RANDOMIZED SPACE-TIME CODING FOR DISTRIBUTED COOPERATIVE COMMUNICATION: FRACTIONAL DIVERSITY

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ABSTRACT

We study the problem of designing distributed space-time codes for cooperative communication. A major challenge in distributed cooperative transmissions is to find a way to coordinate the relay transmissions without requiring extra control information overhead. Most of the previous works on the subject assume each node emulates a predetermined antenna of a multiple-antenna system. However, this requires a centralized "antenna allocation" procedure. Here, we introduce randomized strategies that decentralize the transmission of a space time code from a set of distributed relays. The simple idea we propose is to let each node transmit a random linear combination of the codewords that would be transmitted by all the antennas in a centralized space time coding scheme. We provide different code designs that achieve the diversity order $(\min(N, L))$ when number of nodes N is different than the number of virtual antennas L. In this paper, we focus on the case when $N = L$ and we show that under certain designs the achieved diversity order is fractional.

1. INTRODUCTION

Space-time codes over multiple antenna systems provide diversity and coding gains and hence improve the communication performance over fading channels. However, multiple-antenna systems are impractical for distributed large-scale networks due to size and hardware constraints. Recently, several methods have been proposed for cooperation among relay nodes that are able to provide spatial diversity gains without utilizing multiple transmit antennas at the terminals. Most of the distributed space-time codes proposed for cooperative networks are not truly distributed protocols; they require a central control unit or prior coordination between the nodes [1–4]. In large-scale distributed wireless network, the set of cooperating nodes is unknown or random in most scenarios and the cooperative transmission scheduling is problematic. In [5, 6], the authors propose diversity schemes that do not require the knowledge of the number of cooperating nodes. To the best of our knowledge, the approaches that apply to a decentralized scenario are the ones in [7–9].

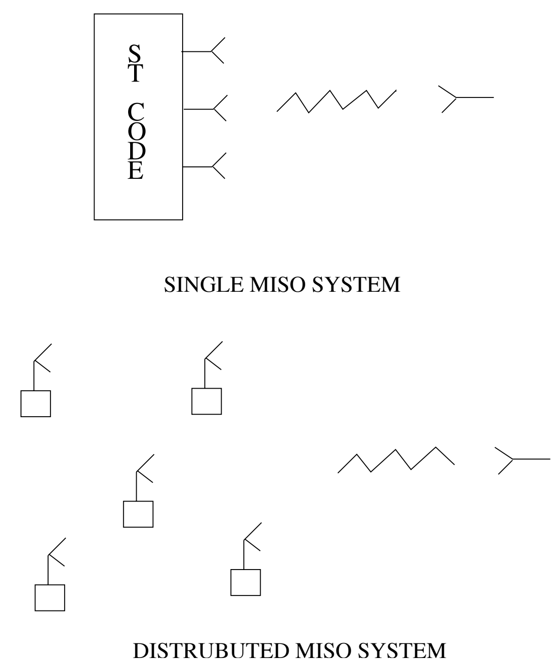

Fig. 1. Single and Distributed MISO systems

We propose a novel and simple methodology to decentralize the relay transmissions and yet obtain diversity and coding gains analogous to those that can be attained on a point-to-point multiple-antenna links. In the proposed scheme, each node uses a *random* linear combination of the columns of an underlying space-time code. Our scheme can be viewed as a randomized version of the distributed space-time codes [4, 6]. The purpose of randomization is, as mentioned before, to eliminate the need for antenna allocation. Similar to [5, 6], the proposed method does not require the knowledge of the number of cooperating nodes. Compared to [7, 9], we are able to characterize precisely the degree of diversity that we should expect.

In [10, 11], we provide designs that achieve full diversity under the condition $N \neq L$, where N is the number of coop-

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erating nodes and L is the number of virtual transmit antennas. We show that, despite code randomization, the proposed scheme achieves full diversity (N) if $N < L$, and the diversity order L is achieved for $N > L$. In this paper, we show that the diversity order depends on the behavior of the *eigenvalue distribution of the matrix $\mathcal{R}\mathcal{R}^H$ around zero*, where $\mathcal{R} = [\mathbf{r}_1\ \mathbf{r}_2 \ldots \mathbf{r}_N]$, and $\mathbf{r}_i$ denotes the randomization coefficient vector for the i'th node. Interestingly, we are able to prove that, in contrast to the multi-antenna systems, certain randomization matrices $\mathcal{R}$ provide fractional diversity orders.

