MAP DETECTION OF NONLINEARLY DISTORTED OFDM SIGNALS

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ABSTRACT

This paper presents the maximum a posteriori (MAP) detector for a multicarrier system with nonlinearity at the transmitter front-end. The analysis is based on block detection of multicarrier symbols. Since implementation of the MAP detector for a system with a large number of subcarriers is far too complex, a sub-optimum algorithm that can be implemented with acceptable complexity is also presented. It will be shown that when the nonlinearity distortion dominates the channel noise, without knowledge of the nonlinearity, the proposed detector outperforms the conventional ML detector.

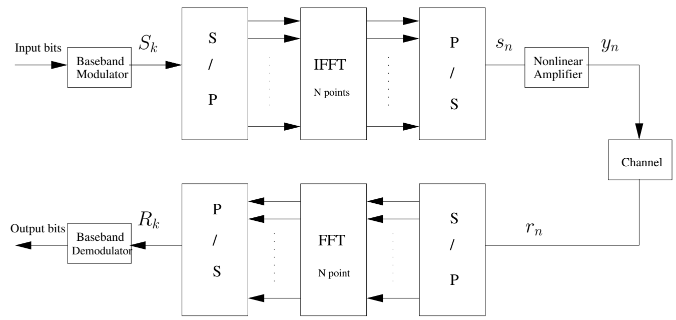

Fig. 1. Block diagram of an OFDM system.

1. INTRODUCTION

Despite the widely-known advantages of multicarrier signals for digital wireless communications, their sensitivity to nonlinearity is a major challenge for system designers. The challenge is two-fold, namely, designing the signals with low envelope fluctuations, and designing the optimum receiver for nonlinearly distorted multicarrier signal.

For an OFDM system over AWGN channel, it can be easily shown that the ML subcarrier-by-subcarrier detection is the optimum detection. This is due to the orthogonality of the Fourier transform samples, which results in zero intercarrier interference (ICI) at the receiver. However, when there is a nonlinearity in the system, because of intercarrier interference, a block OFDM detector is the optimum detector. There are of course complexity issues that make ML block detection of OFDM very difficult.

In multicarrier systems, it is customary to consider the intercarrier interference at the a receiver as a noise-like distortion [1]. In [2] and [3], the nonlinearity distortion is estimated and subtracted from the received signal. The receiver is however an ML receiver for signal plus a Gaussian distributed noise.

In this paper we derive the MAP detector for an OFDM system with a soft limiter at the transmitter front-end. The method is similar to maximum likelihood sequence detection. First, we consider a nonlinear system with no AWGN channel, and derive the MAP detector for this system. This will be the general MAP detector for a nonlinear system with Gaussian input signal. Later, we apply the MAP detector to an OFDM system with nonlinearity and AWGN channel. Our simulations show that at high SNR the proposed MAP detector outperforms the conventional ML detector.

The authors wish to thank Antar Bandyopadhyay, for his help in analysis presented in Section 3.

2. SYSTEM MODEL

The OFDM system under consideration is shown in Figure 1. First, the information bits are mapped into baseband QPSK symbols $\{S_k\}$. In each frame, a block of complex baseband symbols is transformed by taking Inverse Fast Fourier Transform (IFFT) and parallel to serial conversion, to the baseband OFDM signal. The Nyquist rate sampled OFDM signal is described as

$$s_n = \sum_{k=0}^{N-1} S_k e^{(\frac{j2\pi kn}{N})}, \quad n = 0, \dots, N-1, \tag{1}$$

where N is the number of subcarriers.

The nonlinearity under consideration is a soft limiter with the input-output characteristic given by

$$y_n = \begin{cases} x_n, & \text{if } |x_n| \leq A \\ A\exp(j\arctan\frac{\Im(x_n)}{\Re(x_n)}), & \text{if } |x_n| > A, \end{cases} \tag{2}$$

where x_n is the input and y_n is the output signal and A is the saturation level. In order to specify the operating point of the nonlinearity, we define

$$\text{IBO} = 10 \log_{10} \frac{A^2}{\sigma^2}, \tag{3}$$

where σ^2 is the variance of the input signal.

