Constant-modulus Preamble Design for MIMO-OFDM Systems

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***Abstract*— A new preamble design for MIMO-OFDM is proposed in this paper. Based on the Chu sequence, the proposed preamble can be used to achieve frequency offset estimation, fine time offset estimation, and optimal least-squares channel estimation. In addition, since the preamble has constant modulus, it has low peak-to-average power ratio. Furthermore, channel estimation can be done through simple phase and magnitude justification, and enjoys low computational complexity.**

I. Introduction

For multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM), preambles are often employed to estimate the time offset (TO), frequency offset (FO), and multiple channels. Since preambles convey no information, it is important to design them efficiently. In addition, for better efficiency of power amplification, it is also critical to lower the peak to average (PAR) power ratio as much as possible.

Traditionally, different preambles have to be employed to handle those estimation tasks individually (see e.g., [1] and [2]). However, as pointed out by [3], to improve power and bandwidth efficiency, it makes more practical sense to consider designing one preamble for joint TO, FO, and channel estimation. Examples of such designs can be found in [3]–[10]. Particularly, [8] proposes to send training sequences through different antennas in a time division fashion. Although this design is simple and straightforward, it is not bandwidth efficient. [10] proposes a design where the preambles have a repeated structure similar to that in [11]. However, [10] didn't address the problem of fine TO estimation. In [3]–[5], the preamble is composed of several mini-OFDM blocks and the number of mini-OFDM blocks equals the number of transmit antennas. Unlike [10], [3]–[5] considers complete estimation. However, the performance of their fine TO estimation suffers from channel frequency selectivity, and channel estimation is not optimal since more than one cyclic prefixes have to be dropped. In addition, the PAR issue has not been addressed.

In this paper, we propose a novel constant-modulus preamble design for MIMO-OFDM. Constructed from Chu sequences [12], this preamble can be used to achieve FO estimation, fine TO estimation, and optimal least-squares (LS) channel estimation. Compared with the design in [3], the proposed design has the following advantages:

- Fine TO estimation is more accurate;
- Channel estimation is optimal for LS estimator;
- The PAR is lower;
- Channel estimation requires less calculations.

Notations: column vectors (matrices) are denoted by boldface lower (upper) case letters; superscripts ^{T}, and $^\mathcal{H}$ stand for transpose, and conjugate transpose, respectively; $\boldsymbol{I}_K$ denotes the $K \times K$ identity matrix; $\mathbb{Z}$ and $\mathbb{C}$ stand for integer set and complex set, respectively; $[\boldsymbol{x}]_k$ denotes the kth entry of vector $\boldsymbol{x}$.

II. System model

Consider an MIMO-OFDM system with N_t transmit antennas and one receive antenna. The channel between the μth transmit antenna and the receive antenna is represented by $\boldsymbol{h}_\mu := [h_\mu[0], \ldots, h_\mu[L]]^T$. Let $\boldsymbol{t}_\mu := [t_\mu[0], \ldots, t_\mu[K-1]]^T$ be the training sequence which is transmitted through the μth antenna with a cyclic prefix (CP) of length L_{cp} inserted at the front. The kth symbol transmitted at the μth antenna is given by

$$s_\mu[k] = \begin{cases} t_\mu[k] & k = 0, \ldots, K-1 \\ t_\mu[K+k] & k = -L_{\text{cp}}, \ldots, -1 \end{cases}. \tag{1}$$

Let $\tau \in \mathbb{Z}$ and ξ denote the FO and TO, respectively. After discarding the CP, at the kth time instant, the receiver receives

$$x[k;\tau,\xi] = e^{j\frac{2\pi}{K}k\xi} x[k+\tau]$$

where

$$x[n] = \sum_{\mu=1}^{N_t} \sum_{l=0}^{L} h_\mu[l] s_\mu[n-l] + w[n], \tag{2}$$

is the received sample in the absence of both FO and TO offsets, and $w[n]$ stands for i.i.d., zero-mean additive white Gaussian noise (AWGN) with variance σ_w^2. In this work, our goal is to design training sequences $\boldsymbol{t}_\mu$'s, such that reliable time/frequency offset and channel estimates can be acquired from $x[k;\tau,\xi]$'s.

