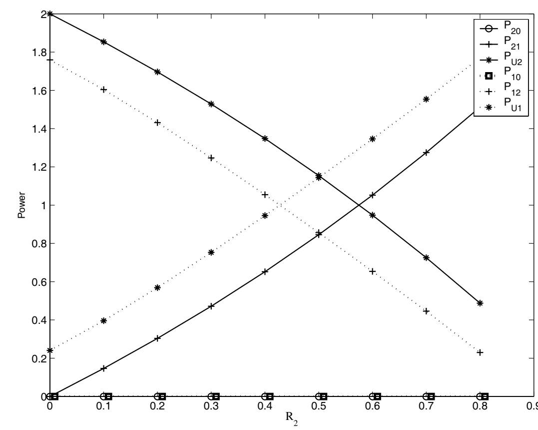

Fig. 4. Power allocation for a channel realization that satisfies the conditions of Case 1.

4. REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Parts I and II," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1948, Nov. 2003.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behaviour," *IEEE Trans. Informat. Theory*, vol. 50, Dec. 2004.

[3] T. M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Informat. Theory*, vol. 25, Sept. 1979.

[4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Informat. Theory*, vol. 51, pp. 3037–3063, Sept. 2005.

[5] O. Kaya and S. Ulukus, "Power control for fading multiple access channels with user cooperation," in *Proc. Int. Conf. Wireless Networks, Communications, Mobile Computing*, (Maui, HI), June 2005.

[6] F. M. J. Willems, E. van der Meulen, and J. P. M. Schalkwijk, "An achievable rate region for the multiple access channel with generalized feedback," in *Proc. Allerton Conf. Commun., Control, Computing*, (Monticello, IL), pp. 284–292, 1983.

[7] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.