DETERMINISTIC ESTIMATION OF CHANNEL PARAMETERS IN ASYNCHRONOUS CDMA COMMUNICATIONS OVER FREQUENCY-SELECTIVE CHANNELS

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ABSTRACT

Designing appropriate user codes for training, we propose a channel gains and delay estimation scheme for asynchronous CDMA systems over frequency-selective channels. Our design arises from a simple but important property of "complex exponentials": a circular shift in a complex exponential vector looks like multiplying it by a unit magnitude complex number.

The main benefit of the algorithm is its independency of near-far ratio which arises from the fact that MUI is deterministically eliminated before estimating the channel parameters. Also, the proposed scheme doesn't need to estimate any statistic (such as a correlation matrix) for parameter estimation. Also since we are only involving in linear computations and search over parameter space, the computational complexity is less than other schemes which usually use subspace-decomposition or other enormous computations.

We first formulate the problem in noise-free case, and then to improve the performance in noisy environments, MMSE as well as ML approaches will be formulated and investigated.

1. INTRODUCTION

So far, much research has been carried out on channel parameters estimation in asynchronous CDMA systems.

Most of the conducted works have been devoted to channel/delay estimation over frequency-flat Rayleigh or Gaussian channels [1]-[3]. But due to the great demand for broadband wireless communication systems, it is important to have estimation methods over frequency-selective channels.

To respond to this demand some works have been reported on channel parameter estimation over frequency-selective channels. In [4] the author proposed an EKF-based method for channel/delay estimation. The method considered the effect of the other users as colored noise and then used EKF to estimate the channel gains/delay. In [5] the authors used an iterative scheme to estimate the channel parameters. Bensley and Aazhang [6] used the EigenValue Decomposition (EVD) of the correlation matrix of observed vector to estimate the parameters. In [7] the authors proposed a statistical maximum likelihood approach for parameter estimation.

The main drawback of all of these schemes (and some other schemes which have not mentioned due to the lack of space) is that they are statistical approaches and hence are very sensitive to near-far ratio and the data which is exploited for estimating of statistics (usually a correlation matrix). On the other hand, some of these algorithms suffer from high computational complexity [5]-[7].

In this paper we propose a method which enjoys the following nice properties:

p1) Since before estimating the channel parameters, the MUI is deterministically eliminated from the received data, the performance of the method is independent of near-far ratio.

p2) Due to the deterministic nature of the main body of the method, it doesn't need to any statistic

p3) The method uses linear computations plus a search over parameter space, and therefore is a low complexity method in comparison with other existing schemes.

The system model will be introduced in section 2. The main body of the training algorithm will be investigated in section 3. Minimum mean square error (MMSE) as well as Maximum likelihood (ML) parameter estimation will be showed in section 4. A brief discussion about complexity can be found in section 5. Section 6 has been devoted to simulation results.

Notation:

Bold upper(lower) letter : Matrix(vector)

$(.)^*$: conjugate; $(.)^T$: transpose; $(.)^H$: Hermitian transpose;

$\mathbf{F}_N$: $N \times N$ FFT matrix with (p,q)th entry $N^{-0.5} e^{-j2\pi(p-1)(q-1)/N}, \forall p,q \in [1,N]$;

$\mathbf{0}_{M\times N}$: all-zero matrix of size $M \times N$

$\mathbf{A}(:,i)$: ith column of $\mathbf{A}$; $\mathbf{A}(i,:)$: ith row of $\mathbf{A}$

$[\mathbf{A}]_{i,j}$: (i,j)th element of matrix $\mathbf{A}$.

$\mathrm{mod}(a,b)$: positive remainder of division a by b.

$\angle c$: phase angle of the complex number c

$|c|$: the magnitude of the complex number c.

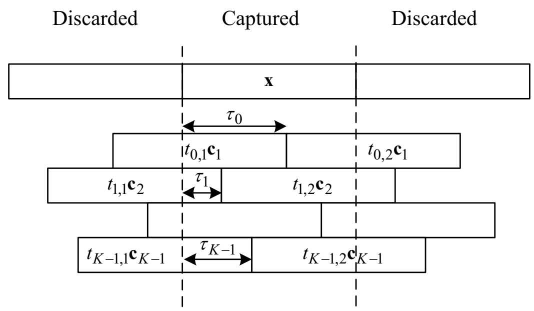

Figure 1: spread training symbols of users and captured frame in the receiver (L=0)