III. Preamble Design

Assume that K is even. Our design of $\boldsymbol{t}_\mu$'s begins with constructing a base sequence $\bar{\boldsymbol{t}} \in \mathbb{C}^{K\times 1}$ with the kth entry

$$\bar{t}[k] = \alpha e^{-j\frac{2\pi}{K}Mk^2}, \tag{3}$$

where $\alpha = \sqrt{\frac{P_T}{N_t K}}$ with P_T being the total power spent on the training, and M is an integer satisfying: c1) M and $K/2$

are co-prime; c2) $MK/2$ is even. Based on $\bar{\boldsymbol{t}}$, $\boldsymbol{t}_\mu$ can be constructed as

$$\boldsymbol{t}_\mu = \boldsymbol{P}((\mu-1)(L+1))\bar{\boldsymbol{t}}, \tag{4}$$

where $\boldsymbol{P}(\tau) \in \mathbb{C}^{K\times K}$ is a permutation matrix. For positive (negative) τ, $\boldsymbol{P}(\tau)\bar{\boldsymbol{t}}$ circularly down- (up-) shifts $\bar{\boldsymbol{t}}$ by τ elements. Note that our training sequences in (4) have constant modulus which is preferable for efficient power amplification.

Under conditions c1) and c2), it can be verified that $\bar{t}[k+K/2] = \bar{t}[k]$ for $k = 0,\ldots,K/2-1$. Since $\boldsymbol{t}_\mu$'s are nothing but circular-shift of the base sequence $\bar{\boldsymbol{t}}$, it can be deduced that $t_\mu[k+\frac{K}{2}] = t_\mu[k]$, for $k = 0,\ldots,K/2-1$. Therefore, all training sequences are composed of two identical halves. By taking advantage of this special structure, coarse TO and accurate FO estimation can be done by using the algorithm proposed in [11]. In this rest of this paper, we will assume perfect coarse TO estimation and ideal FO estimation, and focus our attention to multiple channel estimation and fine TO estimation.

A. Fine TO estimation

After perfect coarse TO estimation and compensation, the residual TO τ_0 falls into the lock-in interval $[L - L_{\text{cp}}, 0]$ [11]. It is the purpose of fine TO estimation to estimate the exact value of this τ_0.

When the TO is within the lock-in interval, the received block $\boldsymbol{x}(\tau,\xi) := [x[0;\tau,\xi],\ldots,x[K-1;\tau,\xi]]^T$ can be written as:

$$\boldsymbol{x}(\tau_0) = \sum_{\mu=1}^{N_t} \tilde{\boldsymbol{H}}_\mu \boldsymbol{P}(\tau_0)\boldsymbol{t}_\mu + \boldsymbol{w}(\tau_0), \tag{5}$$

where $\tilde{\boldsymbol{H}}_\mu \in \mathbb{C}^{K\times K}$ represents a circular channel matrix with its first column being $[h_\mu[0],\ldots,h_\mu[L],0,\ldots,0]^T \in \mathbb{C}^{K\times 1}$, and $\boldsymbol{w}(\tau_0)$ is the noise vector. To proceed, let us denote

$$\tilde{\boldsymbol{t}}_\mu = \boldsymbol{P}(\tau_0)\boldsymbol{t}_\mu = \boldsymbol{P}((\mu-1)(L+1)+\tau_0)\bar{\boldsymbol{t}}, \tag{6}$$

where in writing the second equality, we used the fact $\boldsymbol{P}(\tau_1)\boldsymbol{P}(\tau_2) = \boldsymbol{P}(\tau_1+\tau_2)$. Because of the circular structure of $\tilde{\boldsymbol{H}}_\mu$, we further have

$$\tilde{\boldsymbol{H}}_\mu \boldsymbol{P}(\tau_0)\boldsymbol{t}_\mu = \tilde{\boldsymbol{H}}_\mu \tilde{\boldsymbol{t}}_\mu = \tilde{\boldsymbol{T}}_\mu \boldsymbol{h}_\mu \tag{7}$$

where

$$\tilde{\boldsymbol{T}}_\mu = [\tilde{\boldsymbol{t}}_\mu, \boldsymbol{P}(1)\tilde{\boldsymbol{t}}_\mu, \ldots, \boldsymbol{P}(L)\tilde{\boldsymbol{t}}_\mu]. \tag{8}$$

