Performance of Decode-Based Differential Modulation for Wireless Relay Networks in Nakagami-m Channels

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***Abstract*— We consider a regenerative differential modulation scheme for wireless networks with relays to seek cooperative diversity. We examine the performance of a differential binary phase shift keying (BPSK) modulation scheme, referred to as the *differential decode-and-forward* (DDF), for wireless relay networks composed of one source, one relay and one destination node in Nakagami-m fading channels. A regenerative relay differentially decodes/encodes the received signal and forwards it to the destination. A closed form bit error rate (BER) expression is presented for the piece-wise linear (PL) detector. Both analytical and simulation results show that the proposed DDF scheme is capable of providing diversity gain in Nakagami-m fading channels.**

I. Introduction

Owing to the broadcasting nature of the wireless medium, transmission from a source node in a wireless network may be heard by nodes in the neighborhood. These neighbor nodes may act as *wireless relays* and provide alternative communication routes that give rise to *cooperative diversity*. Practical cooperative techniques were proposed and investigated in [1]. Several low-complexity, uncoded cooperative transmission schemes were discussed in [2] for the case of known channel state information (CSI) to transmitters and receivers, including an *amplify-and-forward* (AF) scheme by which relays amplify the received signal subject to an *instantaneous* power constraint and retransmit it to the destination, and a *decode-and-forward* (DF) scheme that performs hard decisions at the relays before retransmission.

The average bit error rate (BER) and outage probability were examined in [3] for a two-hop, single-relay no diversity system in Nakagami-m fading. A closed-form asymptotic approximation of the average symbol error rate for multi-branch multi-hop AF systems was obtained in [4]. Most of the aforementioned studies focused on coherent detection, assuming the CSI is available at the relays and destination. However, channel estimation is known to be a challenging and costly task, especially in time-selective fading environments.

In this paper, we examine the performance of a differential *decode-and-forward* (DDF) scheme proposed in [5]. We assume Nakagami-m fading, which includes Rayleigh fading as a special case. A closed form BER is derived for the proposed piece-wise linear (PL) detector.

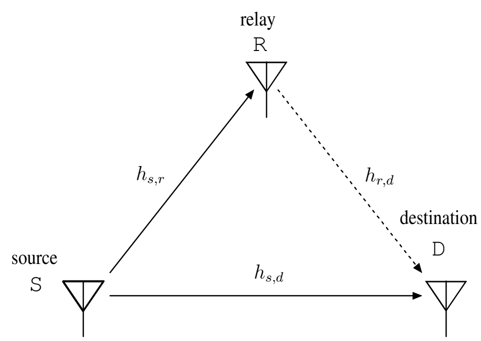

Fig. 1. A cooperative wireless relay system.

II. System Model

A wireless relay network shown in Fig. 1 is composed of one source S, one relay R and one destination D node, where a sequence of symbols are to be transmitted from S to D. To eliminate mutual interference, we assume time-division multiplexing: that divides the transmission into *two* distinct phases. During *phase-I* transmission, S transmits a frame of information bits, while R and D listen. During *phase-II* transmission, S is silent, while R transmits signals to D.

Consider using binary phase shift keying (BPSK) for *phase-I* transmission. The information bits $d(n) \in \{\pm 1\}$ at S are first differentially encoded:

$$s(n) = s(n-1)d(n), \quad n = 1, 2, \cdots, N, \tag{1}$$

where $s(n)$ denotes the signal transmitted from S, $s(0) = 1$ is the initial reference bit, and N is the number of bits within one frame. The baseband signals received at R and D, respectively, are

$$x_r(n) = h_{s,r}s(n) + w_r(n), \quad n = 0, 1, \cdots, N, \tag{2}$$
$$x_d(n) = h_{s,d}s(n) + w_d(n), \quad n = 0, 1, \cdots, N, \tag{3}$$

where $h_{s,r}$ and $h_{s,d}$ denote the corresponding fading coefficients, while $w_r(n)$ and $w_d(n)$ denote the channel noise.

For *phase-II* transmission, the signal received at D is given by

$$y_d(n) = h_{r,d}s_r(n) + u_d(n), \quad n = 0, 1, \cdots, N, \tag{4}$$

where $h_{r,d}$ and $u_d(n)$ denotes the fading and channel noise, respectively, and $s_r(n)$ is the signal transmitted by R.

