OPTIMAL SPECTRUM BALANCING IN MULTI-USER xDSL SYSTEMS WITH ON/OFF POWER LOADING

Jan Vangorp, Paschalis Tsiaflakis, Marc Moonen

Department of Electrical Engineering
Katholieke Universiteit Leuven, Belgium
{Jan.Vangorp, Paschalis.Tsiaflakis}@esat.kuleuven.be

Jan Verlinden, Katleen Van Acker

DSL Experts Team
Alcatel Bell, Belgium
{Jan.VJ.Verlinden}@alcatel.be

ABSTRACT

Optimal Spectrum Balancing (OSB) is a centralized algorithm that optimally allocates transmit power over frequencies in a multi-user DSL environment where crosstalk is a major factor limiting performance. By using a dual decomposition, OSB decouples the spectrum management problem over frequencies. This results in per-tone optimization problems that are solved with an exhaustive search. This exhaustive search, however, has an exponential complexity in the number of users. For scenarios with several users this often becomes computationally intractable. In this paper, this complexity is reduced by limiting the possible power loadings on each tone to ON/OFF loading with an adjustable ON-level. This leads to a simple OSB algorithm with manageable complexity, simple flat transmit spectra and only minor performance degradation.

1. INTRODUCTION

The ever increasing demand for higher data rates forces the use of higher frequencies in xDSL access networks. At these high frequencies, crosstalk between lines is typically 10-15 dB larger than the background noise. This coupling between users is a major source of performance degradation. Therefore, research is focusing on multi-user techniques to mitigate this crosstalk.

Currently, there are two strategies for dealing with crosstalk. One strategy is crosstalk cancellation. Crosstalk cancellation schemes exist which can completely remove crosstalk, without causing noise enhancement. However, these schemes require a high runtime complexity and signal-level coordination between all lines. These requirements are mostly not satisfied, e.g. in an unbundled environment.

In this case, spectrum management can be used to mitigate crosstalk. Here, transmit spectra are chosen such that crosstalk is avoided. Advanced techniques exist that take into account the network topology to determine transmit spectra. Iterative Waterfilling (IW) [1] is one of the first so-called Dynamic Spectrum Management algorithms, demonstrating significant performance gains over static transmit spectra designed for worst case scenarios. However, IW converges to a 'selfish' optimum. As a result, it is suboptimal for highly unbalanced scenarios such as central office (CO) / remote terminal (RT) deployments.

Jan Vangorp is a research assistant with the ESAT/SISTA laboratory.

Paschalis Tsiaflakis is a research assistant with the F.W.O. Vlaanderen.

This research work was carried out at the ESAT laboratory of the Katholieke Universiteit Leuven, in the frame of IUAP P5/22 and P5/11, FWO project nr.G.0196.02, IWT project 030054 and CELTIC/IWT project 040049: 'BANITS 'Broadband Access Networks Integrated Telecommunications' and was partially sponsored by Alcatel-Bell.

Optimal Spectrum Balancing (OSB) [2] is an algorithm that calculates the optimal transmit spectra for any multi-user network topology. By optimizing a weighted rate sum, OSB can make every possible trade off between the data rates of the users. Therefore, every desired point on the rate region can be achieved. However, this can only be done when complete channel information is available (direct channels as well as crosstalk channels), making OSB only suitable with centralized control in a Spectrum Management Center (SMC).

The complexity of OSB is exponential in the number of users. For scenarios with more than 3 users, this often becomes computationally intractable. One possibility to address this complexity is by using an iterative approach, as proposed in [3] [4].

This paper addresses the complexity problem by restricting the transmit spectra to ON/OFF power loading with an adjustable ON-level. This leads to a simple OSB algorithm with manageable complexity, simple flat transmit spectra and only minor performance degradation.

2. OPTIMAL SPECTRUM BALANCING

2.1. System Model

Most current DSL systems use Discrete Multi-Tone (DMT) modulation. The available frequency band is divided in a number of parallel subchannels or tones. Each tone is capable of transmitting data independently from other tones, and so the transmit power and the number of bits can be assigned individually for each tone. This gives a large flexibility in optimally shaping the transmit spectrum.

