Blind Constrained Set-Membership Algorithms with Time-Varying Bounds for CDMA Interference Suppression

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***Abstract*— This work presents blind constrained adaptive filtering algorithms based on the set-membership concept and incorporates time-varying bounds for CDMA interference suppression. Constrained constant modulus (CCM) and constrained minimum variance (CMV) gradient type algorithms designed in accordance with the specifications of the set-membership filtering concept are proposed. Furthermore, the important issue of bound specification is addressed in a new framework that takes into account parameter estimation dependency and multi-access (MAI) and inter-symbol interference (ISI) for multiuser communications. Simulations show that the new algorithms are capable of outperforming previously reported techniques with a smaller number of parameter updates and a reduced risk of overbounding or underbounding.**

I. Introduction

Blind adaptive linear receivers [1], [2] for DS-CDMA systems are effective techniques for interference suppression as they offer an attractive trade-off between performance and complexity, and can be used in situations where a receiver loses track of the desired signal and a training sequence is not available. Some of the most successful blind adaptive techniques in the literature [3], [4], [5] are based on constrained optimization solutions that combine multipath components and suppress MAI. In these works, blind adaptive stochastic gradient (SG) and recursive least squares (RLS) algorithms for multipath environments have been developed based upon the linearly CMV criterion of [3] and the CCM approach [4], [5].

An important research topic is how to implement blind algorithms in a computationally efficient way, while ensuring very good performance. In the literature of adaptive algorithms [6], SG algorithms represent simple and low complexity solutions that are preferred for implementation although their convergence depends on the eigenvalue spread of the covariance matrix of the received vector. In this regard, the existing works on CMV and CCM techniques [3], [4] employ standard SG algorithms with fixed step size that are not efficient with respect to convergence and steady state performance. In wireless networks characterized by non-stationary environments, users frequently enter and exit the system, making it very difficult for the receiver to compute a pre-determined step size and the performance of adaptive receivers for CDMA that use SG algorithms is strongly dependent on the choice of the step size [6]. This suggests the deployment of mechanisms to automatically adjust the step size of an SG algorithm in order to ensure good performance.

Set-membership filtering (SMF) [7], [8], [9], [10] represents a class of recursive estimation algorithms that, on the basis of a pre-determined error bound, seeks a set of parameters that yield bounded filter output errors. In particular, the work on SMF blind algorithms for interference suppression is limited to single-path channels in [11] and to blind equalization for narrow-band systems [12]. The adaptive SMF algorithms usually achieve good convergence and tracking performance due to the use of an adaptive step size for each update and data selective update. However, the performance of SMF techniques depends on the bound specification, which is very difficult to obtain in practice due to the lack of knowledge of the environment. In dynamic scenarios, it is not practical to choose a fixed bound and the risk of overbounding (when the bound is larger than the actual one) and underbounding (when the bound is smaller than the actual one) is significantly increased, leading to performance degradation. These problems suggest the deployment of mechanisms to automatically adjust the bound in order to guarantee good performance and a small update rate. Previous works on time-varying bounds include the approach in [13] that assumes that the "true" error bound is constant and the parameter-dependent error bound recently proposed in [14]. The techniques so far reported do not introduce any mechanism for tracking MAI and ISI and incorporating these estimates in the bound. In this work, we propose set-membership blind adaptive constrained algorithms based on the CMV and CCM criteria. In particular, we derive gradient type CMV and CCM algorithms designed in accordance with the specifications of the set-membership filtering concept. The second contribution is a new low complexity framework for tracking parameter evolution, MAI and ISI that relies on simple estimation techniques and utilizes the proposed set-membership blind CMV and CCM algorithms with a time-varying bound.

This paper is structured as follows. Section II describes the DS-CDMA System Model. Section III briefly reviews linearly constrained receivers and blind adaptive constrained algorithms. Section IV introduces the SM blind algorithms with time-varying bound, Section V proposes parameter dependent and interference dependent bounds. Section VI is devoted to the presentation and discussion of numerical results, while Section VII gives the conclusions.