The paper is organized as follows: In Section 2, we present the proposed scheme. In Section 3, we provide an upper bounds on the average error probability and using this upper bound, we propose sufficient conditions on the eigenvalue distributions to achieve full diversity order. In Section 4, we present the simulations. Finally, we conclude in Section 5.

In the following, $\det(\mathbf{A})$ denotes the determinant of a matrix $\mathbf{A}$. The identity matrix is denoted by $\mathbf{I}$.

2. PROPOSED DIVERSITY SCHEME

Let $\mathbf{s} = [s_0\ s_1 \ldots s_{n-1}]$ be the block of source symbols to be transmitted to the destination. We assume that the message is known perfectly at the active nodes, hence they are responsible for the retransmission to the destination. We will consider the transmission of one block of data for simplicity. In the following, we describe the processing at each node and analyze the decoding performance at the destination.

At each node, the $\mathbf{s}$ is mapped onto a matrix $\mathcal{G}(\mathbf{s})$ as done in standard space-time coding:

$$\mathbf{s} \rightarrow \mathcal{G}(\mathbf{s}),$$

where $\mathcal{G}$ is a $P \times L$ space-time code matrix. Here, L denotes the number of virtual antennas. In our scheme each node transmits a block of P symbols, which is a random linear combination of columns of $\mathcal{G}(s)$. Let $\mathbf{r}_i$ be the $L \times 1$ random vector that contains the linear combination coefficients for the i'th node. Define $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2 \ldots \mathbf{x}_N]$ as the $P \times N$ random code matrix whose rows represent the time and columns represent the space, where $\mathbf{x}_i = \mathcal{G}(\mathbf{s})\mathbf{r}_i$ is the code transmitted by the i'th node. The randomized space time coding can be formulated as the double mapping:

$$\mathbf{s} \rightarrow \mathcal{G}(\mathbf{s}) \rightarrow \mathcal{G}(\mathbf{s})\mathcal{R}, \tag{1}$$

where $\mathcal{R} = [\mathbf{r}_1\ \mathbf{r}_2 \ldots \mathbf{r}_N]$. In the following, the $L \times N$ matrix $\mathcal{R}$ will be referred as the *randomization matrix*. Since each node's processing is intended to be local, $\mathbf{r}_i$'s should be independent for each $i = 1 \ldots N$, and we will also assume that they are identically distributed. This property allows the randomized space-time coding to be implemented in a decentralized fashion. In other words, each node chooses a random set of linear combination coefficients from a given distribution, which does not depend on the node index.

Let $\mathbf{y}$ be the received signal at the destination. We assume that the delay spread of the channel of each user plus the time offset at that user amounts to a coherence bandwidth that is much larger than the transmission bandwidth. Under this assumption, we can write the received signal as

$$\mathbf{y} = \mathcal{G}(\mathbf{s})\mathcal{R}\mathbf{h} + \mathbf{w}, \tag{2}$$

where $\mathbf{w} \sim \mathcal{N}_c(\mathbf{0}, N_0\mathbf{I})$ and $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{\Sigma_h})$. In our analysis, we assume that $\mathbf{\Sigma}_h = \mathbf{I}$, but the results hold as long as the $\mathbf{\Sigma}_h$ is positive definite. In [6], the authors propose that the nodes transmit a deterministic linear combination of the columns of a space-time code matrix; hence, (2) is similar to the system model in [6].

In order to perform coherent decoding, the receiver needs to estimate the channel coefficients. Instead of estimating the channel vector $\mathbf{h}$ and the randomization matrix $\mathcal{R}$ separately, the receiver can estimate the *effective channel* coefficients $\tilde{\mathbf{h}} \triangleq \mathcal{R}\mathbf{h}$. For this, the training data at the transmitters should use the same randomization procedure.