Transmission for a binder of N users can be modelled on each tone k by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k \qquad k = 1 \dots K.$$

The vector $\mathbf{x}_k = [x_k^1, x_k^2, \dots, x_k^N]^T$ contains the transmitted signals on tone k for all N users. $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. $\mathbf{z}_k = [z_k^1, z_k^2, \dots, z_k^N]$ is the vector of additive noise on tone k, containing thermal noise, alien crosstalk, RFI,... The vector $\mathbf{y}_k$ contains the received symbols on tone k.

The transmit power is denoted as $s_k^n \triangleq \Delta_f E\{|x_k^n|^2\}$, the noise power as $\sigma_k^n \triangleq \Delta_f E\{|z_k^n|^2\}$. The vector containing the transmit power of user n on all tones is $\mathbf{s}^n \triangleq [s_1^n, s_2^n, \dots, s_K^n]^T$. The DMT symbol rate is denoted as f_s, the tone spacing as Δ_f.

It is assumed that each modem treats interference from other modems as noise. When the number of interfering modems is large, the interference is well approximated by a Gaussian distribution. Under this assumption the achievable bit loading of user n on tone

k, given the transmit spectra of all modems in the system, is

$$b_k^n \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right), \qquad (1)$$

where Γ denotes the SNR-gap to capacity, which is a function of the desired BER, the coding gain and noise margin. The data rate and total power for user n is

$$R^n = f_s \sum_k b_k^n \quad \text{and} \quad P^n = \sum_k s_k^n.$$

2.2. The Spectrum Management Problem

The spectrum management problem amounts to finding optimal transmit spectra for a bundle of interfering DSL lines, maximizing the sum of the data rates of the users subject to a number of constraints:

$$\begin{array}{lll} \max_{\mathbf{s}^1,\ldots,\mathbf{s}^2} & \sum_{n=1}^N R^n & \\ \text{subject to} & P^n \le P^{n,tot} & n = 1\ldots N \\ & 0 \le s_k^n \le s_k^{n,mask} & n = 1\ldots N, k = 1\ldots K \\ & R^n \ge R^{n,target} & n = 1\ldots N \end{array} \qquad (2)$$

In this equation, the first set of constraints indicate *total power constraints* $P^{n,tot}$ per user. These ensure the user's total power does not exceed the maximum allowed total transmit power. The second set of constraints are *spectral mask constraints* $s_k^{n,mask}$ for each tone to guarantee electromagnetic compatibility with other systems. The last set of constraints are *data rate constraints* $R^{n,target}$ for each user indicating a minimum data rate required by the user.

The rate sum in (2) is a non-convex function. Finding the global optimum can be done by an exhaustive search. However, the total power and data rate constraints are coupled over tones. Therefore, the optimization has to be done over all possible combinations of power loading for tones and users, resulting in an exponential complexity in the number of tones and users $\mathcal{O}(B^{KN})$, where B is the number of possible power loadings. Since the number of tones is large, this is computationally intractable.

2.3. Optimal Spectrum Balancing

In [2] it is shown that the complexity can be reduced by using a dual decomposition. In the dual problem formulation of (2), the constraints coupled over tones are moved to the unconstrained part of the optimization problem by using Lagrange multipliers [5]:

$$\begin{array}{c} \mathbf{s}^{1,opt}, \ldots, \mathbf{s}^{N,opt} = \text{argmax}_{\mathbf{s}^1,\ldots,\mathbf{s}^N} J \\ J = \sum_{n=1}^N \omega_n R^n + \sum_{n=1}^N \lambda_n \left(P^{n,tot} - \sum_{k=1}^K s_k^n\right) \end{array} \qquad (3)$$

$$\begin{array}{lll} \text{subject to} & 0 \le s_k^n \le s_k^{n,mask} & n = 1\ldots N, k = 1\ldots K \\ \text{with} & \lambda_n \ge 0, \omega_n \ge 0 & n = 1\ldots N \end{array}$$