II. DS-CDMA System Model

Consider the downlink connection of a synchronous DS-CDMA system with K users, N chips per symbol and L_p paths. Assuming that the channel is constant during each symbol interval, the received signal after filtering by a chip-pulse matched filter and sampled at chip rate yields the $(M = N + L_p - 1) \times 1$ received vector

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \mathbf{C}_k \mathbf{h}(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i) \quad (1)$$

where $\mathbf{n}(i) = [n_1(i) \ \ldots \ n_M(i)]^T$ is the complex Gaussian noise vector with $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2\mathbf{I}$, where $(\cdot)^T$ and $(\cdot)^H$ denotes transpose and Hermitian transpose, respectively. The operator $E[\cdot]$ stands for ensemble average, $b_k(i) \in \{\pm 1 + j0\}$ is the symbol for user k with $j^2 = -1$, $\boldsymbol{\eta}(i)$ represents the intersymbol interference (ISI), the amplitude of user k is A_k, the channel vector is $\mathbf{h}(i) = [h_0(i) \ \ldots \ h_{L_p-1}(i)]^T$ and the $M \times L_p$ convolution matrix $\mathbf{C}_k$ contains one-chip shifted versions of the signature sequence for user k given by $\mathbf{s}_k = [a_k(1) \ldots a_k(N)]^T$.

III. Linearly Constrained Receivers and Blind Adaptive Constrained Algorithms

Let us describe the design of synchronous blind linearly constrained detectors. Consider the received vector $\mathbf{r}(i)$, and let us introduce the $M \times L_p$ constraint matrix $\mathbf{C}_k$ that contains one-chip shifted versions of the signature sequence for user k:

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix} \quad (1)$$

The receiver design determines an FIR filter $\mathbf{w}_k(i)$ with M coefficients, that provides an estimate of the desired symbol as given by

$$\hat{b}_k(i) = \text{sgn}\Big(\Re\Big[\mathbf{w}_k^H(i)\mathbf{r}(i)\Big]\Big) \quad (2)$$

subject to a set multipath constraints given by $\mathbf{C}_k^H \mathbf{w}_k(i) = \mathbf{h}(i)$ for the CMV case, or $\mathbf{C}_k^H \mathbf{w}_k(i) = \nu \mathbf{h}(i)$ for the CCM case, where ν is a constant to ensure the convexity of the CM-based receiver, $\mathbf{h}(i)$ is the kth user channel vector, $\Re(\cdot)$ selects the real part, sgn$(\cdot)$ is the signum function and the receiver parameter vector $\mathbf{w}_k$ is optimized by the CMV or the CCM criteria, which assume the knowledge of the channel. However, when multipath is present these parameters are unknown and time-varying, requiring channel estimation. Here, we adopt the simple and effective blind adaptive SG channel estimation algorithm of [15]. Next, we briefly describe blind adaptive SG algorithms for estimating the receiver parameters on the basis of the CCM and CMV design criteria.

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A. Constrained Minimum Variance (CMV) SG Algorithm

To derive an CMV-SG algorithm let us consider the cost function

$$J_{MV} = \mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i) + 2\Re\Big[(\mathbf{C}_k^H\mathbf{w}_k(i) - \mathbf{h}(i))^H\boldsymbol{\lambda}\Big] \quad (3)$$

where $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers. An SG solution to (3) can be obtained by taking the gradient terms with respect to $\mathbf{w}_k(i)$, leading to the recursions for the blind estimation of $\mathbf{w}_k(i)$:

$$\mathbf{w}_k(i+1) = \mathbf{\Pi}_k(\mathbf{w}_k(i) - \mu_w\mathbf{r}(i)z_k^*(i)) + \mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{h}(i) \quad (4)$$

where $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i)$, $\mathbf{\Pi}_k = \mathbf{I} - \mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{C}_k^H$ and the channel estimate $\hat{\mathbf{h}}(i)$ obtained in (4) is also employed. We employ a normalized version of the above algorithm whose step size is $\mu_w = \frac{\mu_0}{\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)}$ where μ_0 is the chosen fixed convergence factor. The normalization facilitates the setting of the convergence factor for a wide range of loads.