Traditional space-time codes are designed using the probability error as a performance criterion [12]. In the next section, we will adopt a similar approach for the design of randomized space-time codes.

3. DESIGN OF RANDOMIZED SPACE-TIME CODES

Let $\mathcal{M} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots \mathbf{s}_{|\mathcal{M}|}\}$ be the message set, and $\mathcal{G}_i \triangleq \mathcal{G}(\mathbf{s}_i)$. Let the randomization matrix $\mathcal{R}$ be fixed for every choice of code matrix. Let SNR $= 1/N_0$, where N_0 is the variance of the additive white Gaussian noise per complex dimension (see Eqn. 2). In the following, we assume that the deterministic space-time code matrix $\mathcal{G}$ is $P \times L$ such that $P \geq L$. And the randomization matrix $\mathcal{R}$ is $L \times N$, where N is the number of cooperating nodes. Also, we say an $L \times N$ matrix $\mathbf{A}$ is *full-rank* if rank($\mathbf{A}$) $= \min\{L, N\}$.

Definition The diversity order d^* of a scheme with probability of error $P_e(\text{SNR})$ is defined as

$$d^* = \lim_{\text{SNR}\rightarrow\infty} \frac{-\log P_e(\text{SNR})}{\log \text{SNR}}. \tag{3}$$

We say that the randomized space-time code achieves the *diversity order* d if $d \leq d^*$. Note that $d^* \leq \min\{N, L\}$ for the proposed scheme.

Define $r \triangleq \min\{L, N\}$. In the following, we provide lower bounds to d^*, and when the lower bound is equal to r, we say that the scheme has diversity order $d^* = \min(L, N)$.

Using the union bound, the average probability of error can be upper bounded by the pairwise probability of errors assuming that all source messages $\mathbf{s}_i \in \mathcal{M}$ are equally likely:

$$P_e \leq \frac{1}{|\mathcal{M}|} \sum_{\mathbf{s}_k \in \mathcal{M}} \sum_{\mathbf{s}_i \in \mathcal{M}, i \neq k} \Pr(\mathbf{s}_k \rightarrow \mathbf{s}_i), \tag{4}$$

where $\Pr(\mathbf{s}_k \to \mathbf{s}_i)$ denotes the probability that a transmitted message $\mathbf{s}_k$ is mistaken for another message $\mathbf{s}_i$. Let $\mathbf{s}_k \in \mathcal{M}$ denote the transmitted symbol. For the system given by Eqn. 2, assuming i.i.d. Rayleigh Fading *i.e.,* $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{I})$, the pairwise error probability of coherent detection (averaged over $\{\boldsymbol{\mathcal{R}}, \mathbf{h}\}$) is bounded as,

$$\Pr\{\mathbf{s}_k \to \mathbf{s}_i\} \leq \mathbb{E}_{\boldsymbol{\mathcal{R}}}\left\{\frac{1}{\det(\mathbf{I} + \mathrm{SNR}/4\; \boldsymbol{\mathcal{R}}^H \boldsymbol{\mathcal{A}}_{ki} \boldsymbol{\mathcal{R}})}\right\}, \quad (5)$$

where $\boldsymbol{\mathcal{A}}_{ki} = (\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)^H(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$. Using (4) and (5), the average probability of error P_e is bounded above by $\bar{P}_e$, where

$$\bar{P}_e \triangleq \mathbb{E}_{\boldsymbol{\mathcal{R}}}\left\{\frac{|\mathcal{M}| - 1}{\min_{(k,i)}\{\det(\mathbf{I} + \mathrm{SNR}/4\; \boldsymbol{\mathcal{R}} \boldsymbol{\mathcal{A}}_{ki} \boldsymbol{\mathcal{R}}^H)\}}\right\}. \quad (6)$$

The following theorem allows us to separate the design of the deterministic space-time code $\boldsymbol{\mathcal{G}}$ and the randomization matrix $\boldsymbol{\mathcal{R}}$.