Plugging (8), (7) into (5), (5) can be thus rewritten as

$$\boldsymbol{x}(\tau_0) = \sum_{\mu=1}^{N_t}\sum_{l=0}^{L} h_\mu[l]\boldsymbol{P}(a_{\mu,l}(\tau_0))\bar{\boldsymbol{t}} + \boldsymbol{w}(\tau_0), \tag{9}$$

where $a_{\mu,l}(\tau_0) = (\mu-1)(L+1)+l+\tau_0$.

Let us perform phase rotation and discrete Fourier transform (DFT) on $\boldsymbol{x}(\tau_0)$ to obtain:

$$\boldsymbol{y}(\tau_0) = \boldsymbol{F}\boldsymbol{D}^{\mathcal{H}}\boldsymbol{x}(\tau_0), \tag{10}$$

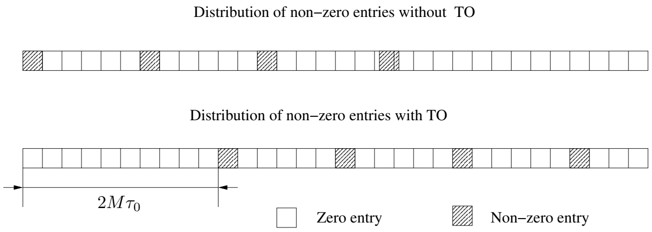

Fig. 1. The distribution of the non-zero entries of $\bar{\boldsymbol{y}}(\tau)$

where $\boldsymbol{F}$ is the DFT matrix, and $\boldsymbol{D}$ represents a constant diagonal matrix with its (k,k)th entry $\exp(-j\frac{2\pi}{K}Mk^2)$. As derived in Appendix, $\boldsymbol{y}(\tau_0)$ can be rewritten as:

$$\boldsymbol{y}(\tau_0) = \alpha\sqrt{K}\sum_{\mu=1}^{N_t}\sum_{l=0}^{L} h_\mu[l]e^{-j\frac{2\pi}{K}Ma_{\mu,l}^2(\tau_0)}\boldsymbol{e}_{(2Ma_{\mu,l}(\tau_0))_K} + \tilde{\boldsymbol{w}}, \tag{11}$$

where $(x)_K$ stands for x modulo K, $\boldsymbol{e}_i$ denotes the ith column of identity matrix $\boldsymbol{I}_K$ with $i = 0,\ldots,K-1$, and $\tilde{\boldsymbol{w}} = \boldsymbol{F}\boldsymbol{D}^{\mathcal{H}}\boldsymbol{w}(\tau_0)$.

Since M and $K/2$ are co-primitive with each other, $\mu = 1,\ldots,N_t$ and $l = 0,\ldots,K-1$, it can be verified that

$$[\boldsymbol{e}_{(2Ma_{\mu,l}(\tau_0))_K}]_{(2Ma_{\nu,m}(\tau_0))_K} = 1 \tag{12}$$

if and only if $\mu = \nu$ and $l = m$. This implies that the non-zero entries of $\boldsymbol{e}_{(Ma_{\mu,l}(\tau_0))_K}$ will not be overlapped for different $\mu = 1,\ldots,N_t$ and $l = 0,\ldots,K-1$. Thus, the kth entry of $\boldsymbol{y}(\tau_0)$ can be represented as

$$[\boldsymbol{y}(\tau_0)]_k = \alpha\sqrt{K}h_\mu[l]e^{-j\frac{2\pi}{K}Ma_{\mu,l}^2(\tau_0)} + [\tilde{\boldsymbol{w}}]_k \quad \text{for } k = (2Ma_{\mu,l}(\tau_0))_K. \tag{13}$$

Otherwise, $[\boldsymbol{y}(\tau_0)]_k = [\tilde{\boldsymbol{w}}]_k$.