For differential detection, the fading channels are assumed (approximately) static over two bit intervals. The envelope of channel, ($|h_{i,j}|$), is assumed to be Nakagami-m distributed [6]:

$$p_{|h_{i,j}|}(r) = \frac{2}{\Gamma(m_{i,j})}\left(\frac{m_{i,j}}{\Omega_{i,j}}\right)^{m_{i,j}} r^{2m_{i,j}-1}e^{-m_{i,j}r^2/\Omega_{i,j}}, \quad (5)$$

where $(i,j) \in \{(s,r),(s,d),(r,d)\}$, $m_{i,j} \geq 0.5$ denotes the fading parameter, and $\Omega_{i,j} = E\{|h_{i,j}|^2\}$. For values of $m_{i,j} < 1$, channel experiences severer fading than Rayleigh; for values of $m_{i,j} > 1$, fading severity is less than Rayleigh. The channel noise $w_r(n)$, $w_d(n)$ and $u_d(n)$ are assumed independent complex Gaussian random variables with zero mean and variance N_0. The *instantaneous SNR* between nodes i and j, denoted by $\gamma_{i,j} = |h_{i,j}|^2/N_0$, has distribution [6]:

$$p_{\gamma_{i,j}}(x) = \frac{1}{\Gamma(m_{i,j})}\left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right)^{m_{i,j}} x^{m_{i,j}-1}e^{-m_{i,j}x/\bar{\gamma}_{i,j}}, \quad (6)$$

where $\bar{\gamma}_{i,j} = \Omega_{i,j}/N_0$ denotes the *average SNR* between nodes i and j. Finally, the channel coefficients are assumed independent of one another and also of the channel noise.

III. A Regenerative DDF Relay Scheme

To facilitate analysis, we briefly introduce the maximum likelihood (ML) and the PL detectors in this section.

A. DDF Transmission at R

At relay R, the received signal is first differentially decoded as follows:

$$\tilde{d}(n) = \text{sign}(\Re\{x_r^*(n-1)x_r(n)\}) \quad n = 1,2,\cdots,N. \quad (7)$$

Next, the decoded bits are re-encoded via a differential encoder:

$$s_r(n) = s_r(n-1)\tilde{d}(n), \quad n = 1,2,\cdots,N, \quad (8)$$

with $s_r(0) = 1$.

B. Differential Detection at D

Substituting (8) into (4), we have

$$y_d(n) = y_d(n-1)\tilde{d}(n) + \tilde{v}(n), \quad (9)$$

where $\tilde{v}(n) \triangleq u_d(n) - u_d(n-1)\tilde{d}(n)$ whose conditional distribution is $\mathcal{CN}(0, 2N_0)$. Since R makes hard decision in (7), either a correct or wrong decision may occur. As a result, the conditional PDF of $y_d(n)$ takes the form of Gaussian mixture:

$$\begin{aligned} p_{y_d(n)}(y) =&(1-\epsilon)\Phi_c(y; y_d(n-1)d(n), 2N_0) \\ &+ \epsilon\Phi_c(y; -y_d(n-1)d(n), 2N_0), \end{aligned} \quad (10)$$

where $\Phi_c(y;\mu,\sigma^2)$ denotes the PDF of a complex Gaussian random variable with mean μ and variance σ^2, and ϵ is the average BER of differential BPSK in Nakagami-m fading channels:

$$\epsilon = \frac{1}{2}\left(\frac{m_{s,r}}{\bar{\gamma}_{s,r} + m_{s,r}}\right)^{m_{s,r}}. \quad (11)$$