B. Constrained Constant Modulus (CCM) SG Algorithm

To derive an CCM-SG algorithm let us consider the cost function

$$J_{CM} = (|z_k|^2 - 1)^2 + 2\Re\Big[(\mathbf{C}_k^H\mathbf{w}_k(i) - \mathbf{h}(i))^H\boldsymbol{\lambda}\Big] \quad (5)$$

where $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i)$ and $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers. An SG solution to (5) can be obtained by taking the gradient terms with respect to $\mathbf{w}_k(i)$ which yields the following parameter estimator:

$$\mathbf{w}_k(i+1) = \mathbf{\Pi}_k(\mathbf{w}_k(i) - \mu_w e_k(i)z_k^*(i)) + \mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{h}(i) \quad (6)$$

where $e_k = (|z_k(i)|^2 - 1)$, $\mathbf{\Pi}_k = \mathbf{I} - \mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{C}_k^H$ and the channel estimate $\hat{\mathbf{h}}(i)$ obtained in (4) is also employed. The normalized version of this algorithm is adopted in order to make easier the choice of the step size, also guaranteing stability. The algorithm utilizes $\mu_w = \frac{\mu_{0_w}(|z_k(i)|+1)}{|z_k(i)|e_k(i)\mathbf{r}^H(i)\mathbf{P}\mathbf{r}(i)}$, where μ_{0_w} is the convergence factor.

IV. Set-Membership Blind Adaptive Constrained Algorithms with Time Varying Error Bounds

In this section, we describe an adaptive filtering framework that gathers set-membership (SM) concept with blind constrained algorithms and introduce simple time-varying error bounds to take into consideration the parameter vector evolution and MAI and ISI. In SM filtering [7], the parameter vector $\mathbf{w}_k(i)$ for user k is designed to achieve a specified bound on the magnitude of an estimated quantity $z_k(i)$. As a result of this constraint, the SM blind adaptive algorithm will only perform filter updates for certain data. Let $\Theta(i)$ represent the set containing all $\mathbf{w}_k(i)$ that yield an estimation quantity upper bounded in magnitude by a time-varying error bound $\gamma(i)$. Thus, we can write

$$\Theta(i) = \bigcap_{(\mathbf{r}(i))\in\mathcal{S}} \{\mathbf{w}_k \in \mathcal{C}^M :| z_k(i) |\leq \gamma(i)\} \quad (7)$$

where $\mathbf{r}(i)$ is the observation vector, $\mathcal{S}$ is the set of all possible data pairs $(b_k(i),\ \mathbf{r}(i))$ and the set $\Theta(i)$ is referred to as the feasibility set, and any point in it is a valid estimate $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i)$. Since it is not practical to predict all data pairs, adaptive methods work with the membership set $\psi_i = \bigcap_{m=1}^{i}\mathcal{H}_m$ provided by the observations, where $\mathcal{H}_m = \{\mathbf{w}_k \in \mathcal{C}^M : |z_k(i)| \leq \gamma(i)\}$. In order to devise an effective SM algorithm, the bound $\gamma(i)$ must be appropriately chosen. Due to the time-varying nature of many practical environments, this bound should also be adaptive and capable of estimating certain characteristics for the SM estimation technique. In what follows, we derive SM-CMV and SM-CCM algorithms for blind parameter estimation that assume time-varying error bounds.

