MAI Impact Suppression with Parallel Interference Cancellation in DS-OCDMA Systems using Prime Codes

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ABSTRACT

In this paper, we study the Parallel Interference Cancellation receiver (PIC) efficiency, in Direct Sequence Optical Code Division Multiple Access system (DS-OCDMA) with Prime Codes (PC). We develop the analytical expression of the error probability in the chip synchronous case, for PC. We show that the PIC receiver permits to suppress totally the effect of Multiple Access Interference (MAI) for Prime Codes, and leads to an error free O-CDMA link in the noiseless case, for whichever PC employed. Simulation results have validated the theoretical analysis. Finally, we show that the PIC receiver permits to reduce the required code length for a given Bit Error Rate.

1. INTRODUCTION

The O-CDMA (Optical-Code Division Multiple Access) inspired from Radio Frequency communications, is nowadays studied for application in optical networks.
Many recent studies concern the use of OCDMA as an alternative scheme of multiple access in optic fiber network, especially for high speed LANs[1]. Thanks to the robustness of CDMA to multi-path fading, the optical CDMA can be extended to optical wireless communications [2].
There have been many approaches for OCDMA implementation. Since coherent optical systems are costly and difficult to implement, the majority of studies concerns the non-coherent optical systems. In incoherent system, the unipolar codes [3] can not be strictly orthogonal. Thus, the system suffers from Multiple Access Interference (MAI). MAI can be considered as the dominant noise source in temporal coding systems employing ideal light sources and electrical coding-decoding functions [4]. In this case, the electrical device bandwidth imposes a limitation on the code length. So, there is a tradeoff between users number, code length and MAI.
To support numbered simultaneous users with short code length, we can employ Prime Codes (PC)[5]. However, such codes suffer from high cross-correlation products which create high MAI. To reduce the MAI effect, a multi-user detection method can be applied at the receiver end.

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We have previously studied in [6] the Parallel Interference Cancellation receiver (PIC) and shown that it is a performing way of improving the performances for a DS-OCDMA system using Optical Orthogonal Codes [6].
In this paper, we study the PIC receiver efficiency to mitigate MAI for PC. We first describe PIC principle. Then, we develop the theoretical expression of the error probability. Moreover, we show that for the optimal threshold levels, the transmission is error free. We validate these results by simulation. The consequence is that the PC code length required for a given BER with a PIC is much smaller than with conventional receiver.

2. SYSTEM DESCRIPTION

2.1. DS-OCDMA system

We consider a non-coherent, synchronous Direct Detection CDMA system.
Each user employs an On/Off Keying (OOK) modulation to transmit independent and equiprobable binary data upon an optical channel. A sequence code is impressed upon the binary data by the encoder. The sequence code is specific to each user, in order to be able to extract the data at the end receiver: the received signal would be compared to the sequence code, then to a threshold level at the comparator for the data recovery.

2.2. Prime Code (PC)

A Prime Sequence (PS) [5] of length P (P a prime number) is defined by $S_i = \{s_{i,0}, s_{i,1}, ..., s_{i,j}, ..., s_{i,P-1}\}$. Each element $s_{i,j}$ is obtained by $s_{i,j} = i \times j$ (mod P), with $i, j \in GF(P)$ the Galois field.
We construct a set of Prime Code (PC) $C_i = \{c_{i,0}, c_{i,1}, ..., c_{i,k}, ..., c_{i,F-1}\}$, whose length is $F = P^2$ from the PS by :

$$c_{i,k} = \begin{cases} 1 & k = jP + s_{i,j} \quad (0 \le j < P) \\ 0 & otherwise \end{cases}$$

We obtain $N=P$ codewords with a length P^2, a weight $W=P$, and with crosscorrelation values no greater than 2 . We will refer to such Prime Code with "PC (P^2,P)".

2.3. Conventional Correlation Receiver(CCR)

We consider throughout the paper that there is no noise contribution and that the synchronisation is perfect. At the receiver end, $r(t)$ is the sum of the users' signals:

$$r(t)=\sum_{k=1}^{N} b_i^{(k)} C_k(t)$$

with $C_k(t)$ the k^{th} user sequence code, and $b_i^{(k)} \in \{0,1\}$ the i^{th} data bit of the k^{th} user.