This dual problem is decoupled across the tones. Therefore, the maximization of (3) can be done by maximizing the weighted rate sum independently on each tone:

$$J = \sum_{k=1}^K \underbrace{\left(\sum_{n=1}^N \omega_n f_s b_k^n - \sum_{n=1}^N \lambda_n s_k^n\right)}_{J_k}. \qquad (4)$$

For given Lagrange multipliers $\omega_1, \ldots, \omega_N$ and $\lambda_1, \ldots, \lambda_N$, the maximization of (3) can be performed by k per-tone exhaustive searches over all possible combinations of transmit power for the users. These searches have an exponential complexity in the number of users N, leading to an $\mathcal{O}(KB^N)$ complexity.

The problem then reduces to finding the Lagrange multipliers $\omega_1, \ldots, \omega_N$ and $\lambda_1, \ldots, \lambda_N$ that enforce the constraints. In [5], an efficient search procedure is presented to find these Lagrange multipliers.

In conclusion, the dual decomposition reduces the complexity from exponential to linear in K: $\mathcal{O}(UKB^N)$, where U is the number of iterations required to find the Lagrange multipliers. With the search algorithm in [5], U is typically $50 - 150$.

3. ON/OFF LOADING

Despite the complexity reduction provided by dual decomposition, OSB is often still too complex for scenarios with more than 3 users. The reason is the per-tone exhaustive search which has an exponential complexity in the number of users: B^N. The iterative approach of [3] [4] reduces this complexity to INB, where I is the number of iterations to converge.

In this paper, the complexity is combated by reducing B. Originally, for OSB, typical values for B are 14 in case of bit loading and 60 in case of power loading. By limiting the transmit spectra to ON/OFF power loading, $B = 2$, the complexity is reduced from B^N to 2^N. The ON-level will be adjusted for each individual user. This ON/OFF power loading problem corresponds to (3) with the spectral mask constraints replaced by

$$s_k^n \in \{0, s_k^{n,ON}\} \text{ with } s_k^{n,ON} \le s_k^{n,mask}. \qquad (5)$$

This ON/OFF power loading results in simple transmit spectra, similar to what is used in current ADSL systems. In order to perform the reduced per-tone exhaustive searches, $s^{n,ON}$ has to be defined. This ON-level should not violate the spectral mask constraint and the resulting spectra should not violate the total power constraint.

3.1. Fixed ON-Level

The safest definition for the ON-level is

$$s_k^{n,ON} = \min\left(\frac{P^n}{K}, s_k^{n,mask}\right). \qquad (6)$$

By choosing this ON-level, the total power constraints are automatically satisfied. Therefore, no λ Lagrange multipliers have to be searched to enforce the total power constraints. This is presented as **algorithm 1**.

Algorithm 1 Multi-user ON/OFF loading with fixed ON-level

for $k = 1$ to K **do**
  $s_k^{1,opt}, \ldots, s_k^{n,opt} = \text{argmax}_{s_k^1,\ldots,s_k^n} \omega_1 b_k^1 + \ldots + \omega_n b_k^n$
    with $s_k^n \in \{0, s_k^{n,ON}\}$
end for

However, for this fixed ON-level, if some tones are switched off, not all available power is allocated. This unused power could then be redistributed to the tones that are switched on.

Because of the lack of this power redistribution, ON/OFF loading with a fixed ON-level performs worse than IW. In **figure 1(b)**, the performance of ON/OFF loading with a fixed ON-level is compared to IW for the scenario shown in **figure 1(a)**.