Similarly, the conditional PDF of $x_d(n)$ is

$$p_{x_d(n)}(x) = \Phi_c(x; x_d(n-1)d(n), 2N_0). \quad (12)$$

Note the independency of $x_d(n)$ and $y_d(n)$, and the mixture distribution shown in (10). As a result, the ML detector takes a nonlinear form. Specifically, it can be shown that the *ML detector* for DDF is

$$f(t_1) + t_0 \underset{-1}{\overset{1}{\gtrless}} 0, \quad (13)$$

where

$$f(t_1) = \ln\frac{(1-\epsilon)e^{t_1} + \epsilon}{\epsilon e^{t_1} + 1 - \epsilon}, \quad (14)$$

$$t_1 = \frac{q_1}{N_0},\ q_1 = y_d^*(n-1)y_d(n) + y_d(n-1)y_d^*(n), \quad (15)$$

$$t_0 = \frac{q_0}{N_0},\ q_0 = x_d^*(n-1)x_d(n) + x_d(n-1)x_d^*(n). \quad (16)$$

The nonlinear function $f(t_1)$ effectively "clips" large inputs to $\pm\ln[(1-\epsilon)/\epsilon]$ and is approximately linear in between. In particular, it was shown that $f(t_1)$ can be approximated by a piece-wise linear (PL) function [7]:

$$f_{PL}(t_1) \triangleq \begin{cases} -T_1, & t_1 \leq -T_1 \\ t_1, & -T_1 \leq t_1 \leq T_1 \\ T_1, & t_1 \geq T_1 \end{cases} \quad (17)$$

where $T_1 = \ln[(1-\epsilon)/\epsilon]$ assuming $\epsilon < 0.5$. This leads to the following *PL detector*:

$$f_{PL}(t_1) + t_0 \underset{-1}{\overset{1}{\gtrless}} 0, \quad (18)$$

which is easier to implement than the ML detector (13).

IV. Performance Analysis

A. Average BER of PL Detector

A BER analysis of the ML detector (13) for the proposed DDF scheme is prohibitively complex due to its nonlinear nature. Instead, we derive the average BER of the PL detector (18), which closely matches the performance of the ML detector (see Section V). As a result, our analysis also provides a useful tool for the assessment of the ML detector.

The analysis is complicated by a decision statistic that involves quadratic forms in Gaussian variates. Closed form expressions of the distributions of such quadratic forms, in general, are known only via series expansion [8].

Due to the symmetric nature of the problem, we can assume without loss of generality that $d(n) = 1$ is transmitted from S. A close examination of the PL detector indicates that the error event can be represented using three mutually exclusively events. Specifically, the conditional BER of the PL detector is

$$\begin{aligned} P_e(\gamma_{s,d},\gamma_{r,d}) = &\Pr\{t_0 - T_1 < 0 | t_1 < -T_1, d(n) = 1\} \\ &\times \Pr\{t_1 < -T_1 | d(n) = 1\} \\ &+ \Pr\{t_0 + T_1 < 0 | t_1 > T_1, d(n) = 1\} \\ &\times \Pr\{t_1 > T_1 | d(n) = 1\} \\ &+ \Pr\{t_0 + t_1 < 0, -T_1 \leq t_1 \leq T_1 | d(n) = 1\}. \end{aligned} \quad (19)$$

Due to the space limitation, we skip rigorous mathematical derivations, and we can show that the conditional BER can be written as

$$P_e(\gamma_{s,d},\gamma_{r,d}) = (P_{e1}P_{e2}+P_{e3}P_{e4}+P_{e7})(1-\epsilon) + (P_{e1}P_{e3}+P_{e2}P_{e4}+P_{e8})\epsilon, \tag{20}$$

where

$$P_{e1}(\gamma_{s,d}) = 1-\frac{e^{-2\gamma_{s,d}}}{2}\sum_{k=0}^{\infty}\sum_{n=0}^{k}\frac{2^{k-n}\gamma_{s,d}^{k}}{k!(k-n)!} \times\Gamma(k+1-n,T_1), \tag{21}$$

$$P_{e2}(\gamma_{r,d}) = \frac{e^{-\gamma_{r,d}-T_1}}{2}, \tag{22}$$

$$P_{e3}(\gamma_{r,d}) = \frac{e^{-2\gamma_{r,d}}}{2}\sum_{k=0}^{\infty}\sum_{n=0}^{k}\frac{2^{k-n}\gamma_{r,d}^{k}}{k!(k-n)!} \times\Gamma(k+1-n,T_1), \tag{23}$$