A. Set-Membership CMV Gradient Type Algorithms

The set-membership estimation criterion for the CMV design criterion corresponds to find a parameter vector $\mathbf{w}_k$ for user k that limits the instantaneous symbol estimate z_k subject to certain constraints not to exceed a specified time-varying error bound $\gamma(i)$. In other words, the SM-CMV approach corresponds to

$$J_{MV}(\mathbf{w}_k, \mathbf{r}(i)) = |\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 \leq \gamma^2(i) \quad (8)$$

for all $\mathbf{r}(i) \in \mathcal{S}$. The SM-CMV solution is the set

$$\Theta(i) = \bigcap_{(\mathbf{r}(i))\in\mathcal{S}} \{\mathbf{w}_k \in \mathcal{C}^M : J_{MV}(\mathbf{w}_k, \mathbf{r}(i)) \leq \gamma^2(i)\} \quad (9)$$

and the set of parameter vectors that ensure the SM-CMV criterion for $\mathbf{r}(i)$, that is the constraint set, is defined by

$$\mathcal{H}_i = \{\mathbf{w}_k \in \mathcal{C}^M : J_{MV}(\mathbf{w}_k, \mathbf{r}(i)) \leq \gamma^2(i)\} \quad (10)$$

The constraint set is the set of all $\mathbf{w}_k$ that satisfy $|\mathbf{w}_k^H(i)\mathbf{r}(i)| \leq \gamma(i)$. By substituting the CMV-SG recursion of (4) in (8), we have

$$|\mathbf{w}_k^H(i)\mathbf{r}(i)| = |\mathbf{w}^H(i)\mathbf{\Pi}_k\mathbf{r}(i) - \mu_w z_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i) +$$

$$\mathbf{h}^H(i)(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{C}_k^H\mathbf{r}(i)| = |z_k(i) - \mu_w z_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)| \quad (11)$$

Using the above expression we obtain the following condition:

$$|z_k(i) - \mu_w z_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)| = \gamma(i) \quad (12)$$

and the variable step size:

$$\mu_w = \left(\frac{1 - \sqrt{\gamma(i)}}{|z_k(i)|}\right)\frac{1}{\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)} \quad (13)$$

The resulting SM-CMV algorithm incorporates a variable step size mechanism, exhibits data-selective update and is described by:

$$\mathbf{w}_k(i+1) = \mathbf{\Pi}_k(\mathbf{w}_k(i) - \mu_w z_k^*(i)\mathbf{r}(i)) + \mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{h}(i) \quad (14)$$

where

$$\mu_w(i) = \begin{cases} \left(\frac{1-\sqrt{\gamma(i)}}{|z_k(i)|}\right)\frac{1}{\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)} & \text{if } |z_k(i)| \geq \gamma(i) \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

B. Set-Membership CCM Gradient Type Algorithms

The set-membership estimation criterion combined with the CCM design criterion is to seek a parameter vector $\mathbf{w}_k$ for user k that limits the instantaneous CM cost function subject to certain constraints not to exceed a specified time-varying error bound $\gamma(i)$. In other words, the SM-CCM approach corresponds to

$$J_{CM}(\mathbf{w}_k, \mathbf{r}(i)) = (|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2 \leq \gamma^2(i) \quad (16)$$

for all $\mathbf{r}(i) \in \mathcal{S}$. The SM-CCM solution is the set

$$\Theta(i) = \bigcap_{(\mathbf{r}(i))\in\mathcal{S}} \{\mathbf{w}_k \in \mathcal{C}^M : J_{CM}(\mathbf{w}_k, \mathbf{r}(i)) \leq \gamma^2(i)\} \quad (17)$$

and the set of parameter vectors that ensure the SM-CCM criterion for $\mathbf{r}(i)$, that is the constraint set, is defined by

$$\mathcal{H}_i = \{\mathbf{w}_k \in \mathcal{C}^M : J_{CM}(\mathbf{w}_k, \mathbf{r}(i)) \leq \gamma^2(i)\} \quad (18)$$