Theorem 1 *Let $r = \min\{L, N\}$. Assume the scheme satisfies the following condition:*

C1) The Rank Criterion for $\boldsymbol{\mathcal{G}}$: *For any pair of space-time code matrix $\{\boldsymbol{\mathcal{G}}_k, \boldsymbol{\mathcal{G}}_i\}$, the matrix $(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$ is full-rank, i.e., of rank L.*

Then, P_e is bounded as

$$P_e \leq \max(1, \alpha^{-L})(|\mathcal{M}| - 1)\, \underbrace{\mathbb{E}_{\boldsymbol{\mathcal{R}}}\left\{\frac{1}{\det(\mathbf{I} + \mathrm{SNR} \boldsymbol{\mathcal{R}} \boldsymbol{\mathcal{R}}^H)}\right\}}_{\triangleq \tilde{P}_e}, \quad (7)$$

where $\alpha = \min_{(k,i)} \lambda_{k,i}^2/4$ such that $\lambda_{k,i}$ is the smallest singular value of $(\boldsymbol{\mathcal{G}}_k - \boldsymbol{\mathcal{G}}_i)$. Moreover, the diversity order of $\tilde{P}_e$ (Eqn. 7) and the diversity order of $\bar{P}_e$ (Eqn. 6) are the same, i.e.,

$$\lim_{\mathrm{SNR} \to \infty} \frac{-\log \bar{P}_e}{\log \mathrm{SNR}} = \lim_{\mathrm{SNR} \to \infty} \frac{-\log \tilde{P}_e}{\log \mathrm{SNR}} \quad (8)$$

Proof Proof is skipped for brevity. See [10] for details.

The condition C1 is equivalent to the rank criterion for the deterministic space-time codes [12] that achieves maximum diversity. There is a vast literature on the design of deterministic space-time codes which can be utilized directly. In this context, by choosing an existing space-time code matrix $\boldsymbol{\mathcal{G}}$ designed for multiple-antenna system, we can easily satisfy C1. In the rest of the paper, we assume the deterministic space-time code $\boldsymbol{\mathcal{G}}$ satisfies the condition C1 in Theorem 1.

Definition We say that a non-negative function $f(\mathrm{SNR}) = \Theta(x^\alpha)$ as $x \to 0$ if there exist $\epsilon > 0$ and positive constants $0 < c_1 < c_2$ such that $|x| < \epsilon$ implies $c_1 x^\alpha \leq f(\mathrm{SNR}) \leq c_2 x^\alpha$.

Note that, the upper bound in (7) can be expressed in terms of eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$. In the following, we will express the diversity order of $\tilde{P}_e$ in terms of the asymptotic expansion of the pdf of the eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$.

Theorem 2 *Let $S = \{\lambda_1, \ldots, \lambda_r\}$ be the unordered set of eigenvalues of $\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H$. Let $f(\lambda_1, \ldots, \lambda_r)$ be the joint density for the eigenvalues. Assume there exists constants $\{\gamma_{ik} > -1, k = 1 \ldots r, i = 1 \ldots M\}$ and $\{C_i > 0, i = 1 \ldots M\}$ for some M such that*

$$f(\lambda_1, \ldots, \lambda_L) = \Theta\Big(\sum_{i=1}^{M} C_i \lambda_1^{\gamma_{i1}} \lambda_2^{\gamma_{i2}} \ldots \lambda_{ir}^{\gamma_{ir}}\Big) \quad (9)$$

is satisfied for $\lambda_1, \lambda_2, \ldots, \lambda_r$ in a small neighborhood of the origin. Then the $\tilde{P}_e$ has diversity order d where

$$d = \min_k \sum_{i=1}^{r} \min(1, 1 + \gamma_{ki}). \quad (10)$$

And the scheme also achieves this diversity order.

Proof Proof is skipped for brevity. See [10] for details.

Theorem 2 fully characterizes the diversity order of $\tilde{P}_e$. In addition, Theorem 2 presents an interesting result, which is the possibility of fractional diversity orders. Depending on the values of $\{\gamma_{ik}\}$, the diversity order d can be rational number. If $\tilde{P}_e$ is tight then the diversity order of the randomized space-time codes is fractional! Note that, if Eqn. 9 is satisfied for $\gamma_{ik} > 0, \forall i, k$, then the scheme achieves diversity order $d = \min(N, L)$.

