BIT AND POWER ALLOCATION FOR GOODPUT OPTIMIZATION IN CODED OFDM SYSTEMS

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ABSTRACT

We focus on resource allocation in a coded orthogonal frequency division multiplexing system. A frame-oriented transmission with convolutional coding and Viterbi decoding is considered. We suppose that only error-free frames are kept by the receiver, other frames are thrown away and have to be retransmitted. As a consequence, the only meaningful criterion for evaluating the performance of such a system is the error-free rate, or goodput. This criterion is a good compromise between the rate and bit error probability criterions. We here present a formulation for the goodput of the system and propose a bit and power allocation algorithm maximizing it. The proposed algorithm is then compared with other existing resource allocation strategies. Simulation results show that the proposed algorithm significantly improves the performance.

1. INTRODUCTION

Multicarrier modulation, known as orthogonal frequency division multiplexing (OFDM), has become very popular in wireless communication [1]. It deals efficiently with frequency selective channels. The spectrum is divided into orthogonal narrowband subchannels, known as subcarriers. Multiple data substreams are transmitted in parallel through these subcarriers. Since different subcarriers experience different channel gains, allocating bits and power adaptively may improve the performance (assuming that the channel state information is available at the transmitter side). This problem has been widely studied, but most solutions are allocating resources in order to minimize the bit error rate (BER) under a rate constraint, or to maximize the rate under a BER constraint.

A particular user of a communication system is only interested in one quantity: the error-free rate, or goodput, achieved by the system. In fact, it is meaningless to achieve a high rate if it is received with many errors. Similarly, it is not interesting to achieve a very low BER if the corresponding rate is ridiculously low. The real challenge is to come to the best compromise between rate and BER criterions. This is exactly what the goodput criterion expresses. In [2], the goodput was used as criterion to dynamically select the best physical layer mode for IEEE 801.11a wireless LANs. This has been extended to a multi-user scheme in [3].

In this paper, we propose a bit and power allocation algorithm maximizing the goodput of a coded OFDM system. A frame-oriented transmission with convolutional coding and Viterbi decoding is considered. The rest of the paper is organized as follows. We start in section 2 by giving the system model. Section 3 mathematically formulates the allocation problem, while the proposed algorithm is presented in section 4. It is then simulated and compared with other allocation strategies in section 5. Finally, we conclude in section 6.

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2. SYSTEM MODEL

The communication system considered in this paper is depicted in Fig. 1. The information bits are first convolutionally encoded, then transmitted through an adaptive OFDM system, and finally Viterbi decoded. In this section, first, the adaptive OFDM system is described and, then, a bound for the frame error rate of the whole coded system is presented.

The OFDM system is adaptive in the sense that power and bits are adaptively allocated to the subcarriers, depending on the channel state (see Fig. 1). Denoting by N the number of subcarriers, we have the following model for the received signal on the kth subcarrier

$$r_k = \sqrt{p_k}\,\Omega_k\,s_k + n_k \qquad k = 1, \ldots, N \tag{1}$$

where p_k is the power allocated to the kth subcarrier, and Ω_k is the channel gain on the kth subcarrier. The N channel gains are given by the N-point FFT of the frequency selective channel impulse response. The noise samples n_k are assumed to be i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance σ_n^2. Finally, s_k is the data symbol transmitted on the kth subcarrier. We consider QAM symbols with unit variance. We will denote by m_k the number of bits in the constellation used on the kth subcarrier. After equalization, hard-decision is taken on the received signal

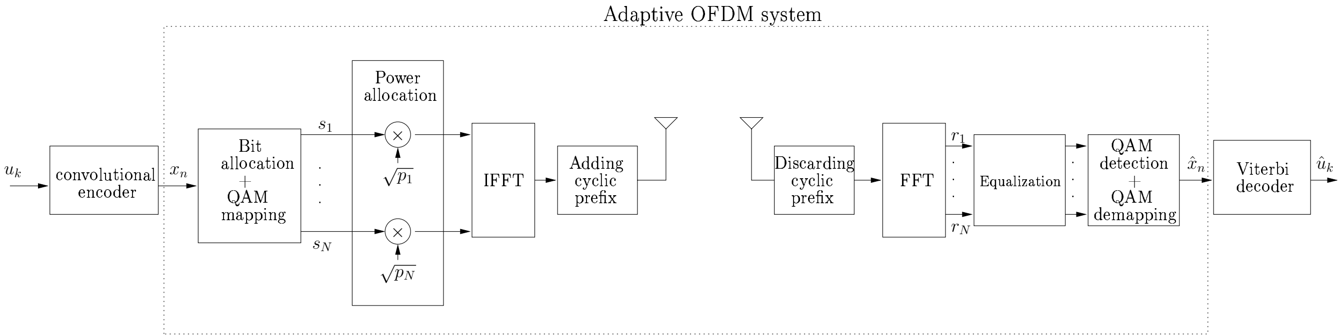

Fig. 1. System structure: convolutional encoder, adaptive OFDM system, and Viterbi decoder.