The resulting signal is passed through a complex additive white Gaussian noise (AWGN) channel. The signal at the receiver is

$$r_n = y_n + w_n = s_n + d_n + w_n, \tag{4}$$

where d_n is the nonlinear distortion which is dependent on s_n, and w_n is the AWGN noise.

At the receiver, after serial to parallel conversion, each OFDM frame is converted into a sequence of distorted QPSK symbols $\mathbf{R} = \{R_0, \dots, R_{N-1}\}$ by Fast Fourier Transform (FFT) as

$$R_k = \sum_{n=0}^{N-1} r_n e^{\frac{-j2\pi nk}{N}}, \quad k = 0, \dots, N-1. \tag{5}$$

3. THEORETICAL ANALYSIS

In this section, we study the effects of the nonlinearity. We consider the nonlinear block in Figure 1, input to which is the sequence of complex symbols $\mathbf{s} = \{s_0, \dots, s_{N-1}\}$ and $\mathbf{y} = \{y_0, \dots, y_{N-1}\}$ is the complex output symbols.

By the central limit theorem and with a large number of subcarriers N, the signal at the output of the IFFT block, i.e. $\mathbf{s} = \{s_0, \dots, s_{N-1}\}$, is a complex $N(0, \sigma^2)$ distributed with variance equal to the power of the signal. Without loss of generality, we assume that the variance of the input sequence $\mathbf{s}$ is unity and only one frame is being transmitted. The MAP detector for the sequence $\mathbf{y} = \{y_0, \dots, y_{N-1}\}$ can be written as

$$\hat{\mathbf{s}} = \arg \max_i \{p(\mathbf{s}_i, \mathbf{y})\}. \tag{6}$$

Subscript i refers to the different OFDM symbols. For a QPSK modulation with N number of subcarriers $i = 1, \dots, 4^N$. This detector is not feasible due to the high number of signal alternatives, when N is large. However, the implementation issue is discussed in Section 4.

Because of the nonlinearity, the samples of the transmitted sequence $\mathbf{s}$ that lie outside the circle of radius A in the complex plane, get compressed and lie uniformly on its perimeter. Let us define G_1 to be the set of unclipped samples with total number of elements g_1, and G_2 to be the set of clipped samples with a total number of elements g_2, with $g_1 + g_2 = N$. Note that the samples s_n are uncorrelated and for a large number of subcarriers they are approximately Gaussian distributed. Hence, samples s_n are approximately independent. Therefore, we can write

$$\begin{aligned} p(\mathbf{s}_i, \mathbf{y}) &\approx \prod_{n=0}^{N-1} p(s_{i,n}, y_n) \\ &= \prod_{n \in G_1} p(s_{i,n}, y_n \big| |s_{i,n}| \le A) p(|s_{i,n}| \le A) \\ &\times \prod_{n \in G_2} p(s_{i,n}, y_n \big| |s_{i,n}| > A) p(|s_{i,n}| > A). \end{aligned} \tag{7}$$

The joint probability of $s_{i,n}$ and y_n is equivalent to the probability of having the distortion $|d_{i,n}| = |y_n - s_{i,n}|$. Let us model the envelope of nonlinear distortion $|d_{i,n}|$. The probability of having distortion $|d_{i,n}|$ is same as the probability of having $|s_{i,n}| = |d_{i,n}| + A$ given $|s_{i,n}| > A$.

Since the input signal to the nonlinear amplifier is complex Gaussian, its envelope will be Rayleigh distributed. So, we can write

$$\Pr(|s_{i,n}| > A) = e^{-\frac{A^2}{2}}. \tag{8}$$

Now consider

$$\begin{aligned} \Pr(|s_{i,n}| \le t \big| |s_{i,n}| > A) &= 1 - \Pr(|s_{i,n}| > t \big| |s_{i,n}| > A) \\ &= 1 - \frac{\Pr(|s_{i,n}| > t)}{\Pr(|s_{i,n}| > A)} \\ &= 1 - e^{-\frac{t^2 - A^2}{2}}, \end{aligned} \tag{9}$$