Using the expression in (16) and rearranging the terms, we can verify that the constraint set is non-convex and comprises two parallel hyper-strips in the parameter space. The constraint set is the set of all $\mathbf{w}_k$ that satisfy

$$\sqrt{1 - \gamma(i)} \leq |\mathbf{w}_k^H(i)\mathbf{r}(i)| \leq \sqrt{1 + \gamma(i)} \quad (19)$$

From the above conditions we consider two cases: i) $|\mathbf{w}_k^H(i)\mathbf{r}(i)| \leq \sqrt{1 + \gamma(i)}$ and ii) $|\mathbf{w}_k^H(i)\mathbf{r}(i)| \geq \sqrt{1 - \gamma(i)}$. By substituting the CCM-SG recursion obtained in (6) into $|\mathbf{w}_k^H(i)\mathbf{r}(i)|$, we get

$$|\mathbf{w}_k^H(i)\mathbf{r}(i)| = |z_k(i) - \mu_w e_k(i)z_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)| \quad (20)$$

Using the above expression we have for case i):

$$|z_k(i) - \mu_w e_k(i)z_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)| = \sqrt{1 + \gamma(i)} \quad (21)$$

and thus:

$$\mu_w = \left(\frac{1 - \sqrt{1 + \gamma(i)}}{|z_k(i)|}\right)\frac{1}{e_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)} \quad (22)$$

Using (19) we have for case ii):

$$|z_k(i) - \mu_w e_k(i) z_k(i) \mathbf{r}^H(i) \mathbf{\Pi}_k \mathbf{r}(i)| = \sqrt{1-\gamma(i)} \quad (23)$$

and thus:

$$\mu_w = \left(\frac{1-\sqrt{1-\gamma(i)}}{|z_k(i)|} \right) \frac{1}{e_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k \mathbf{r}(i)} \quad (24)$$

The resulting SM-CCM algorithm is described by:

$$\mathbf{w}_k(i+1) = \mathbf{\Pi}_k(\mathbf{w}_k(i) - \mu_w e_k(i) z_k^*(i)\mathbf{r}(i)) + \mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1}\mathbf{h}(i) \quad (25)$$

where

$$\mu_w(i) = \begin{cases} \left(\frac{1-\sqrt{1+\gamma(i)}}{|z_k(i)|}\right) \frac{1}{e_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)} & \text{if } |z_k(i)| \geq \sqrt{(1+\gamma(i)} \\ \left(\frac{1-\sqrt{1-\gamma(i)}}{|z_k(i)|}\right) \frac{1}{e_k(i)\mathbf{r}^H(i)\mathbf{\Pi}_k\mathbf{r}(i)} & \text{if } |z_k(i)| \leq \sqrt{(1-\gamma(i)} \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

V. Time-varying Bounds based on Parameter Dependency and Interference

This section is devoted to the description of time-varying bounds based on parameter dependency and the power of MAI and ISI.

A. Parameter Dependent Bound

Here, we describe a parameter dependent bound (PDB), that is similar to the one proposed in [14] and considers the evolution of the parameter vector $\mathbf{w}_1$. It is introduced a recursion for tracking and computing a bound for the SM adaptive algorithms:

$$\gamma(i+1) = \beta\gamma(i) + (1-\beta)\sqrt{\alpha||\mathbf{w}_1||^2 \hat{\sigma}_v^2(i)} \quad (27)$$

where $\alpha||\mathbf{w}_1(i)||^2\hat{\sigma}_v^2(i)$ is the variance of the inner product of $\mathbf{w}_1(i)$ with $\mathbf{n}(i)$ that provides information on the evolution of $\mathbf{w}_1$, α is a tuning parameter and $\hat{\sigma}_v^2(i)$ is an estimate of the noise power.