The received signal $r(t)$ is multiplied by the code sequence corresponding to the desired user $C_k(t)$, and the result is integrated. We get the decision variable value $Z_i^{(k)}$:

$$\begin{aligned} Z_i^{(k)} &= \int_0^T r(t).C_k(t)dt = W.b_i^{(k)} + \sum_{j=1, j\neq k}^{N} b_i^{(j)}.\int_0^T C_k(t).C_j(t)dt \qquad (1) \\ &= W.b_i^{(k)} + I_{CCRk} \end{aligned}$$

The term I_{CCRk} is the interference due to all the undesired users (MAI). The decision variable value is compared to the threshold level S_T of the decision device and an estimation of the transmitted bit, $\hat{b}_i^{(k)}$, is given. An error can occur only when $b_i^{(k)}$ is a zero data and the MAI term is greater than the threshold level value S_T.

Yang and Kwong [5] have reported the analytical expression of the error probability P_{ECCR} for the ideal chip synchronous case for a threshold value S_T :

$$P_{ECCR}=\frac{1}{2}-\frac{1}{2}\sum_{i_1=0}^{S_T-1}\sum_{i_2=0}^{\lfloor (S_T-1-i_1)/2 \rfloor}\left[\frac{(N-1)!}{i_1!i_2!(N-1-i_1-i_2)!}\left(\frac{p_1}{2}\right)^{i_1}\left(\frac{p_2}{2}\right)^{i_2}\left(1-\frac{p_1}{2}-\frac{p_2}{2}\right)^{N-1-i_1-i_2}\right]$$

with p_1, p_2 the average probabilities of having one and two overlaps respectively between 2 codes sequences,defined as:

$$p_1+2p_2=1 \text{ and } p_2=\frac{(P+1)(P-2)}{6P^2}$$

2.4. Parallel Interference Cancellation Receiver

To present the PIC principle, we assume the first user to be the desired one. All the users are supposed to have the same transmitting energy so there is no strongest interfering signal.

The aim of the PIC [6] is to reproduce the interference term due to all interfering users and to remove it from the received signal. The PIC first detects the $N-1$ undesired users employing the conventional correlation receiver defined in the previous part with a threshold level S_u. The estimated interference is built by spreading the estimated data with the corresponding code sequence, and removed from the received signal $r(t)$.

The bit sent by the desired user (#1) is evaluated with a conventional correlation receiver with a threshold level S_d.

The signal $s(t)$ applied to the entry of the receiver is:

$$s(t)=r(t)-\sum_{j=2}^{N}\hat{b}_i^{(j)}C_j(t)=b_i^{(1)}C_1(t)+\sum_{j=2}^{N}(b_i^{(j)}-\hat{b}_i^{(j)}).C_j(t)$$

3. THEORETICAL ANALYSIS

In this section, the expression of the error probability for the PIC receiver, will be demonstrated in a synchronous case, for PC (P^2,P), and for N simultaneous users. We consider the same threshold levels S_u $(0<S_u\leq W)$ for the $(N-1)$ undesired users' receivers. We consider that the threshold level for the desired user #1 is S_d $(0<S_d\leq W)$.

In a general way, the error probability can be written as:

$$P_E=\frac{1}{2}P(\hat{b}_i^{(1)}=1/b_i^{(1)}=0)+\frac{1}{2}P(\hat{b}_i^{(1)}=0/b_i^{(1)}=1)$$

The decision-making concerning $\hat{b}_i^{(1)}$ is related to user#1's decision variable, which is expressed as:

$$\begin{aligned} Z_i^{(1)} &= W.b_i^{(1)}+\sum_{j=2}^{N}(b_i^{(j)}-\hat{b}_i^{(j)})\int_0^T C_1(t).C_j(t)dt \\ &= W.b_i^{(1)}+\sum_{j=2}^{N} I_i^{(j)} = W.b_i^{(1)}+I \end{aligned}$$

I is called the interfering term. Errors on user #1's data are due to this term.

Indeed, for an undesired user *#j*, $I_i^{(j)}$ is the interference term due to user $\#j$ on user #1. $(b_i^{(j)}-\hat{b}_i^{(j)})$ is either null when there is no error, or non null when there is an error. With CCR, an error can occur only when the sent data is a "0", and $(b_i^{(j)}-\hat{b}_i^{(j)})=-1$. As $\int_0^T C_1(t).C_j(t).dt$ is either null, equal to 1 or equal to 2, $I_i^{(j)}$ is either null, equal to "-1", or "-2".