Let $\bar{\boldsymbol{y}}(\tau_0)$ denote the signal part of $\boldsymbol{y}(\tau_0)$, i.e., $\boldsymbol{y}(\tau_0) = \bar{\boldsymbol{y}}(\tau_0) + \tilde{\boldsymbol{w}}$. Regarding (13), two remarks are in order:

- At most $N_t(L+1)$ non-zero entries exist in $\bar{\boldsymbol{y}}(\tau_0)$, and each of them corresponds to a certain channel tap for a certain transmit antenna;
- By definition of $a_{\mu,l}(\tau_0)$, when $\tau_0 = 0$, non-zero entries of $\bar{\boldsymbol{y}}(0)$ are located at $(2M[(\mu-1)(L+1)+l])_K$. If TO $\tau_0 \neq 0$, however, the locations of these non-zero entries are changed to $(2M[(\mu-1)(L+1)+l]+2M\tau_0)_K$, as we illustrate in Fig. 1 where $L+1 = 2$, $N_t = 2$, $M = 5$, and $\tau_0 = 1$.

Based on (13), the cost function for the fine TO estimation can be established as

$$J_{\text{syn}}(d) = \sum_{\mu=1}^{N_t}\sum_{l=0}^{L} |[\boldsymbol{y}(\tau_0)]_{(2M[(\mu-1)(L+1)+l]+2d)_K}|^2, \tag{14}$$

where $d = 0,\ldots,K/2-1$. Assume that the length of training sequences satisfy $K \geq 2N_t(L+1)$. Because M and $K/2$ are co-primitive with each other, it can be proved that all channel taps $h_\mu(l)$ are included into the sum of (14) if and only if

$$d = (M\tau_0)_{K/2}. \tag{15}$$

Obviously, $d \in [0, K/2-1]$. Due to the co-primitivity between M and $K/2$, for any specific d, there exists an unique integer $\tau_0 \in (-K/4, K/4]$ that satisfies (15). This implies that τ_0 can be uniquely obtained from d. Since the length of training sequence K is often much longer than the guard interval, it is almost for sure that $\tau_0 \in (-K/4, K/4]$ after coarse TO estimation.

Let:

$$\hat{d} = \arg\max_{d} J_{\text{syn}}(d). \quad (16)$$

The fine TO estimate $\hat{\tau}_0$ can be directly calculated from (15).

If the noise and inter block interference are not considered, based on (13), the maximum value of $J_{\text{syn}}(d)$ in (14) turns out to be

$$J_{\text{syn}}(M\hat{\tau}_0) = \frac{P_t}{N_t}\sum_{\mu=1}^{N_t}\sum_{l=0}^{L}|h_\mu[l]|^2. \quad (17)$$

It can be verified that, $J_{\text{syn}}(M\hat{\tau}_0)$ is exact the total received signal power.

B. Optimal channel estimation

Unlike traditional methods [3], [4] where channel estimation process is separated from the synchronization, the proposed new preamble design enable us to obtain the channel estimates directly during the fine TO estimation.

Indeed, after $\hat{\tau}_0$ is obtained, the channel estimates of lth tap for the μth transmit antenna can be calculated from (13) as

$$\hat{h}_\mu[l] = \frac{1}{\alpha\sqrt{K}}e^{j\frac{2\pi}{K}Ma^2_{\mu,l}(\hat{\tau}_0)}[\boldsymbol{y}(\tau_0)]_{(Ma_{\mu,l}(\hat{\tau}_0))_K}. \quad (18)$$

Assume that the synchronization is correct, i.e, the residual TO $\tau_0 \in [L-L_{cp}, 0]$, $\hat{\tau}_0 = \tau_0$, and $\hat{\xi} = \xi$. The noise vector in (11) satisfy $(E)[\tilde{\boldsymbol{w}}\tilde{\boldsymbol{w}}^H] = \sigma_w^2\boldsymbol{I}_K$. It can be deduced that the MSE of channel estimate for (18) can be represented as

$$\begin{aligned} J_{prop} &= \textstyle\sum_{\mu=1}^{N_t}\sum_{l=0}^{L}|\hat{h}_\mu[l]-h_\mu[l]|^2 \\ &= \sigma_w^2 N_t^2(L+1)/P_T. \end{aligned} \quad (19)$$

According to [13], this result is exactly the lower bound that can be achieved by the LS estimator, which implies that the proposed training sequence is optimal in the LS sense.