2. SYSTEM MODEL

Consider an asynchronous CDMA system, consists K users communicating over a frequency-selective channel.
Let's denote the propagation delay of kth user by τ_k which is a multiple of chip period (the users are chip-synchronized).
Also suppose that the FIR channel tap-weights between kth user and the receiver are denoted by $\{h_k[l]\}_{l=0}^{L}$, where the delay-spread and therefore L is known for the transmitter and receiver, both.
The aim of this paper is to deterministically estimate the propagation delays τ_k and channel tap-weights $\{h_k[l]\}_{l=0}^{L}$.
We adopt the following assumption:
a1) $\tau_k < K - L; \quad \forall k$
This is an acceptable assumption for "not very reach scattering enviroments" where we have $L << K$.
Here we set the spreading factor equal to the number of users to show that we are going to attain the maximum bandwidth efficiency too.
Denoting the spreading code of kth user by $\mathbf{c}_k := \begin{bmatrix} c_{k,0} & c_{k,1} & \cdots & c_{k,K-1} \end{bmatrix}^T$, and adopting **a1**, the input-output relation of system can be written as

$$\mathbf{x} = \sum_{k=0}^{K-1} (\mathbf{C}_{k,1} t_{k,1} + \mathbf{C}_{k,2} t_{k,2}) \mathbf{h}_k + \mathbf{v} \tag{1}$$

where $\mathbf{h}_k := \begin{bmatrix} h_k[0] & h_k[1] & \ldots & h_k[L] \end{bmatrix}^T$ and

$$\mathbf{C}_{k,1} = \begin{bmatrix} \mathbf{0}_{\tau_k \times (L+1)} \\ \mathbf{C}'_{k,1} \end{bmatrix}, \ \mathbf{C}_{k,2} = \begin{bmatrix} \mathbf{C}'_{k,2} \\ \mathbf{0}_{(K-\tau_k-L)\times(L+1)} \end{bmatrix} \tag{2}$$

are signatures corresponding to two consecutive training symbols of kth user, which in $\mathbf{C}'_{k,1}$ is an $(K-\tau_k)\times(L+1)$ Toeplitz matrix with first column $\begin{bmatrix} c_{k,0} & c_{k,1} & \cdots & c_{k,K-1-\tau_k} \end{bmatrix}^T$ and first row $\begin{bmatrix} c_{k,0} & \mathbf{0}_{1\times L} \end{bmatrix}$ and $\mathbf{C}'_{k,2}$ is an $(L+\tau_k)\times(L+1)$ Toeplitz matrix whose first column is $\begin{bmatrix} c_{k,K-\tau_k} & c_{k,K-\tau_k+1} & \cdots & c_{k,K-1} & \mathbf{0}_{1\times L} \end{bmatrix}^T$ and first row is $\begin{bmatrix} c_{k,K-\tau_k} & c_{k,K-\tau_k-1} & \cdots & c_{k,K-\tau_k-L} \end{bmatrix}$, and $\mathbf{v}$ is AWGN vector and $t_{k,1}$ and $t_{k,2}$ are two consecutive training bits. Fig. 1 shows the relative position of spread symbols and the captured frame in the receiver (vector $\mathbf{x}$) for frequency-flat channels ($L = 0$).

3. TRAINING STRATEGY

We accomplish the channel/delay estimation during $2K$ symbol periods, comprising K symbol pairs, where the codes used for spreading the symbols of any pair is the same, and for the next pair of training symbols, a different spreading code will be assigned.
In the receiver we totally collect one symbol (K chips) from each pair and discard the remained K chips.(take a look at fig.1 again)
Denoting the code used for spreading the ith pair ($2i$th and $(2i+1)$th symbols) of kth user by $\mathbf{c}_k[i]$, we choose

$$\mathbf{c}_k[i] := \mathbf{f}_k[i]; \qquad \begin{matrix} k = 0,1,\ldots,K-1 \\ i = 0,1,\ldots,K-1 \end{matrix} \tag{3}$$

where

$$\mathbf{f}_k[i] := \mathbf{f}_{\mathrm{mod}(k+i,K)} \tag{4}$$

which in $\mathbf{f}_j := \mathbf{F}_K(:, j+1)$.
By this we can rewrite the ith capture symbol in the receiver by

$$\mathbf{x}[i] = \sum_{k=0}^{K-1} (\mathbf{C}_{k,1}[i] t_{k,2i} + \mathbf{C}_{k,2}[i] t_{k,2i+1}) \mathbf{h}_k + \mathbf{v} \tag{5}$$

and if we set $t_{k,2i+1} = t_{k,2i} = +1$ we will have

$$\mathbf{x}[i] = \sum_{k=0}^{K-1} (\mathbf{C}_{k,1}[i] + \mathbf{C}_{k,2}[i]) \mathbf{h}_k + \mathbf{v} \tag{6}$$