to recover the QAM symbol s_k. Let us denote by ρ_k the bit error probability on the kth subcarrier, associated with this hard-decision making. The approximate BER expression for QAM constellations given in [4] will be used in this paper:

$$\rho_k = c_1 \exp\left(-\frac{c_2\, \Omega_k^2\, p_k}{(2^{m_k}-1)\sigma_n^2}\right) \tag{2}$$

with $c_1 = 0.2$, and $c_2 = 1.6$.

Let us now consider the whole communication system in Fig.1, taking also into account convolutional coding and Viterbi decoding. A packet-oriented transmission is considered. The information bits are organized in frames of N_f bits. Only error-free frames are kept by the receiver, other frames are thrown away and have to be retransmitted. In order to evaluate performance of this system, we will further need an expression for the frame success probability (FSR). In [5], a lower bound was given for the FSR under the assumption of binary convolutional coding and hard-decision Viterbi decoding with independent errors:

$$FSR(\rho, l) \geq \left[1 - P_u(\rho)\right]^l, \tag{3}$$

where l is the frame length in coded bits, ρ is the BER associated with the hard-decision making.The union bound $P_u(\rho)$ of the first-event probability is given by

$$P_u(\rho) = \sum_{d=d_{free}}^{\infty} a_d\, P_d(\rho), \tag{4}$$

where d_{free} is the free distance of the convolutional code, a_d is the total number of paths at distance d from the correct path, and $P_d(\rho)$ is the probability that an incorrect path at distance d from the correct path is chosen by the Viterbi decoder. For a hard-decision decoder, $P_d(\rho)$ is given by

$$P_d(\rho) = \begin{cases} \sum_{v=(d+1)/2}^{d} \binom{d}{v} \rho^v\, (1-\rho)^{d-v}, & \text{if } d \text{ is odd} \\ \frac{1}{2}\binom{d}{d/2}\rho^{d/2}(1-\rho)^{d/2} \\ +\sum_{v=d/2+1}^{d} \binom{d}{v} \rho^v\, (1-\rho)^{d-v}, & \text{if } d \text{ is even.} \end{cases} \tag{5}$$

3. PROBLEM FORMULATION

As introduced before, we are interested in allocating resource (power and bits) in order to improve the goodput achievable by the system described in section 2. The mathematical formulation of this problem is presented in this section.

In the considered communication system, only error-free frames are kept by the receiver, other frames are thrown away and have to be retransmitted. As a consequence, when evaluating the performance of such system, the only meaningful criterion is the number of error-free frames transmitted by unit of time, or error-free rate. Let's use, as unit of time, the duration of an OFDM symbol. Typically, one OFDM symbol corresponds to only a fraction of a frame, in terms of number of information bits transmitted. Therefore, we want to maximize, per OFDM symbol, the number of information bits transmitted that will belong to an error-free frame, or goodput (GP), which can be expressed as follows:

$$\max_{m_k, p_k} GP = \left(r \sum_{k=1}^{N} m_k\right)\; FSR\left(\rho, \frac{1}{r} N_f\right) \tag{6}$$

subject to

$$\sum_{k=1}^{N} p_k \leq P_T \tag{7}$$

$$m_k \in \mathcal{M}, \quad k = 1, \ldots, N \tag{8}$$

where r is the rate of the convolutional code, and P_T is the total power available for the OFDM symbol. Recall that m_k is the number of bits in the constellation used on the kth subcarrier. The set $\mathcal{M}$ is defined as the union of the possible constellations (in bits) together with 0 (no transmission). In this paper, we consider three possible constellations: 4-QAM, 16-QAM and 64-QAM. We have

$$\mathcal{M} = \{0, 2, 4, 6\}.$$

In the expression (6) measuring the goodput, the first parenthesis expresses the number of information bits in the OFDM symbol. This quantity is then multiplied by the probability that these bits belong to a frame that will be received without any error. Note that the size of a coded frame is $\frac{1}{r}N_f$. It is also important to note that, even though one frame is spread over several OFDM symbols, only one bit error probability ρ is used in the FSR expression. This hypothesis is valid if consecutive OFDM symbols achieve similar bit error rates and thus if the channel does not change drastically over OFDM symbols belonging to the same frame. In other words, this paper assumes channels with slow fading.