When $I_i^{(j)}$ is non-null, the user #j is called "interfering user". The interfering users are the ones that send a '0' detected as a '1', and have at least one common chip with user #1.

As $I_i^{(j)}$ is either null or negative, $I=\sum_{j=2}^{N} I_i^{(j)}$ is always negative or null, so the decision variable $Z_i^{(1)}$ is less (or equal) than it should be. So, there can be errors only for $b_i^{(1)}=1$. Therefore, the expression of the error probability can be simplified to:

$$P_e=\frac{1}{2}P(\hat{b}_i^{(1)}=0/b_i^{(1)}=1)$$

From now on, we consider the case $b_i^{(1)}=1$.

In this case, P_e can be expressed as a function of the probabilities of 2 events:

- the probability P_{i1} for an undesired user #j who sent "0" to create an interference of "-1" on user #1, when $b_i^{(1)}=1$,
- the probability P_{i2} for an undesired user #j who sent "0" to create an interference of "-2" on user #1, when $b_i^{(1)}=1$.

For the determination of P_{i1}, and P_{i2}, we consider that $b_i^{(1)}=1$, and that N_1 undesired users sent a 1.

3.1. Expression of P_{i1}

P_{i1} is the probability for an undesired user who send a "0" to create an interference of "-1" on user #1. Consequently, user #j must verify 2 conditions:

- he has one chip in common with user #1's code
- his datum is detected as a "1" instead of a "0".

So, we can write:

$$P_{i1} = P(\int_0^T C_1(t).C_j(t).dt = 1 \cap Z_i^{(j)} \geq S_u / (b_i^{(j)} = 0 \cap b_i^{(1)} = 1))$$
$$= p_1 \times P(Z_i^{(j)} \geq S_u / (\int_0^T C_1(t).C_j(t).dt = 1 \cap b_i^{(j)} = 0 \cap b_i^{(1)} = 1))$$

When $\int_0^T C_1(t).C_j(t).dt = 1$, there is an overlapping between user #1's and user #j's codes. Consequently, the user #1 (who send a datum "1") generates an interference of value +1 on user #j. Thus, the contribution I'_{CCRj} of the others users (i.e. the N_1 undesired users who send a "1") must be greater than $S_u - 1$. Thus:

$$P(Z_i^{(j)} \geq S_u / (\int_0^T C_1(t).C_j(t).dt = 1 \cap b_i^{(j)} = 0 \cap b_i^{(1)} = 1)$$
$$= P(I_{CCRj} \geq S_u / (\int_0^T C_1(t).C_j(t).dt = 1 \cap b_i^{(1)} = 1) = P(I'_{CCRj} \geq S_u - 1)$$

An undesired user who send a 1, generates an interference of "+1" and of "+2" on user #j, with the probability p_1 and p_2 respectively. So the probability to have i_1 and i_2 users interfering with a value "+1" and "+2" respectively, among the N_1 undesired users who send a "1" is described by a trinomial rule, and can be expressed as:

$$\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2}$$

Moreover, we have:

$$P(I'_{CCRj} \geq S_u - 1) = 1 - P(I'_{CCRj} < S_u - 1)$$

and $I'_{CCRj} = i_1 + 2i_2$ when considering i_1 and i_2 users interfering with a value "+1" and "+2" respectively. So :

$$P(I'_{CCRj} \geq S_u - 1) = 1 - P(i_1 + 2i_2 < S_u - 1)$$
$$= 1 - \sum_{i_1=0}^{S_u-2} \sum_{i_2=0}^{\lfloor (S_u-2-i_1)/2 \rfloor} \left[\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2} \right]$$

Consequently, we finally get :

$$P_{i1} = p_1 \times \left(1 - \sum_{i_1=0}^{S_u-2} \sum_{i_2=0}^{\lfloor (S_u-2-i_1)/2 \rfloor} \left[\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2} \right] \right) \quad (2)$$

3.2. Expression of P_{i2}

P_{i2} is the probability for an undesired user who send a "0" to create an interference of "-2" on user #1.