where $\mathbf{C}_{k,1}[i]$ and $\mathbf{C}_{k,2}[i]$ are defined just like (2).
Due to (2)-(4), we can easily find that

$$\mathbf{C}_{k,1}[i] + \mathbf{C}_{k,2}[i] = e^{j\omega(k+i)\tau_k} \mathbf{C}_k[i] \tag{7}$$

where $\mathbf{C}_k[i]$ is a $K \times (L+1)$ Toeplitz matrix whose first row is $\begin{bmatrix} c_{k,0}[i] & c_{k,K-1}[i] & c_{k,K-2}[i] & \cdots & c_{k,K-L}[i] \end{bmatrix}$, first column is $\begin{bmatrix} c_{k,0}[i] & c_{k,1}[i] & c_{k,2}[i] & \cdots & c_{k,K-1}[i] \end{bmatrix}^T$ and as we former mentioned in (3), $c_{k,p}[i] := K^{-0.5} \exp\{j\omega(k+i)p\}$ where $\omega = 2\pi/K$

Substituting (7) into (6) yields

$$\mathbf{x}[i] = \sum_{k=0}^{K-1} e^{j\omega(k+i)\tau_k} \mathbf{C}_k \mathbf{h}_k + \mathbf{v} \tag{8}$$

Now, to estimate the channel tap-weights between mth user and receiver, we first left-multiply (8) by $\mathbf{c}_{m,i}^H = \mathbf{f}_{\text{mod}(m+i,K)}^H$, which yields:

$$y_m[i] = e^{j\omega(m+i)\tau_m} H_m^{-1}[\text{mod}(m+i,K)] + w_m[i] \qquad i = 0,1,\cdots,K-1 \quad m = 0,1,\ldots,K-1 \tag{9}$$

where $H_m^{-1}[p]$ is the *inverse discrete Fourier transform* (IDFT) of $h_m[l]$ is defined as

$$H_m^{-1}[p] := \sum_{l=0}^{L} h_m[l] e^{j\omega pl}; \quad p = 0,1,\cdots,K-1 \tag{10}$$

and $w_m[i] := \mathbf{f}_{\text{mod}(m+i,K)}^H \mathbf{v}$ is still additive Gaussian noise.
Now, applying the DFT to (9), we will have

$$\begin{aligned} z_m[n] &:= [\mathbf{F}_K \mathbf{y}_m]_{n+1,1} \\ &= h_m[n-\tau_m] e^{j\omega nm} + \overline{w}_m[n] \end{aligned} \tag{11}$$

Where $\mathbf{y}_m := [y_m[0] \quad y_m[1] \quad \cdots \quad y_m[K-1]]^T$ and $\overline{w}_m[n]$ is the DFT of $w_m[i]$.

4. PARAMETER ESTIMATION

4.1. Noise-Free Case

To have a more obvious interpretation, we first consider the noise-free scenario, where (11) have to rewrite as

$$z_m[n] = h_m[n-\tau_m] e^{j\omega nm}; \quad n = 0,1,\ldots,K-1 \tag{12}$$

where $L+1$ points out of K points $\{z_m[n]\}_{n=0}^{K-1}$ are corresponding to $\{h_m[l]\}_{l=0}^{L}$ and therefore are nonzero, and the left $K-L-1$ points equal zeros:

$$z_m[n] = \begin{cases} 0 & 0 \le n < \tau_m \\ h_m[n-\tau_m] e^{j\omega nm} & \tau_m \le n \le \tau_m + L \\ 0 & \tau_m + L < n \le K-1 \end{cases} \tag{13}$$

Therefore to estimate the propagation delay, we should check for the nonzero values of $z_m[n]$, where due to (13) the first nonzero $z_m[n]$ will be the τ_mth one. Therefore we can easily estimate τ_m as the first value of time index, n, which for $z_m[n] \neq 0$. In the other words we have

$$\tau_m = \min_n \arg(z_m[n] \neq 0) \tag{14}$$

4.2. Noisy Case: MMSE parameters estimation

In noisy case, (13) must be rewritten as

$$z_m[n] = \begin{cases} \overline{w}_m[n] & 0 \le n < \tau_m \\ h_m[n-\tau_m] e^{j\omega nm} + \overline{w}_m[n] & \tau_m \le n \le \tau_m + L \\ \overline{w}_m[n] & \tau_m + L < n \le K-1 \end{cases} \tag{15}$$

Where as we mentioned former, $\overline{w}_m[n]$ is additive white Gaussian noise.
Due to (15), here we can't estimate the propagation delay by checking for first nonzero $z_m[n]$.
Instead, we exploit the following minimum mean-square error (MMSE) criteria for estimating the propagation delay:

$$\hat{\tau}_m^{(MMSE)} = \arg\min_\tau \left(\sum_{n \notin [\tau, \tau+L]} |z_m[n]|^2 \right) \tag{16}$$