C. Why Fine TO Estimation First

In our scheme, channel estimation is done right after fine TO estimation. In what follows, we explain why this order makes sense.

Let us ignore the noise. According to (6) and (7), (5) can be rewritten as

$$\boldsymbol{x}(\tau_0) = \boldsymbol{P}(\tau_0)\boldsymbol{T}\boldsymbol{h}, \quad (20)$$

where $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_{N_t}^T]^T$ and $\boldsymbol{T} = [\boldsymbol{T}_1, \ldots, \boldsymbol{T}_{N_t}]$ with $\boldsymbol{T}_\mu$ representing a $K \times (L+1)$ column-wise circulant matrix whose first column is $\boldsymbol{t}_\mu$. Assume that the channel estimation is carried out first and $\hat{\boldsymbol{h}}(\tau_0) = \boldsymbol{T}^{\mathcal{H}}\boldsymbol{x}(\tau_0)$ is obtained. Because each training sequence is composed of two identical halves, to guarantee unbiased channel estimation, it is required that $K \geq 2N_t(L+1)$. Let us consider the case of $K = 2N_t(L+1)$. Based on the definition of the training sequences in (4), it can be verified that $\boldsymbol{T}^{\mathcal{H}}\boldsymbol{T} = I$ and $\boldsymbol{T}^{\mathcal{H}}$ is actually the pseudo-inverse matrix of $\boldsymbol{H}$. With (4), it can then be deducted that $\hat{\boldsymbol{h}}(\tau_0) = \boldsymbol{P}_h(\tau_0)\boldsymbol{h}$ with $\boldsymbol{P}_h \in \mathbb{C}^{(K/2)\times(K/2)}$ denoting a permutation matrix. In general, τ_0 can not be identified through $\hat{\boldsymbol{h}}(\tau_0)$ directly. Of course, if there are addition conditions such as the first channel tap has the largest power or the L in (4) is much larger than the real channel order, it is possible to find τ_0 and obtain the real $\boldsymbol{h}$. However, as one can imagine, additional conditions place additional restrictions. Therefore, it is better to get fine TO first. For the case of $K > 2N_t(L+1)$, similar result can be obtained.

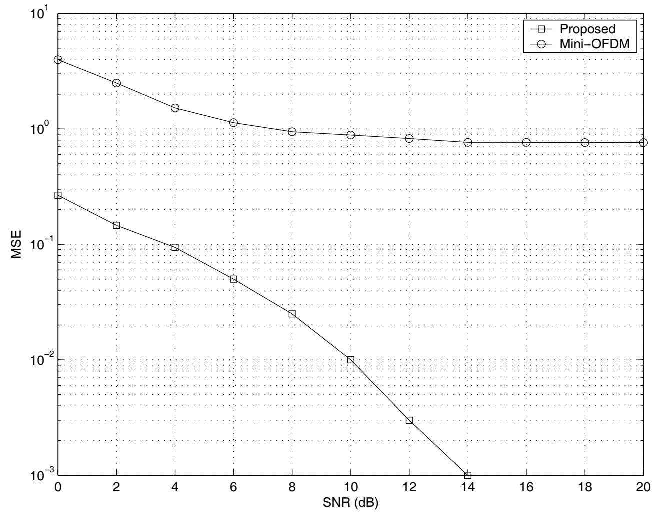

Fig. 2. MSE comparison of TO estimation

IV. Simulations

In this section, the performance of the proposed preamble design is tested and compared with the mini-OFDM method in [3], [4]. The simulated system has two transmit antennas and one receive antenna. 1000 Monte Carlo simulations are implemented. The channels are assumed to be frequency-selective Rayleigh faded and the generated taps are independent zero-mean complex Gaussian variables. Let $\sigma_\mu^2[l]$ denote the expected power for the lth channel tap of the μth transmit antenna. In the simulation, $\sigma_\mu^2[l] = ce^{\beta l}$, where $c = 1/\sum_{l=0}^{L}e^{-\beta l}$ and β is chosen to be 1.