B. Parameter and Interference Dependent Bound

We develop an interference estimation and tracking procedure to be combined with the PDB outlined above and incorporated in a time-varying error bound for SM recursions. The MAI and ISI power estimation scheme, shown in Fig. 1, employs both the RAKE receiver and the linear receiver described in (2) for subtracting the desired user signal from $\mathbf{r}(i)$ and estimating MAI and ISI power levels. With the adopted channel estimation algorithm [15] that models the channel as an FIR filter, we design the blind linear receiver and obtain the detected symbol $\hat{b}_k$, which is combined with an amplitude estimate $\hat{A}_k$ for substracting the desired signal from the output x_k of the RAKE. Then, the difference d_k between the desired signal and x_k is used to estimate MAI and ISI power.

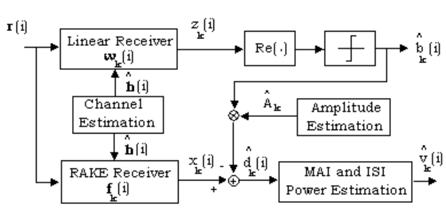

Fig. 1. Block diagram of the MAI and ISI power estimation scheme.

Let us consider a RAKE receiver, whose parameter vector $\mathbf{f}_k$ equals the effective signature sequence at the receiver, i.e. $\mathbf{f}_k(i) = \mathbf{C}_k\mathbf{h}(i) = \tilde{\mathbf{s}}_k$. The output of the RAKE for user 1 (desired one) is:

$$x(i) = \mathbf{f}_1^H(i)\mathbf{r}(i) = \underbrace{A_1 b_1(i)\mathbf{f}_1^H(i)\tilde{\mathbf{s}}_1(i)}_{\text{desired signal}} + \underbrace{\sum_{k=2}^{K} A_k b_k(i)\mathbf{f}_1^H(i)\tilde{\mathbf{s}}_k(i)}_{\text{MAI}} + \underbrace{\mathbf{f}_1^H(i)\boldsymbol{\eta}(i)}_{\text{ISI}} + \underbrace{\mathbf{f}_1^H(i)\mathbf{n}(i)}_{\text{noise}} \quad (28)$$

where $\mathbf{f}_1^H\tilde{\mathbf{s}}_1 = 1$ and $\mathbf{f}_1^H\tilde{\mathbf{s}}_j = \rho_{1,j}$ for $j \neq 1$. The second-order statistics of the output of the RAKE in (28) are described by:

$$E[|x(i)|^2] = A_1^2 \underbrace{E[|b_1(i)|^2]}_{\to 1} + \underbrace{\sum_{k=2}^{K}\sum_{l=2}^{K} A_k^2 E[b_k(i)b_l^*(i)]\mathbf{f}_1^H\tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_l^H\mathbf{f}_1}_{\to\sum_{k=2}^{K}\mathbf{f}_1^H\tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^H(i)\mathbf{f}_1} + \mathbf{f}_1^H E[\boldsymbol{\eta}(i)\boldsymbol{\eta}^H(i)]\mathbf{f}_1 + \underbrace{\mathbf{f}_1^H E[\mathbf{n}(i)\mathbf{n}^H(i)]\mathbf{f}_1}_{\to\sigma^2\mathbf{f}_1^H\mathbf{f}_1} \quad (29)$$

From the analysis above, we conclude that through the second-order statistics one can identify MAI, ISI and the noise terms. Therefore, our strategy is to obtain instantaneous estimates of the MAI, the ISI and the noise from the output of a RAKE receiver, subtract the detected symbol in (2) from this output (using the more reliable linear receiver ($\mathbf{w}_1$)) and to track the interference (MAI + ISI + noise). Let us define the difference between the output of the RAKE receiver and the detected symbol:

$$d(i) = x(i) - \hat{A}_1\hat{b}_1 \approx \underbrace{\sum_{k=2}^{K} A_k b_k(i)\mathbf{f}_1^H(i)\tilde{\mathbf{s}}_k(i)}_{\text{MAI}} + \underbrace{\mathbf{f}_1^H(i)\boldsymbol{\eta}(i)}_{\text{ISI}} + \underbrace{\mathbf{f}_1^H(i)\mathbf{n}(i)}_{\text{noise}} \quad (30)$$