$$P_{e4}(\gamma_{s,d}) = \frac{e^{-\gamma_{s,d}-T_1}}{2}, \tag{24}$$

$$P_{e7}(\gamma_{s,d},\gamma_{r,d}) = \frac{1}{2}e^{-\gamma_{r,d}} - \frac{1}{2}e^{-\gamma_{r,d}-T_1} - \frac{e^{-2\gamma_{s,d}-\gamma_{r,d}}}{4}\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{k-n}\frac{2^{k-n-m-1}\gamma_{s,d}^{k}}{k!m!} \times\gamma(m+1,2T_1) + \frac{e^{-\gamma_{s,d}-2\gamma_{r,d}}}{8} \times\sum_{k=0}^{\infty}\sum_{n=0}^{k}\frac{\gamma_{r,d}^{k}}{k!(k-n)!}\gamma(k-n+1,2T_1), \tag{25}$$

$$P_{e8}(\gamma_{s,d},\gamma_{r,d}) = \frac{e^{-2\gamma_{r,d}}}{2}\sum_{k=0}^{\infty}\sum_{n=0}^{k}\frac{2^{k-n}\gamma_{r,d}^{k}}{k!(k-n)!} \times\gamma(k-n+1,T_1) - \frac{e^{-2\gamma_{s,d}-2\gamma_{r,d}}}{8} \times\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\sum_{n=0}^{k}\sum_{j=0}^{i}\sum_{m=0}^{i-j}\frac{\gamma_{r,d}^{k}\gamma_{s,d}^{i}}{k!(k-n)!i!m!2^{j+m-i}} \times\gamma(k-n+m+1,2T_1) + \frac{1}{8}e^{-\gamma_{s,d}-\gamma_{r,d}}(1-e^{-2T_1}), \tag{26}$$

and the upper and lower incomplete Gamma function are defined as $\Gamma(a,x)=\int_x^{\infty}e^{-t}t^{a-1}dt$ and $\gamma(a,x)=\int_0^{x}e^{-t}t^{a-1}dt$, respectively. Notice that the conditional BER only depends on the instantaneous SNRs $\gamma_{r,d}$ and $\gamma_{s,d}$, and the error probability ϵ at R.

The average BER for DDF is obtained by averaging (20) using the distributions (6) of $\gamma_{r,d}$ and $\gamma_{s,d}$:

$$\bar{P}_e = \int_0^{\infty}\int_0^{\infty}P_e(\gamma_1,\gamma_2)p_{\gamma_{s,d}}(\gamma_1)p_{\gamma_{r,d}}(\gamma_2)d\gamma_1 d\gamma_2. \tag{27}$$

A close examination of (21)–(26) reveals that the two-dimensional integration in (27) is separable. Using [9, Eqn. (3.351.3)], we arrive at the following closed form expression of the average BER for DDF:

$$\bar{P}_e = (\bar{P}_{e1}\bar{P}_{e2}+\bar{P}_{e3}\bar{P}_{e4}+\bar{P}_{e7})(1-\epsilon) + (\bar{P}_{e1}\bar{P}_{e3}+\bar{P}_{e2}\bar{P}_{e4}+\bar{P}_{e8})\epsilon, \tag{28}$$

where

$$\bar{P}_{e1} = 1-\frac{1}{2\Gamma(m_{s,d})}\left(\frac{m_{s,d}}{2\bar{\gamma}_{s,d}+m_{s,d}}\right)^{m_{s,d}}\sum_{k=0}^{\infty}\sum_{n=0}^{k} \frac{2^{k-n}(k+m_{s,d}-1)!\bar{\gamma}_{s,d}^{k}\Gamma(k+1-n,T_1)}{k!(k-n)!(2\bar{\gamma}_{s,d}+m_{s,d})^{k}}, \tag{29}$$

$$\bar{P}_{e2} = \frac{(m_{r,d}-1)!}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}e^{-T_1}, \tag{30}$$

$$\bar{P}_{e3} = \frac{1}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}\sum_{k=0}^{\infty}\sum_{n=0}^{k} \frac{2^{k-n}(k+m_{r,d}-1)!\bar{\gamma}_{r,d}^{k}\Gamma(k+1-n,T_1)}{k!(k-n)!(2\bar{\gamma}_{r,d}+m_{r,d})^{k}}, \tag{31}$$