Theorem 3 *Consider the randomized space-time code utilizing a uniform phase randomization matrix $\boldsymbol{\mathcal{R}} = [\mathbf{r}_1\ \mathbf{r}_2]$ such that $\mathbf{r}_k = [\exp(j\theta_{k1})\ \exp(j\theta_{k2})]^t$, where $\theta_{kn} \overset{i.i.d.}{\sim} U(-\pi, \pi)$ for $n, k \in \{1, 2\}$. Assume that condition C1 is satisfied. Then, the diversity order of the scheme is $d^* = 1.5$.*

Proof Proof is avoided for brevity. See [10] for details.

4. SIMULATIONS

In this section, we present the performance of proposed *randomized* distributed space-time codes. We obtain the average probability of error through Monte-Carlo methods and validate the conclusions drawn in our analysis. We compare the performance of randomized schemes with the centralized space-time codes for different values of N and L. In the following, we assume the nodes channel gains to the destination are i.i.d., *i.e.,* $h_k \sim \mathcal{N}_c(0, 1)$.

In the following, we evaluate the performance of Alamouti scheme under different randomization methods and compare it with a centralized space-time coding. Let $\mathbf{r}_i$ be the i'th column of the randomization matrix $\boldsymbol{\mathcal{R}}$. The randomization is done in there different ways:

i) *Uniform phase randomization*: each element of $\mathbf{r}_i$ is equal to $e^{j\theta}$ where θ is a random variable uniformly distributed in $[0, 2\pi)$.

ii) *Gaussian randomization*: $\mathbf{r}_i$'s are zero-mean independent complex Gaussian vectors with covariance $\mathbf{I}$.

iii) *Uniform spherical randomization*: $\mathbf{r}_i$'s are chosen as zero-mean independent complex Gaussian vectors with covariance $\mathbf{I}$, and then normalized to have the norm $||\mathbf{r}_i|| = 1$.

In the centralized Alamouti, half of the nodes choose to serve as the first antenna, and the other half choose to serve as the second antenna (if N is odd, at one of the nodes the power is equally distributed between two virtual antennas). The transmission power of each node is normalized so that the comparison among the schemes is fair.

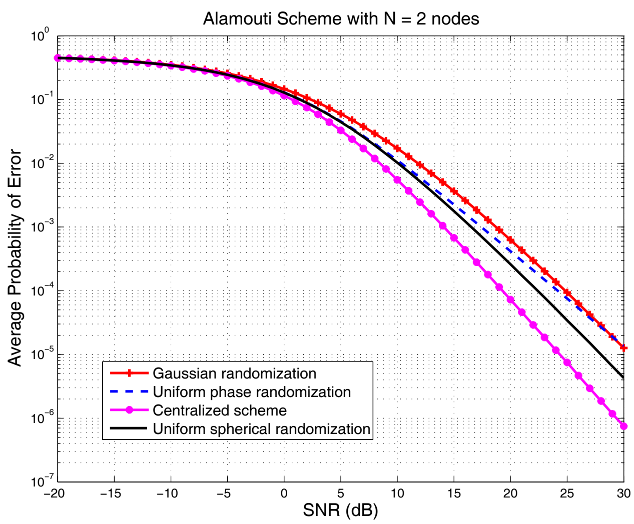

Fig. 2. Average Probability of Error versus SNR (dB) under the Alamouti scheme where L = 2 and N = 2

In Figures 2, we plot the average probability of error with respect to SNR $= 1/N_0$ for $N = 2$. For $N = 2$, the centralized scheme has diversity order $d^* = 2$ and the performance is better than the decentralized schemes. We observe that the randomized schemes have diversity order $1 \leq d^* \leq 2$. From theoretical analysis, we know that the diversity order of uniform phase randomization is $d^* = 1.5$.

5. CONCLUSION

In this paper, we proposed a decentralized space-time coding protocol for distributed networks. Our scheme is based on independent randomization done at each node. We showed that performance of the scheme in terms of diversity depends on the eigenvalue distribution of the randomization matrix around zero. Furthermore, we presented examples where the diversity order is fractional for $N = L$, where N is the number of nodes and L is the number of virtual antennas.

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