By taking expectations on $|d(i)|^2$ and taking into account the assumption that MAI, ISI and noise are uncorrelated we have:

$$E[|d(i)|^2] \approx \sum_{k=2}^{K}\mathbf{f}_1^H(i)\tilde{\mathbf{s}}_k(i)\tilde{\mathbf{s}}_k^H(i)\mathbf{f}_1(i) + \mathbf{f}_1^H(i)E[\boldsymbol{\eta}(i)\boldsymbol{\eta}^H(i)]\mathbf{f}_1(i) + \sigma^2\mathbf{f}_1^H(i)\mathbf{f}_1(i) \quad (31)$$

where the above equation represents the interference power. Based on time averages of the instantaneous values of the interference power, we introduce the following algorithm to estimate and track $E[|d(i)|^2]$:

$$\hat{v}(i+1) = \beta\hat{v}(i) + (1-\beta)|d(i)|^2 \quad (32)$$

where β is a forgetting factor. To incorporate parameter dependency and interference power for computing a more effective bound, we propose the parameter and interference dependent bound (PIDB):

$$\gamma(i+1) = \beta\gamma(i) + (1-\beta)\Big(\sqrt{\tau\ \hat{v}(i)} + \sqrt{\alpha||\mathbf{w}_1||^2\hat{\sigma}_v^2(i)}\Big) \quad (33)$$

where $\hat{v}(i)$ is the estimated interference power in the multiuser system and τ is a parameter that must be set.

VI. Simulation Experiments

In this section we assess the bit error rate (BER) performance of the adaptive algorithms, i.e., the CMV-SG, the CCM-SG, the SM-CMV and the SM-CCM with and without the proposed time-varying bounds. The DS-CDMA system employs Gold sequences of length $N = 31$. With respect to the time-varying bounds, we consider the parameter dependent bound (PDB) approach of (27) and the parameter and interference dependent bound (PIDB) technique outlined in (33). The channels experienced by different users are identical since we focus on a downlink scenario and the channel coefficients are $h_l(i) = p_l\alpha_l(i)$, where $\alpha_l(i)$ ($l = 0, 1, \ldots, L_p - 1$) are obtained with Clarke's model [16]. We show the results in terms of the normalized Doppler frequency f_dT (cycles/symbol) and use three-path channels with relative powers given by 0, -3 and -6 dB, where in each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips. The channel estimator of [15] models the channel as an FIR filter and we employ a filter with 6 taps as an upper bound for the experiments.

The algorithms are assessed in a non-stationary environment where users enter and exit the system, as depicted in Fig. 2. The system starts with 2 interferers with 7 dB above the desired user's power level and 5 interferers with the same power level of the desired one, which corresponds to the signal-to-noise ratio $E_b/N_0 = 15$ dB. At 1000 symbols, 2 interferers with 10 dB above the desired signal power level and 1 with the same power level enter the system, whereas 1 interferers with 7 dB above the desired signal power level leaves it. At 2000 symbols, 1 interferer with 10 dB above, and 5 interferers with the same power level of the desired signal exit the system, whilst 1 interferer with 15 dB above the desired user enters the system. The results for 100 runs show that the proposed SM-CCM algorithm achieves the best performance, followed by the proposed SM-CMV recursion, the CCM-SG and the CMV-SG methods. Note that in a near-far scenario the eigenvalue spread of the covariance matrix of the received vector $\mathbf{r}(i)$ is large, affecting the convergence performance of the SG algorithms with fixed step size and making it very difficult to compute a pre-determined value [6] for the step size. Conversely, the SM algorithms are able to deal with near-far situations since they adopt variable step size mechanisms, ensuring good tracking performance in dynamic scenarios. In addition, due to their data-selective update feature the SM algorithms can save significant computational resources as they only require parameter updates for about 20% of the time.