$$\bar{P}_{e4} = \frac{(m_{s,d}-1)!}{2\Gamma(m_{s,d})}\left(\frac{m_{s,d}}{\bar{\gamma}_{s,d}+m_{s,d}}\right)^{m_{s,d}}e^{-T_1}, \tag{32}$$

$$\bar{P}_{e7} = \frac{(m_{r,d}-1)!}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}(1-e^{-T_1}) - \frac{(m_{r,d}-1)!}{8\Gamma(m_{s,d})\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{2\bar{\gamma}_{s,d}+m_{s,d}}\right)^{m_{s,d}} \times\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{k-n} \frac{2^{k-n-m}(k+m_{s,d}-1)!\bar{\gamma}_{s,d}^{k}\gamma(m+1,2T_1)}{k!m!(2\bar{\gamma}_{s,d}+m_{s,d})^{k}} + \frac{(m_{s,d}-1)!}{8\Gamma(m_{s,d})\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{\bar{\gamma}_{s,d}+m_{s,d}}\right)^{m_{s,d}} \times\left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}\sum_{k=0}^{\infty}\sum_{n=0}^{k} \frac{(k+m_{r,d}-1)!\bar{\gamma}_{r,d}^{k}\gamma(k-n+1,2T_1)}{k!(k-n)!(2\bar{\gamma}_{r,d}+m_{r,d})^{k}}, \tag{33}$$

$$\bar{P}_{e8} = \frac{1}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}\sum_{k=0}^{\infty}\sum_{n=0}^{k} \frac{2^{k-n}(k+m_{r,d}-1)!\bar{\gamma}_{r,d}^{k}\gamma(k+1-n,T_1)}{k!(k-n)!(2\bar{\gamma}_{r,d}+m_{r,d})^{k}} - \frac{1}{8\Gamma(m_{s,d})\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{2\bar{\gamma}_{s,d}+m_{s,d}}\right)^{m_{s,d}} \times\left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\sum_{n=0}^{k}\sum_{j=0}^{i}\sum_{m=0}^{i-j} \frac{(k+m_{r,d}-1)!(i+m_{s,d}-1)!\bar{\gamma}_{r,d}^{k}(2\bar{\gamma}_{s,d})^{i}}{(2\bar{\gamma}_{r,d}+m_{r,d})^{k}(2\bar{\gamma}_{s,d}+m_{s,d})^{i}} \tag{34}$$

$$\times \frac{\gamma(k+m+1-n, 2T_1)}{k!(k-n)!i!m!2^{j+m}} + \frac{(m_{s,d}-1)!}{8\Gamma(m_{s,d})}$$
$$\times \frac{(m_{r,d}-1)!}{\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{\bar{\gamma}_{s,d}+m_{s,d}}\right)^{m_{s,d}}$$
$$\times\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}+m_{r,d}}\right)^{m_{r,d}}(1-e^{-2T_1}).$$

B. Alternative Average BER of PL Detector

Note that t_0 in (16) is a quadratic form of Gaussian variates $x_d(n-1)$ and $x_d(n)$. At the high SNR regime, we may ignore the cross term of the Gaussian noise components of $x_d(n-1)$ and $x_d(n)$ (see [6, p. 273]) for details) and approximate the conditional distribution of t_0 as $\mathcal{N}(2\gamma_{s,d}d(n), 4\gamma_{s,d})$, where $\mathcal{N}(\mu, \sigma^2)$ denotes a real Gaussian random variable with mean μ and variance σ^2. Likewise, at high SNR, we can approximate the distribution of t_1 as a Gaussian mixture:

$$\begin{aligned} p_{t_1}(y) \approx &(1-\epsilon)\Phi(y; 2\gamma_{r,d}d(n), 4\gamma_{r,d}) \\ &+ \epsilon\Phi(y; -2\gamma_{r,d}d(n), 4\gamma_{r,d}), \end{aligned} \tag{35}$$

where $\Phi(q;\mu,\sigma^2)$ denotes the PDF of a real Gaussian random variable with mean μ and variance σ^2. Based on the above approximations and error events defined in (19), the conditional BER can be expressed as