where $t \in (A, \infty)$ is the realization of the random variable $|s_{i,n}|$. By differentiating equation (9) with respect to t, we get the conditional pdf as

$$p(t \big| |s_{i,n}| > A) = t e^{-\frac{t^2 - A^2}{2}}. \tag{10}$$

From above we can get the pdf of $|d_{i,n}|$ by substituting $t = |d_{i,n}| + A$, since A is a constant

$$p(|d_{i,n}|) = (|d_{i,n}| + A) e^{-\frac{|d_{i,n}|^2 + 2A|d_{i,n}|}{2}}, \tag{11}$$

where $|d_{i,n}| \in (0, \infty)$. We can now write

$$\begin{aligned} p(s_{i,n}, y_n \big| |s_{i,n}| > A) &= p(|d_{i,n}|) \\ &= (|d_{i,n}| + A) e^{-\frac{|d_{i,n}|^2 + 2A|d_{i,n}|}{2}}. \end{aligned} \tag{12}$$

Now consider

$$\prod_{n \in G_1} p(s_{i,n}, y_n \big| |s_{i,n}| \le A) = \begin{cases} 1, & \text{if } s_{i,n} = y_n, n \epsilon G_1 \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

and also

$$
\begin{aligned}
&\prod_{n\in G_2} p(s_{i,n}, y_n \big| |s_{i,n}| > A) \\
&= \prod_{n\in G_2} (|d_{i,n}| + A) e^{-\frac{1}{2}((|d_{i,n}|+A)^2 - A^2)} \\
&= (A^{g_2} + A^{g_2-1} \sum_{n\in G_2} |d_{i,n}| + A^{g_2-2} \sum_{n\in G_2} |d_{i,n}||d_{i,n-1}| \\
&+ \cdots) \times \exp(-\frac{1}{2} \sum_{n\in G_2} ((|d_{i,n}| + A)^2 - A^2)). \qquad (14)
\end{aligned}
$$

Taking natural logarithm on both side of (14) we get

$$
\begin{aligned}
&\ln(\prod_{n\in G_2} p(s_{i,n}, y_n \big| |s_{i,n}| > A)) \\
&= \ln(A^{g_2} + A^{g_2-1} \sum_{n\in G_2} |d_{i,n}| + A^{g_2-2} \sum_{n\in G_2} |d_{i,n}||d_{i,n-1}| \\
&+ \cdots) - \frac{1}{2}(\sum_{n\in G_2} (|d_{i,n}|)^2 + 2A \sum_{n\in G_2} (|d_{i,n}|)). \qquad (15)
\end{aligned}
$$

Since logarithm varies slower as compared to the square and the linear terms, the only terms which have large influence on (15) are $\sum_{n\in G_2}(|d_{i,n}|)^2$ and $\sum_{n\in G_2}(|d_{i,n}|)$. Simulation results have shown that the term $\sum_{n\in G_2}|d_{i,n}|$ has larger influence on (15) at low IBOs. But when IBO is increased, very few samples will be clipped and the effect of $\prod_{n\in G_2} p(s_{i,n}, y_n \big| |s_{i,n}| > A)p(|s_{i,n}| > A)$ in the (7) decreases. In other words, it means that at high IBO most of the time output signal will be undistorted and have the distribution which tends to become Gaussian. Under the condition of low IBO, in order to maximize (15), the detector needs to select $\mathbf{s}_i$ or equivalently i which minimizes $\sum_{n\in G_2} |d_{i,n}|$. If we take into account (13), this results in the minimization of $\sum_{n=0}^{N-1} |d_{i,n}|$. This is the Manhattan distance between $\mathbf{s}_i$ and $\mathbf{y}$. Hence, in order to maximize (6), receiver needs to find $\mathbf{s}_i$ which has the minimum Manhattan distance from $\mathbf{y}$. All the analysis has been carried out in the time domain without considering the effect of channel noise. As will be shown by simulations, above analysis is also valid in the presence of low noise in the system.