4. THE PROPOSED ALGORITHM

In this section, we propose an algorithm for solving the problem introduced in section 3. We first present how power is allocated, for a given bit allocation. Then, making use of that result, a bit allocation algorithm is described.

4.1. Power allocation strategy

The expressions (3) - (5) for the FSR, suppose the existence of a single value for the BER ρ resulting from how resources are allocated in the adaptive OFDM system. Moreover, this value was supposed constant over the whole frame. Thanks to the slow fading hypothesis, this will be satisfied if we impose a constant BER on each OFDM symbol. This is precisely the basic idea of our power allocation strategy.

Let us suppose a given bit allocation $m_1, m_2, \ldots, m_N$. Like it has just been said, we are looking for the power allocation $p_1, \ldots, p_N$ such that the BER is constant over all subcarriers:

$$\rho_k = \rho, \quad \forall k \in \{1, \ldots, N\} : m_k \neq 0$$

under the power constraint (7). This equation system has the following closed-form solution, using (2):

$$p_k = \begin{cases} \frac{P_T}{\sum_{j=1}^{N} \frac{2^{m_j}-1}{2^{m_k}-1} \frac{\Omega_k^2}{\Omega_j^2}}, & \text{if } m_k \neq 0 \\ 0 & \text{if } m_k = 0 \end{cases} \tag{9}$$

In fact, no power should be allocated to a subcarrier which does not carry any bit.

4.2. Bit allocation algorithm

From last section, we are able to compute the power allocation (9) for a given bit allocation, and deduce a value for the BER (2) and for the goodput (6). From that, it is easy to construct an algorithm allocating both bits and power. We start with a null bit allocation on each subcarrier. We then proceed iteratively. At each step, we propose the allocation of two more bits on the kth subcarrier, for each $k \in \{1, \ldots, N\}$. Thanks to last section, we can associate with each of these N proposals, a new power allocation and thus a new goodput value. We choose the proposal with highest new goodput value, but only if this value is greater than the value that was reached at the previous step (otherwise the algorithm stops). The algorithm can be written as follows:

1. Set $\mathcal{K} = \{1, \ldots, N\}$, set $\mathcal{M} = \{0, 2, 4, 6\}$
2. Set $m_k = 0, \forall k \in \mathcal{K}$. Set $GP = 0$.
3. Using equations (9) and (2) - (6), $\forall k \in \mathcal{K}$ compute $GP_k(m_k + 2)$, the goodput value achieved if the constellation on the kth subcarrier is increased by 2 bits. If $m_k = 6$, set $GP_k = 0$, since we chose 64-QAM as the highest constellation size.
4. Select $k^* = \max_k GP_k$.
5. If $GP_{k^*} > GP$, $GP \leftarrow GP_{k^*}$ and $m_{k^*} \leftarrow m_k + 2$, and go to step 3. Else, algorithm finishes.

5. PERFORMANCE EVALUATION

In this section, the performance of the proposed algorithm are evaluated and compared to other resource allocation strategies.

The simulation parameters are the following: $N_f = 128$, $N = 32$ and $\sigma_n^2 = 1$. The convolutional code used has memory order 2, rate $r = 1/2$, and generator polynomial [6,7] in octal notation. Moreover, we consider 7 taps long channel impulse responses. The taps are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance such that the impulse response has unitary mean energy. All curves that will be presented result from an average over several channel realizations.

The proposed algorithm is first compared with the case where uniform power and bit strategies are chosen. In this case, the same constellation is used on all N subcarriers and the power P_T is uniformly allocated to all subcarriers. Fig. 2 shows the achievable goodput (in bits/OFDM symbol) as a function of P_T/σ_n^2, and compares the proposed scheme with the uniform power and bit allocation for the three constellations considered in this paper (4-QAM, 16-QAM or 64-QAM). We see that, for uniform power and bit allocation, one constellation is best for a given P_T/σ_n^2 region (for example, 64-QAM is best above 40 dB). Most importantly, it is shown that the proposed algorithm by far outperforms uniform power and bit allocation. The improvement reaches value such as more than 30 bits per OFDM symbol, which is significant. Note that at very high P_T/σ_n^2, uniform power allocation with 64-QAM on each subcarrier tends to be optimal. Also note that the fact that the proposed algorithm reaches a maximum value is only due to the fact that 64-QAM was imposed as maximum constellation.