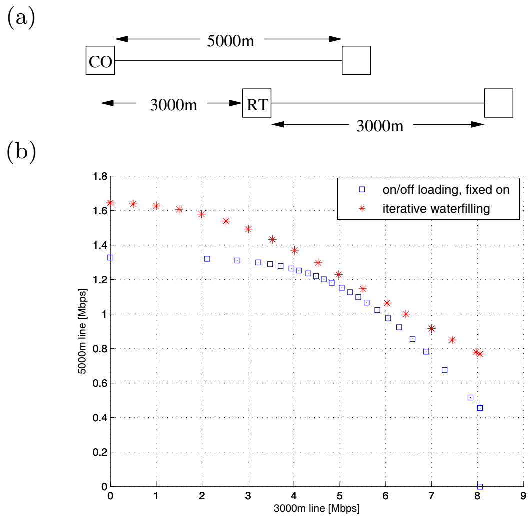

Fig. 1. 2-user ADSL CO/RT scenario

3.2. Adaptive ON-Level

The performance of ON/OFF loading can be greatly improved by redistributing the power of tones that are switched off to the other tones. To do so, one can iterate over the procedure with a fixed ON-level, in each iteration updating the ON-level as

$$s_k^{n,ON} = \min\left(\frac{P^n}{(\#\text{ active tones})^n}, s_k^{n,mask}\right), \tag{7}$$

where $(\#\ active\ tones)^n$ represents the number of active tones for user n.

However, in some cases, the per-tone exhaustive search does not voluntarily switch off tones. E.g. in a CO/RT scenario, CO lines do not cause significant crosstalk on the RT lines. Thus these tones are not switched off by the per-tone exhaustive search. Even though they carry very few bits, the performance would benefit from the redistribution of power from high frequencies to lower frequencies.

This can be resolved by setting a threshold on the minimum number of bits that a tone should carry. Tones which do not reach this minimum are forced to switch off. This allows power which yields only few bits to be redistributed to tones where the power would yield more bits.

Tones which are forced to switch off are never activated again to improve the convergence of the iterative procedure, presented in **algorithm 2**.

Algorithm 2 Multi-user ON/OFF loading with adaptive ON-level

$t_k^{n,\text{usable}} = 1, \quad n = 1\dots N, k = 1\dots K$
repeat
  $s_k^{n,ON} = \min\left(\frac{P^n}{(\#\text{ active tones})^n}, s_k^{n,mask}\right)$
  for $k = 1$ to K **do**
    $s_k^{1,opt}, \dots, s_k^{n,opt} = \text{argmax}_{s_k^1,\dots,s_k^n}\, \omega_1 b_k^1 + \dots + \omega_n b_k^n$
      with $s_k^n \in \{0, s_k^{n,ON} \cdot t_k^{n,\text{usable}}\}$
    $\forall n$: **if** $b_k^n < threshold$ **then** $t_k^{n,\text{usable}} = 0, s_k^{n,opt} = 0$
  end for
until convergence

In **figure 2**, rate regions are shown for various values of the threshold for the scenario in figure 1(a).

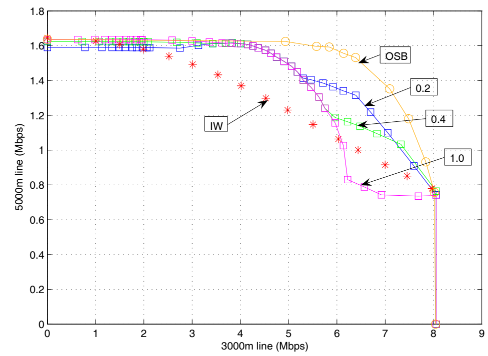

Fig. 2. 2-user ADSL CO/RT rate regions, adaptive ON-level

The procedure typically converges in 4-5 iterations. Note that a good value for the threshold depends on the point of the rate region. Low thesholds give good performance for high rates on the RT lines, high tresholds give good performance for high rates on the CO lines. So the threshold depends on the scenario as well as on the target rates.

3.3. Adaptive Threshold

To adaptively select a good threshold for a specific scenario and given target rates, algorithm 2 can be repeated for a set of thresholds $th_i, i = 1\dots T$. This increases the complexity by the number of thresholds. However the complexity increase can be reduced by applying the thresholds in ascending order. For each new threshold, one can then start from the converged transmit spectra of the previous threshold by passing the current threshold and for each user the tones that are still available to algorithm 2. This procedure is presented in **algorithm 3**.