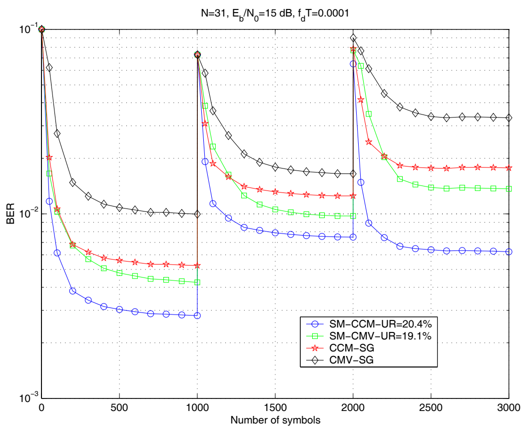

Fig. 2. BER performance of the algorithms versus number of symbols for a non-stationary scenario. The parameters are $\gamma = 1.3$ for the SM-CMV, $\gamma = 0.65$ for the SM-CCM.

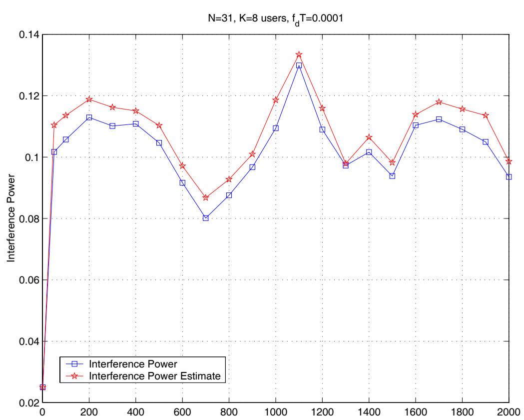

Fig. 3. Performance of the interference power estimation and tracking at $E_b/N_0 = 12dB$.

At this point, we wish to evaluate the effectiveness of the proposed algorithms when operating with the time-varying bounds. To this end, we carried out an experiment, depicted in Fig. 3, where the proposed algorithm estimates of the MAI and ISI powers are compared to the actual interference power. The results show that the proposed algorithm is very effective for estimating and tracking the interference power in dynamic environments, as depicts Fig. 3.

The same scenario illustrated in Fig. 2 is considered for the SM algorithms with time-varying error bounds, as shown in Fig. 4. The results indicate that the new time-varying bounds are capable of improving the performance of SM algorithms, while further reducing the number of updates. In particular, the algorithms with the PIDB approach achieve the best performance, followed by the algorithms with the PDB method and the SM recursions with fixed bounds. With respect to the update rates, the PIDB technique results in the smallest number of updates, followed by the PDB approach and the fixed bound.

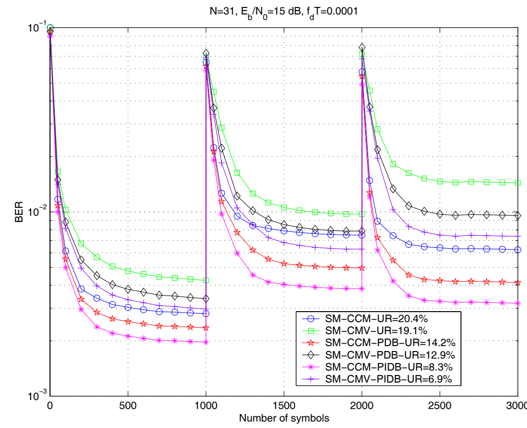

Fig. 4. BER performance of the algorithms versus number of symbols for a non-stationary scenario with time-varying bounds. The parameters are $\alpha = 8$, $\tau = 0.35$ and $\beta = 0.95$ for the time-varying bounds.

VII. Conclusions

We have proposed set-membership blind adaptive algorithms for DS-CDMA interference suppression using constrained optimization techniques along with the CM and MV cost functions. A new framework for SM estimation that takes into account parameter estimation dependency and MAI and ISI for multiuser communications was also introduced to provide a time-varying bound. Simulations experiments have shown that the new SM blind algorithms outperform previously reported techniques for several scenarios.

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