The proposed scheme is now compared to another classi-

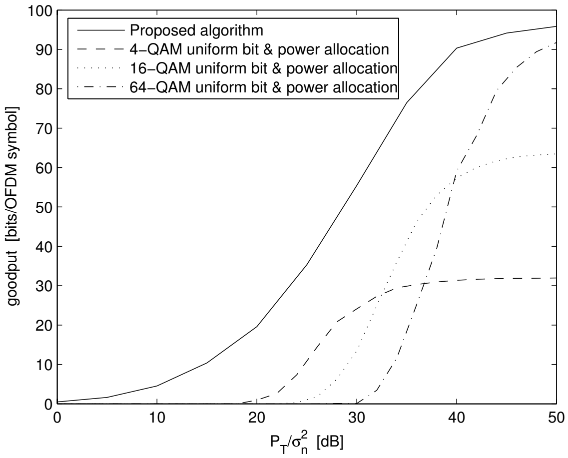

Fig. 2. Goodput comparison between the proposed algorithm and a uniform bit and power allocation.

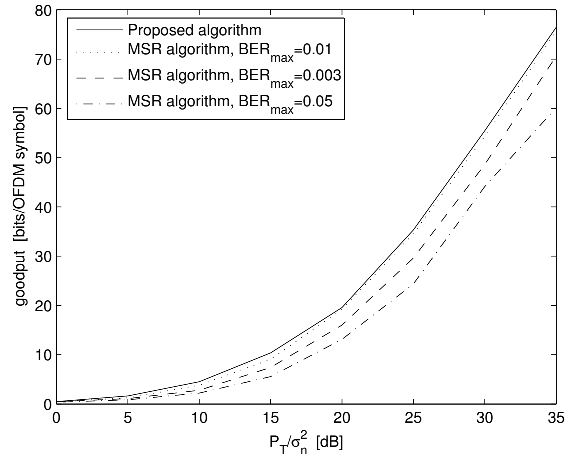

Fig. 3. Goodput comparison between the proposed algorithm and the MSR algorithm for different BER constraint values.

cal resource allocation strategy. In [6], a bit and power allocation algorithm was proposed in order to maximize the total rate of the system, under a maximum BER constraint (BER_{max}). We will denote this algorithm by MSR (for maximum sum rate), it solves the following problem:

$$\max_{m_k, p_k} \sum_{k=1}^{N} m_k$$

subject to (7), (8), and $\rho_k \leq \text{BER}_{\text{max}} \quad \forall k = 1, \ldots, N$.
Fig. 3 compares the MSR algorithm with the proposed algorithm, in terms of achievable goodput. We see that, when $\text{BER}_{\text{max}} = 0.01$, the MSR algorithm performs close to the proposed algorithm. However, if we choose a different value for the parameter BER_{max}, the MSR algorithm diverges from what the proposed algorithm achieves (see curve for $\text{BER}_{\text{max}} = 0.05$ and 0.003). Note that, when you take into account the very small duration of one OFDM symbol, a goodput increase of 1 bit per OFDM symbol is a significant improvement.

6. CONCLUSIONS

We have considered the problem of allocating bits and power in a convolutionally coded adaptive OFDM system with Viterbi decoding. The objective was to maximize the error-free rate, or goodput. This criterion has been chosen because it reaches a compromise between the rate and BER criterions, and because it really expresses what a user is looking for in a communication system. We presented a formulation of the goodput, under the assumption of channels with slow fading. An iterative algorithm was then proposed to solve the allocation problem. This algorithm has been shown to outperform significantly the uniform bit and power allocation strategy. We also compared it with an MSR algorithm whose objective is the maximization of the rate under a BER constraint. It has been found that the MSR algorithm is only slightly outperformed by the proposed algorithm for some given value of the BER constraint, but significantly outperformed by the proposed scheme for other values. While, when using the MSR algorithm, we have to look for the best value for the BER constraint, the proposed algorithm automatically chooses the best value to optimize the goodput. As a conclusion, the proposed algorithm is a promising solution.

7. REFERENCES

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