Algorithm 3 Multi-user ON/OFF loading with adaptive threshold

$t_k^{n,\text{usable}} = 1, \quad n = 1\dots N, k = 1\dots K$
for $th_i, \quad i = 1$ to T **do**
  $[\mathbf{s}^{1,th_i} \dots \mathbf{s}^{N,th_i}, t^{\text{usable}}]$ = algorithm2(th_i, t^{usable})
end for
select th where $\mathbf{s}^{1,th} \dots \mathbf{s}^{N,th}$ has highest weighted rate sum
$\mathbf{s}^{1,opt}, \dots, \mathbf{s}^{n,opt} = \mathbf{s}^{1,th} \dots \mathbf{s}^{N,th}$

The performance of the ON/OFF loading algorithm with adaptive threshold is compared to the OSB algorithm in the following simulations, where a set of 7 predefined thresholds is used. All scenarios use a line diameter of 0.5 mm (24 AWG), the maximum transmit power is 20.4 dBm. The SNR gap Γ is set to 12.9 dB, corresponding to a target symbol error probability of 10^{-7}, coding gain of 3 dB and a noise margin of 6 dB. The tone spacing $\Delta_f = 4.3125$ kHz and the DMT symbol rate $f_s = 4$ kHz [6] [7].

In **figure 3(b)** the rate regions are shown for the downstream ADSL CO/RT scenarios of **figure 3(a)**. In this case, the extra constraint of flat spectra in ON/OFF loading results in a 15% performance loss compared to OSB. The corresponding reduction in complexity is $\frac{U}{I_2}(\frac{B}{2})^N$, which in this 2-user case corresponds to a factor of 250.

In **figure 4(b)** the rate regions are shown for the upstream VDSL scenario of **figure 4(a)**. Both the rate regions with and without a

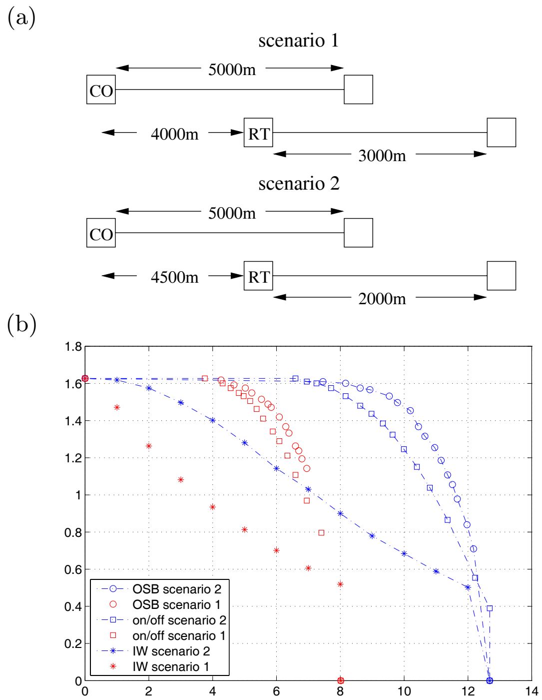

Fig. 3. 2-user ADSL CO/RT scenario, adaptive threshold

spectral mask of -60 dBm/Hz are shown. In this case, ON/OFF loading results in a 10% performance loss compared to OSB.