4. PROPOSED ALGORITHM AND SIMULATIONS

Since the number of signal alternatives is too high for implementation of MAP detection (see Section 3), we now propose a sub-optimal greedy algorithm to implement the finding from above. We present this algorithm in the context of QPSK modulation, but it can be modified for other modulation techniques. For clarity, let us denote received sequence in the time domain as $\mathbf{r} = \{r_0, r_2, \ldots, r_{N-1}\}$ and demodulated OFDM sequence in the frequency domain as $\mathbf{R} = \{R_0, R_2, \ldots, R_{N-1}\}$. First, we need to identify the regions where compression of modulated baseband symbols has occurred because of nonlinearity. In the case of QPSK, the received symbols close to real and imaginary axis are more likely to be erroneous. Now the algorithm can be described in the following steps.

1. Find the weakest subcarrier R_k in the sequence $\mathbf{R}$. By weakest, we mean the subcarrier which is closest to the decision boundaries.
2. Find two neighboring QPSK symbols Q_1 and Q_2 in the constellation which are nearest to the symbol R_k.
3. Make a hard decision on the sequence R and denote the resulting sequence as $\mathbf{A} = [A_0, A_2, \ldots, A_k, \ldots, A_{N-1}]$. Note that symbols in $\mathbf{A}$ are QPSK symbols.
4. Create two replicas of $\mathbf{A}$, namely $\mathbf{A}_1$ and $\mathbf{A}_2$.
5. Replace the k^{th} symbols in $\mathbf{A}_1$ and $\mathbf{A}_2$ by the symbols Q_1 and Q_2 respectively.
6. Take N point IFFT of $\mathbf{A}_1$ and $\mathbf{A}_2$ and denote them by $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively.
7. Compute the Manhattan distance D_1 and D_2 between $\mathbf{r}$ and $\mathbf{r}_1$, i.e, $\sum_{n=0}^{N-1} |r_n - r_{1,n}|$ and between $\mathbf{r}$ and $\mathbf{r}_2$, i.e $\sum_{n=0}^{N-1} |r_n - r_{2,n}|$ respectively .
8. If $D_1 > D_2$, replace A_k by Q_1, otherwise by Q_2.
9. If required, check next subcarrier closest to the decision boundaries in the sequence $\mathbf{R}$ and repeat the algorithm iteratively from step 2.
10. Demodulate the sequence $\mathbf{A}$ to get the information bits.

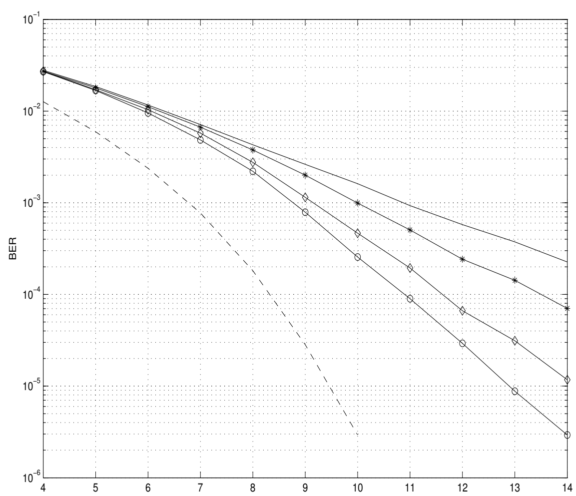

Fig. 2. BER vs SNR of the proposed method for an AWGN channel and a soft limiter with IBO=0 dB for a QPSK modulated signal. (–) without correction, (- -) without clipping, (*) 1 subcarrier correction, (◇) 3 subcarriers correction, (○) 5 subcarriers correction.