To improve the performance of our MMSE estimator, we can do the training several times, lets say Q times, and then use the following criteria to achieve an improved estimation

$$\hat{\tau}_m^{(MMSE)} = \arg\min_\tau \left(\sum_{q=1}^{Q} \sum_{n \notin [\tau, \tau+L]} |z_m[n;q]|^2 \right) \tag{17}$$

Where $z_m[n;q]$ is the $z_m[n]$ in the qth train.
After estimating the propagation delay, the magnitude and phase angle of channel gains can be respectively estimated by the following formulae

$$\left|\hat{h}_m[l]\right| = \frac{1}{Q} \sum_{q=1}^{Q} \left| z_m[l+\hat{\tau}_m;q] \right| \tag{18}$$

$$\angle \hat{h}_m[l] = \sum_{q=1}^{Q} \angle \{ z_m[l+\hat{\tau}_m;q] e^{-j\omega m(l+\hat{\tau}_m)} \} \tag{19}$$

4.3. Noisy Case: ML parameter estimation

We can also have a maximum likelihood (ML) estimation for propagation delay as

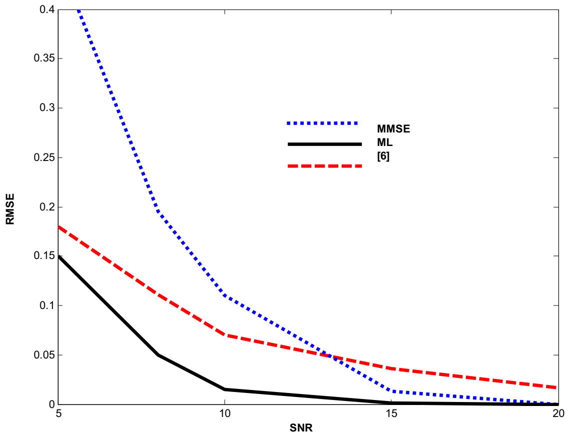

Fig.2 : Root-mean-square error (RMSE) of delay estimation versus signal-to-noise ratio

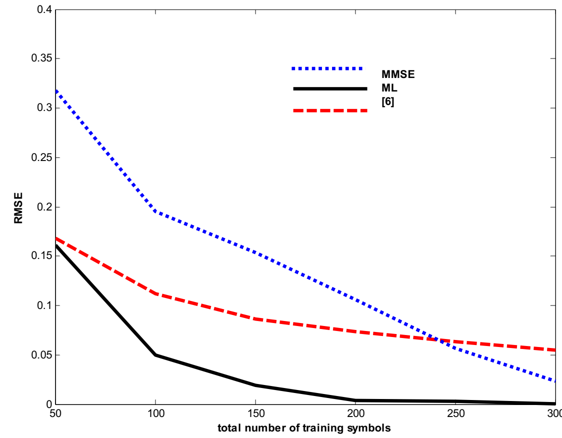

Fig.3 : Root-mean-square error (RMSE) of delay estimation versus total number of training symbols

$$\hat{\tau}_m^{(ML)} = \arg\max_{\tau_m} f\left(\bigcap_{(n,q)=(0,1)}^{(K-1,Q)} z_m[n;q] \mid \tau_m \right) \qquad (20)$$

where $f(x \mid y)$ is the conditional probability density function (PDF).

5. COMPUTATIONAL COMPLEXITY

One of the advantages of the proposed method is its low computational complexity.

It can be easily observed that the method comprises the linear computations and search over parameter space, which is obviously lighter than other (usually subspace-based) existing schemes which involve heavy subspace decomposition operations and the search over parameter space.

6. SIMULATION RESULTS

Here we have considered a 10 user asynchronous CDMA system as our simulation example.

The underlying channel is frequency-selective with three paths.

As the opponent method, we have considered the subspace-based method introduced in [6].

Figure 2, shows a performance comparison for different signal-to-noise ratios for 100 training symbols.

Figure 3 shows the same for different number of training symbols and 8db signal-to-noise ratio.

Note that for our ML and MMSE approaches, the total number of training symbols equals QK .

As it can be observed the proposed ML method outperforms [6], where MMSE approach exhibits week performance in low signal-to-noise ratios and small number of training symbols.

7. CONCLUDING REMARKS

By appropriate designing of user codes and planning of training strategy which arises from properties of complex exponential vectors, we achieved an approach for channel/delay estimation in CDMA systems over frequency-selective channels.

Due to the deterministic nature of our proposed approach, the performance is independent of near-far ratio as well as estimated statistics.

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