For the proposed method, the parameter is chosen as $K = 256$, $L = 4$, and $L_{\text{cp}} = 16$. Therefore, the total length of preamble is $K + L_{\text{cp}} = 272$. To achieve a fair comparison , the preamble for the mini-OFDM method also has the length of 272. Since there are two transmit antennas, a preamble contains two mini-OFDM symbols, and each mini-OFDM symbol includes a CP of length 16 and a training sequence of length 120.

Test 1 *(TO estimation comparison)* In this test, we compare the MSE of fine TO estimation. Due to the special structure of the training sequences in the proposed method, the estimation of TO will not be affected by the unknown channel dispersion. From Fig. 2, it can be observed that the proposed method

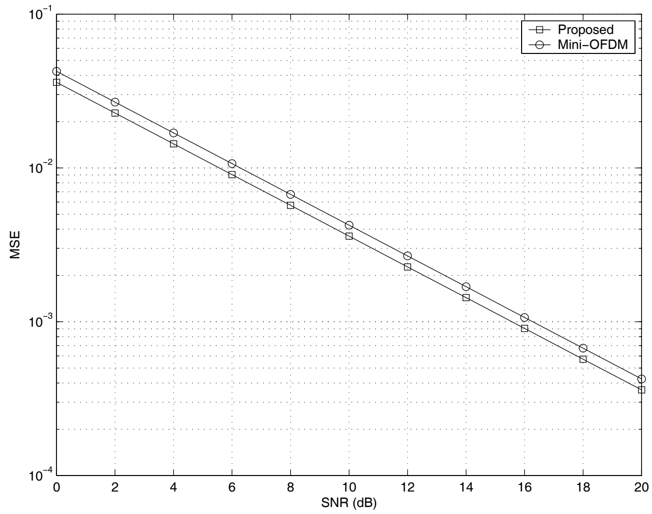

Fig. 3. MSE comparison of channel estimation

can obtain much more accurate TO than that of mini-OFDM preamble design in [3], [4].

Test 2 *(Channel estimation comparison)* In this test, LS estimator is used for the proposed design. The comparison of the channel estimation MSE performance can be found in Fig. 3. As we have proved, the proposed training sequence is optimal for the LS estimator. Therefore, it enjoys the optimal MSE performance. For the mini-OFDM method in [3], the MSE is higher because more than one CP are discarded during the channel estimation.

V. Conclusion

In the paper, a novel preamble design is proposed for OFDM systems with multiple transmit antennas. With this new preamble design, one can achieve FO estimation, fine TO estimation as well as optimal channel estimation. In addition, this design has the property of constant-modulus, which guarantees the power efficiency of power amplifier. The simulations show that, the proposed method can achieve more accurate estimation than current methods.

Appendix
Derivation of (11)

Let us consider a vector $\boldsymbol{P}(n)\bar{\boldsymbol{t}}$ with $n \in \mathbb{Z}$, which is the $|n|$-times up or down-ward circular-shift of $\bar{\boldsymbol{t}}$. According to (3), the kth entry of $\boldsymbol{P}(n)\bar{\boldsymbol{t}}$ can then be denoted by

$$\begin{aligned} [\boldsymbol{P}(n)\bar{\boldsymbol{t}}]_k &= \bar{t}[(k-n)_K] \\ &= e^{-j\frac{2\pi}{K}M(k-n)_K^2} \\ &= e^{-j\frac{2\pi}{K}Mn^2} e^{-j\frac{2\pi}{K}Mk^2} e^{j\frac{2\pi}{K}(2Mn)k} \end{aligned} \tag{21}$$

where $(x)_K$ stands for x modulo K. Note tat, the last equation is obtained based on conditions c1) and c2). As a consequence, the kth entry of $[\boldsymbol{P}(n)\bar{\boldsymbol{t}}]_k$ is decomposed into three parts. Since the last part $e^{j\frac{2\pi}{K}(2Mn)k}$ can be regarded as the $(k,(2Mn)_K)$th entry (note: start from $(0,0)$th entry) of $(\sqrt{K})\boldsymbol{F}^H$ with $\boldsymbol{F}^H$ the inverse Fourier transform matrix, (21) can be written into a matrix-vector form