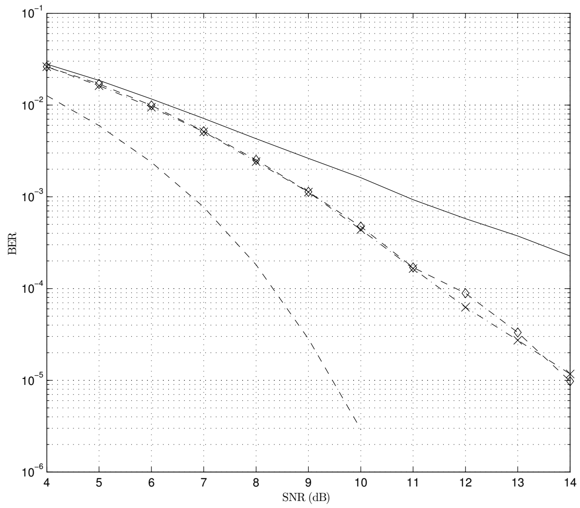

Fig. 3. Comparison of Euclidean and Manhattan distance (3 subcarriers correction), IBO=0 dB. (–) no correction, (- -) no clipping, ($\times$) Manhattan distance, ($\diamond$) Euclidean distance with the knowledge of nonlinearity at the receiver.

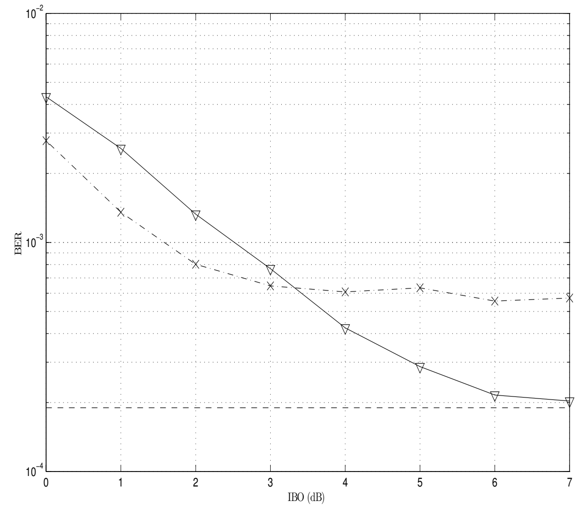

Fig. 4. BER vs IBO for SNR=8 dB (3 subcarriers correction). ($\times$) Manhattan distance cancellation, (∇) no cancellation, (- -) no clipping.

Next, we present simulation results of the proposed algorithm. The number of subcarriers in all simulations has been kept constant to $N = 256$. Figure 2 shows BER versus SNR curves for different number of subcarriers chosen for correction. Improvement greater than 3 dB between system with no correction and system with 5 subcarriers correction is visible around BER of 10^{-4}. Figure 3 compares the improvement given by using Manhattan distance as a distortion criterion with respect to Euclidean distance with knowledge of nonlinear function at the receiver and using the same algorithm (3 subcarriers correction) for both distances. Note that for Euclidean distance to give gain, the knowledge of nonlinearity is necessary at the receiver [2]. Without this knowledge, Euclidean distance performs same as a system without cancellation, while Manhattan distance without knowledge of nonlinearity gives significant improvement at SNR higher than 7 dB, but at low IBO. This is approximately the same as Euclidean distance with the knowledge of nonlinearity. Next, we compare the BER vs IBO curves given by the proposed algorithm and the system without cancellation. As seen from the curves, in Figure 4, this algorithm performs better than conventional detector, when the IBO is low. It is visible that as IBO is increased, the Manhattan distance starts to perform worse than the Euclidean distance. This is because, with increasing IBO the effect of $\prod_{n \in G_2} p(s_{i,n}, y_n \big| |s_{i,n}| > A) p(|s_{i,n}| > A)$ in (7) starts to decrease. The reason is that the clipping level A increases and the signal at the output of nonlinearity tends to become more Gaussian as explained in Section 3. But with this system when IBO is more than 4 dB the effect of nonlinearity is not profound.

5. CONCLUSION

In this paper, we derived the maximum a posteriori detector for OFDM symbols with nonlinearity. The MAP detector works on the received signal before the FFT at the receiver. We also presented a sub-optimum algorithm to implement the proposed detector. It was shown that when the system operates at low IBO, the Manhattan distance is a better measure than the Euclidean distance because it does not require the knowledge of nonlinearity at the receiver.This was also shown by several simulations.

6. REFERENCES

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