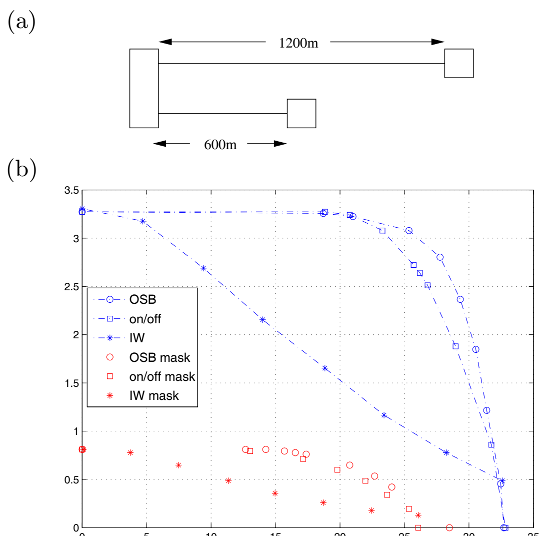

Fig. 4. 2-user VDSL scenario, adaptive threshold

For all scenarios the performance of iterative waterfilling (IW) is also shown. Significant performance gains can be seen from the proposed ON/OFF loading scheme over IW. Note that the objective of spectrum management is to protect long lines from being overpowered by crosstalk. For a fixed rate on these long lines, a performance gain of up to 400% can be seen on the short lines.

4. ITERATIVE SEARCH

The algorithms of the previous section can be combined with the iterative search algorithm presented in [3] [4] (Iterative Spectrum Balancing, ISB). Instead of performing an exhaustive search over all users at once, each user iteratively performs a 1-D search to optimize the weighted rate sum, while other users are kept fixed. Because of the significant difference in power levels used in ON/OFF loading, this iterative approach gives the same results as a full exhaustive search in most cases.

An overview of the complexities of various algorithms discussed is shown in **table 1**. I_1 is the nuber of iterations needed to converge to an adaptive ON-level, typically 4-5, I_2 is the number of iterations needed to converge in the adaptive threshold case, typically 10-12. In the ISB case, I_{ISB} iterations are performed in the per tone search, typically 2-3.

Table 1. Complexity ON/OFF loading

	OSB	ISB
full complexity	$\mathcal{O}(UKB^N)$	$\mathcal{O}(UKI_{ISB}BN)$
fixed on	$\mathcal{O}(K2^N)$	$\mathcal{O}(KI_{ISB}2N)$
adaptive on	$\mathcal{O}(I_1K2^N)$	$\mathcal{O}(KI_1I_{ISB}2N)$
adaptive threshold	$\mathcal{O}(I_2K2^N)$	$\mathcal{O}(KI_2I_{ISB}2N)$

5. CONCLUSION

In this paper, ON/OFF loading was considered as a means of reducing the complexity of Optimal Spectrum Balancing (OSB). By only allowing 2 user-adjustable PSD levels, the per-tone exhaustive search inherent to OSB becomes more tractable. Moreover, the extra constraint for flat PSD's results in simple transmit spectra, similar to what is currently used in ADSL. An algorithm is presented to adjust the ON-level for this ON/OFF loading, such that the total power constraint is always satisfied.

Simulations show that this low-complexity algorithm performs whithin 10-15% of the optimal performance of OSB.

6. REFERENCES

[1] W. Yu, G. Ginis and J. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," *IEEE J. Sel. Area. Comm.*, vol. 20, no. 5, pp. 1105–1115, Jun. 2002.

[2] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden and T. Bostoen, "Optimal Multi-user Spectrum Management for Digital Subscriber Lines," *accepted for IEEE Trans. Comm.*

[3] R. Lui and W. Yu, "Low-Complexity Near-Optimal Spectrum Balancing for Digital Subscriber Lines," in *ICC*, May 2005.

[4] R. Cendrillon, M. Moonen, "Iterative Spectrum Balancing for Digital Subscriber Lines," in *ICC*, May 2005.

[5] P. Tsiaflakis, J. Vangorp, M. Moonen, J. Verlinden, K. Van Acker and R. Cendrillon, "An efficient Lagrange Multiplier search algorithm for Optimal Spectrum Balancing in crosstalk dominated xDSL systems," *submitted to IEEE J. Sel. Area. Comm.*, 2005.

[6] *Asymmetric digital subscriber line transceivers2 (ADSL2)*, ITU-T Std. G.992.3, 2002.

[7] *Transmission and Multiplexing (TM); Access transmission systems on metallic access cables; Very high speed Digital Subscriber Line (VDSL); Functional Requirements*, ETSI Std. TS 101 270-1, Rev. V.1.3.1, 2003.