$$\begin{aligned} \boldsymbol{P}(n)\bar{\boldsymbol{t}} &= \sqrt{K}e^{-j\frac{2\pi}{K}Mn^2}\boldsymbol{D}[\boldsymbol{F}^H]_{(2Mn)_K} \\ &= \sqrt{K}e^{-j\frac{2\pi}{K}Mn^2}\boldsymbol{D}\boldsymbol{F}^H\boldsymbol{e}_{(2Mn)_K} \end{aligned} \tag{22}$$

where $\boldsymbol{D}$ is a diagonal matrix with the $(k,k)th$ entry being $e^{-j\frac{2\pi}{K}Mk^2}$, and $\boldsymbol{e}_i$ is the ith column of $\boldsymbol{I}_K$ with $i = 0,\ldots,K-1$.

Replacing n by $a_{\mu,l}(\tau_0)$ and substituting (22) into (10) and (9), we arrive at (11).

References

[1] E. Zhou, X. Zhang, H. Zhao, and W. Wang, "Synchronization algorithm for MIMO OFDM systems," in *Proc. Wireless Commun. and Networking Conf.*, vol. 1, New Orleans, LA, Mar. 2005, pp. 18–22.

[2] J. Kin and D. Hong, "New preamble structure for ofdm systems with multiple transmit antennas," in *Proc. VTC*, Los Angeles, CA, May 2004, pp. 17–19.

[3] G. L. Stuber, J. R. Barry, S. W. McLaughlin, Y. Li, M. A. Ingram, and T. G. Pratt, "Broadband MIMO-OFDM wireless communications," *Proceedings of the IEEE*, vol. 92, pp. 271–294, Feb. 2004.

[4] A. N. Mody and G. L. Stuber, "Synchronization for MIMO OFDM systems," in *Proc. GLOBECOM*, vol. 1, San Antonio, TA, Nov. 2001, pp. 25–29.

[5] ——, "Receiver implementation for a mimo ofdm system," in *Proc. GLOBECOM*, vol. 1, Taipei, Taiwan, Nov. 2002, pp. 17–21.

[6] E. G. Larsson and J. Li, "Preamble design for multiple-antenna OFDM-based WLANs with null subcarriers," *IEEE Signal Processing Lett.*, vol. 8, pp. 285–288, Nov. 2001.

[7] J. Li, J. Li, and P. Stoica, "A MIMO system with backward compatibility for OFDM based WLANs," in *Proc. Signal Processing Advances in Wireless Commun.*, vol. 1, San Francisco,CA, June 2003, pp. 130–134.

[8] A. V. Zelst and T. C. W. Schenk, "Implementation of a MIMO OFDM-based wireless LAN system," *IEEE Trans. Signal Processing*, vol. 52, pp. 483–494, Feb. 2004.

[9] M. Oh, X. Ma, G. B. Giannakis, and D. Park, "Cooperative synchronization and channel estimation in wireless sensor networks," in *Proc. Signals, Systems and Computers*, vol. 1, Pacific Grove, CA, Nov. 2003, pp. 238–242.

[10] Y. Ogawa, K. Nishio, T. Nishimura, and T. Ohgane, "Channel and frequency offset estimation for a MIMO-OFDM system," in *Proc. VTC*, vol. 2, Los Angeles, CA, Sept. 2004, pp. 1523–1527.

[11] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.*, vol. 45, pp. 1613–1621, Dec. 1997.

[12] D. C. Chu, "Polyphase codes with good periodic correlation properties," *IEEE Trans. Inform. Theory*, vol. 18, pp. 531–532, July 1972.

[13] J. Coon, M. Beach, and J. McGeehan, "Optimal training sequence for channel estimation in cyclic-prefix-based single-carrier systems with transmit diversity," *IEEE Signal Processing Lett.*, vol. 11, pp. 729–732, Sept. 2004.