With the same demonstration than for P_{i1}, we get :

$$P_{i2} = p_2 \times \left(1 - \sum_{i_1=0}^{S_u-3} \sum_{i_2=0}^{\lfloor (S_u-3-i_1)/2 \rfloor} \left[\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2} \right] \right) \quad (3)$$

3.3. Expression of P_e

P_e is the error probability for user #1. P_e can be written:

$$P_e = \frac{1}{2} P(Z_i^{(1)} < S_d / b_i^{(1)} = 1)$$

We first consider that N_1 undesired users sent a "1". The probability to have exactly N_1 undesired users who sent a "1" is:

$$P(N_1) = \binom{N-1}{N_1} (1/2)^{N_1} \times (1/2)^{N-1-N_1} = \binom{N-1}{N_1} (1/2)^{N-1} \quad (4)$$

Thus

$$P_e = \frac{1}{2} \sum_{N_1=0}^{N-1} P(N_1) \times P(Z_i^{(1)} < S_d / N_1 \cap b_i^{(1)} = 1) \quad (5)$$

$$P(Z_i^{(1)} < S_d / N_1 \cap b_i^{(1)} = 1) = P(W + I < S_d)$$

On considering N_{21} and N_{22} users creating interference of "-1" and "-2" on user #1 respectively, we can write:

$$P(Z_i^{(1)} < S_d / N_1 \cap b_i^{(1)} = 1) = P(W - N_{21} - 2N_{22} < S_d)$$
$$= P(N_{21} + 2N_{22} \geq W - S_d + 1)$$

Moreover, we have shown that an undesired user who send a 0, generates an interference of "-1" and "-2" on user #1, with the probability P_{i1} and P_{i2} respectively. So the probability to have N_{21} and N_{22} users interfering with a value "-1" and "-2" respectively, among the $N - 1 - N_1$ undesired users who send a "0", can be expressed as:

$$\frac{(N-1-N_1)!}{N_{21}! N_{22}! (N-1-N_1-N_{21}-N_{22})!} P_{i1}^{N_{21}} P_{i2}^{N_{22}} (1 - P_{i1} - P_{i2})^{N_1 - 1 - N_{21} - N_{22}}$$

Thus, we get :

$$P(Z_i^{(1)} < S_d / N_1 \cap b_i^{(1)} = 1) = 1 - P(N_{21} + 2N_{22} < W - S_d + 1)$$
$$= 1 - \sum_{N_{21}=0}^{W-S_d} \sum_{N_{22}=0}^{\lfloor (W-S_d-N_{21})/2 \rfloor} \quad (6)$$
$$\left[\frac{(N-1-N_1)!}{N_{21}! N_{22}! (N-1-N_1-N_{21}-N_{22})!} P_{i1}^{N_{21}} P_{i2}^{N_{22}} (1 - P_{i1} - P_{i2})^{N_1 - 1 - N_{21} - N_{22}} \right]$$

We finally obtain the expression of the error probability from (4), (5) and (6) :

$$P_e = \left(\frac{1}{2}\right)^N \sum_{N_1=0}^{N-1} \binom{N-1}{N_1} \times \left(1 - \sum_{N_{21}=0}^{W-S_d} \sum_{N_{22}=0}^{\lfloor (W-S_d-N_{21})/2 \rfloor} \right. \quad (7)$$
$$\left. \left[\frac{(N-1-N_1)!}{N_{21}! N_{22}! (N-1-N_1-N_{21}-N_{22})!} P_{i1}^{N_{21}} P_{i2}^{N_{22}} (1 - P_{i1} - P_{i2})^{N_1 - 1 - N_{21} - N_{22}} \right] \right)$$