$$\begin{aligned} &P_e(\gamma_{s,d},\gamma_{r,d}) \approx Q\left(\frac{2\gamma_{s,d}-T_1}{2\sqrt{\gamma_{s,d}}}\right) \\ &\times\left[(1-\epsilon)Q\left(\frac{2\gamma_{r,d}+T_1}{2\sqrt{\gamma_{r,d}}}\right)\right. \\ &\left.+\epsilon Q\left(\frac{T_1-2\gamma_{r,d}}{2\sqrt{\gamma_{r,d}}}\right)\right] + Q\left(\frac{T_1+2\gamma_{s,d}}{2\sqrt{\gamma_{s,d}}}\right) \\ &\times\left[(1-\epsilon)Q\left(\frac{T_1-2\gamma_{r,d}}{2\sqrt{\gamma_{r,d}}}\right) + \epsilon Q\left(\frac{T_1+2\gamma_{r,d}}{2\sqrt{\gamma_{r,d}}}\right)\right] \\ &+\int_{-T_1}^{T_1} dy p_{t_1}(y) Q\left(\frac{2\gamma_{s,d}+y}{2\sqrt{\gamma_{s,d}}}\right), \end{aligned} \tag{36}$$

where $Q(\cdot)$ denotes the standard Gaussian Q function. Averaging the conditional BER using the distribution of $\gamma_{s,d}$ and $\gamma_{r,d}$ yields the approximate average BER, which has the same form of (27). This alternative BER expression requires numerical integration.

V. Numerical Results and Remarks

We consider a *symmetric* scenario where the average SNRs of all hops are identical: $\bar{\gamma}_{s,d} = \bar{\gamma}_{s,r} = \bar{\gamma}_{r,d}$. We compare our cooperative system to a conventional non-cooperative system that involves direct transmission from S to D with differential BPSK (DBPSK). For fair comparison, we set $\bar{\gamma}_{s,d} = \bar{\gamma}_{s,r} = \bar{\gamma}_{r,d} = 0.5E_b/N_0$, where E_b denotes the energy per bit, so that the sum of the transmitted energy from both S and R for the cooperative system is identical to that of the non-cooperative system. Fig. 2 shows the exact and the alternative average BERs for the PL detector, computer simulation results for the PL and the ML detector are also plotted in Fig. 2, along with the average BER of the non-cooperative differential BPSK for Rayleigh fading ($m_{i,j} = 1$). It is seen that our proposed DDF scheme achieves diversity gain, which depends on the fading parameter $m_{i,j}$. As $m_{i,j}$ becomes larger, the diversity gain increases. We observe that our analytical BERs (both exact and alternative) for the PL detector are in agreement with simulation results, and the performance of the PL detector is very close to that of the ML detector. Thus, we are in favor of the PL detector, and may use the analytical BER of the PL detector to assess the performance of the ML detector.

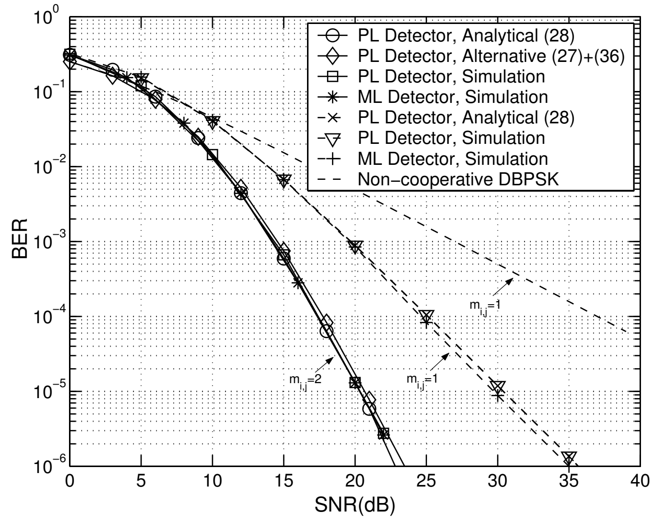

Fig. 2. BER performance of DDF in Nakagami-m fading channels($m_{i,j} = 1$ and $m_{i,j} = 2$).

VI. Acknowledgment

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