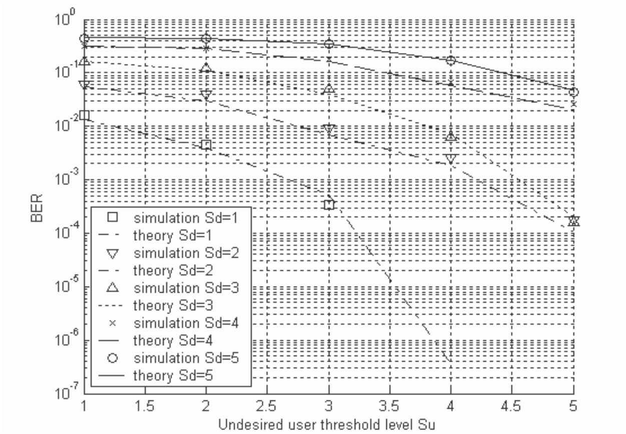

Fig 1: Simulated and theoretical PIC performances for PC (25,5) with N=5

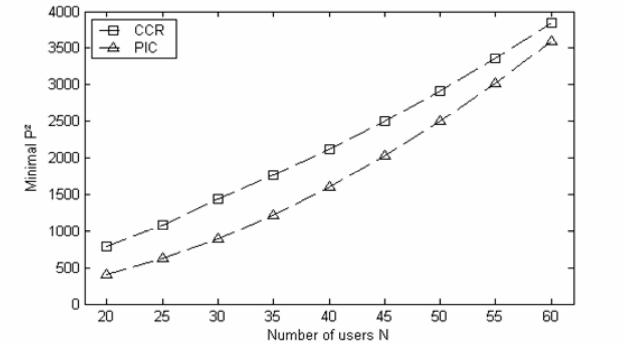

Fig 2: Minimal code length P^2 for BER<10^{-9} for PC with CCR and PIC

4. RESULTS EXPLOITATION

Fig.1 shows the comparison between theoretical and simulated results for the PIC receiver for a PC(25,5) with N=5 users, versus the threshold level values of respectively the desired and undesired users' CCR. We can first point out that the theoretical expression (7) correctly describes the PIC receiver performances.

In addition to that, we can observe that the BER decreases when S_u increases and S_d decreases. Thus, we can deduce that $S_u=P$ and $S_d=1$ are the optimal threshold levels. Indeed, for CCR, the lower error probability is obtained for $S_u=P$, so for such threshold level, the non-desired users are better estimated. Moreover, as the interference on the desired user is negative, the optimal threshold for the desired user is the smallest positive number, that is $S_d=1$.

Furthermore, we can observe that for $S_u=5$ and $S_d=1$, the theoretical and simulated BER are null. It can be verified that, for all the values of P, the BER is null when considering the optimal threshold levels. Indeed, in order to have undesired users detected as "*1*" instead of "*0*",we must have for each of these users: $I_{CCRj} \geq S_u=P$, on considering the optimal threshold level. As the maximum interference contribution from one given user is *2*, there must be at least $\lceil P/2 \rceil$ users having sent a "*1*" to create errors on some undesired users. Moreover, there is an error on the desired user if $I<S_d-P$, i.e. $|I|\geq P$ when considering the optimal threshold level $S_d=1$. Therefore there must be at least $\lceil P/2 \rceil$ undesired user interfering on the desired user. On the whole, to fit with the conditions for error with the PIC receiver, there must be at least $\lceil P/2 \rceil$ users that sent a "*0*",and $\lceil P/2 \rceil$ users that sent a "*1*", so there must be at least $2\times\lceil P/2 \rceil$. As P is a prime number, there must be at least $P+1$ different active users in the network. But, there are at most only P possible users in the code set. Thus, errors can never occur. Thus, in spite of the high MAI due to high cross-correlation value, the PIC receiver applied to any PC leads to an error free transmission link in the noiseless case.

In addition to that, we can remark that for $S_u=4$ and $S_d=1$, the theoretical analysis predicts a non null BER whereas simulated BER seems to be null (we get no error for 10^{10} bits). This is due to the fact that we did not take into account in our theoretical analysis that there is one user in each PC family set who is always well detected (and thus can never be an interfering user for the desired user) and whose maximum interference on the undesired users is "1".

In order to evaluate the PIC benefit with PC, we have plotted on fig. 2 the minimal code length P^2 required to have a BER <10^{-9} for the optimal threshold levels, as a function of users number N. We can observe that the PIC receiver permits to decrease the code length required compared to CCR of about 40%. This decrease is significant and permits to have more flexibility regards to the electronic bandwidth.

5. CONCLUSION

We have evaluated the Parallel Interference Cancellation receiver (PIC) efficiency in a DS-OCDMA link with Prime Codes (PC). The analytic expression of the error probability in the case of Prime Codes has been established. From numerical calculation and simulation, we have proved the reliability of the theoretical analysis. Moreover, we have shown theoretically that, in spite of the high cross-correlation value of the PC, the PIC receiver permits to remove the Multiple Access Interference (MAI) and permits to obtain an error free transmission link, in the noiseless case, for whichever Prime Code. Finally, we have shown that by reducing the code length, the PIC receiver brings flexibility regards to the electronic bandwidth.

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