MMSE OPTIMISATION FOR LS CHANNEL ESTIMATION IN WIDEBAND DS–CDMA RAKE RECEIVERS

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ABSTRACT

It is well established that the quality of the channel estimate plays a crucial role in the performance of a DS–CDMA receiver. This paper addresses the problem of optimising the channel estimate for a wideband DS–CDMA rake receiver. A simplified adaptive least squares scheme is proposed as a channel estimator, with only one adjustable parameter, the averaging window size. Then, the mean squared error of the channel estimate is analytically extracted. The estimation error is found to consist of two antagonistic (with respect to the window size) components: a) the estimation noise (due to the transmission channel noise, inter–chip interference and multiple–access interference), and b) the estimation distortion (due to the limited ability of the adaptive algorithm to track the non–stationary channel). An MMSE channel estimator is finally proposed, exhibiting the best trade–off between the two antagonistic estimation error components.

1. INTRODUCTION

The third generation (3G) universal mobile telecommunication system (UMTS) [1] is based on a wideband direct sequence code division multiple access (DS–CDMA) radio interface [2]. It has been demonstrated that imperfect channel estimation has dramatic effects on the performance of both a conventional rake receiver [3] and serial interference cancellation [4] or parallel interference cancellation [5] multi–user detectors.

In this paper we propose a MMSE optimisation scheme for the channel estimator of a rake receiver, suitable for the low signal–to–noise plus interference ratios (SNIR) and fast fading environments likely to be experienced in a 3G wideband DS-CDMA system. A realistic framework was employed in order to demonstrate the performance of the channel estimation scheme for the frequency division duplex (FDD) UMTS terrestrial radio access (UTRA) uplink.

The rest of this paper is organised as follows. Section 2 presents the proposed channel estimation scheme in the context of a conventional rake receiver. In section 3 we extract the variance of the estimation error due to noise, while section 4 derives the variance of the estimation error due to the distortion and lag. The overall channel estimation error is studied in section 5, while section 6 presents some useful experimental results. Finally, conclusions are drawn in section 7.

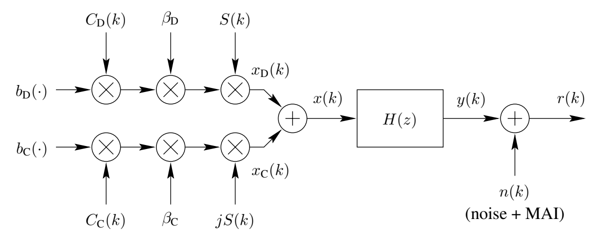

Fig. 1. System model.

2. SYSTEM MODEL

Consider the CDMA system in fig. 1. The binary data symbols $b_{\mathrm{D}}(\cdot)$ are multiplied with a binary data channelisation code $C_{\mathrm{D}}(k)$, then with an amplitude coefficient β_{D}. Finally a complex scrambling code $S(k)$ is applied to form the transmitted data chip sequence $x_{\mathrm{D}}(k)$. Accordingly, the control chip sequence $x_{\mathrm{C}}(k)$ is formed by the multiplication of the binary control symbols $b_{\mathrm{C}}(\cdot)$ with the control channelisation code $C_{\mathrm{C}}(k)$, the control amplitude coefficient β_{C} and the scrambling code rotated by $\pi/2$.

The transmitted signal $x(k)$ passes through a FIR channel with tap coefficients $\mathbf{h} = [h_0,\ h_1,\ \ldots\ h_N]$. Assuming that $H(z)$ is stationary, the received signal is

$$r(k) = \sum_{i=0}^{N} x_{\mathrm{C}}(k-i)h_i\ + \sum_{i=0}^{N} x_{\mathrm{D}}(k-i)h_i\ + n(k) \qquad (1)$$

where $n(k)$ is noise plus multiple–access interference. We can assume that $n(k)$ is a zero–mean complex Gaussian i.i.d. sequence with variance σ_{N}^2.

Consider now the CDMA receiver in fig. 2, consisting of a rake receiver and a chip–rate channel estimator. The con-

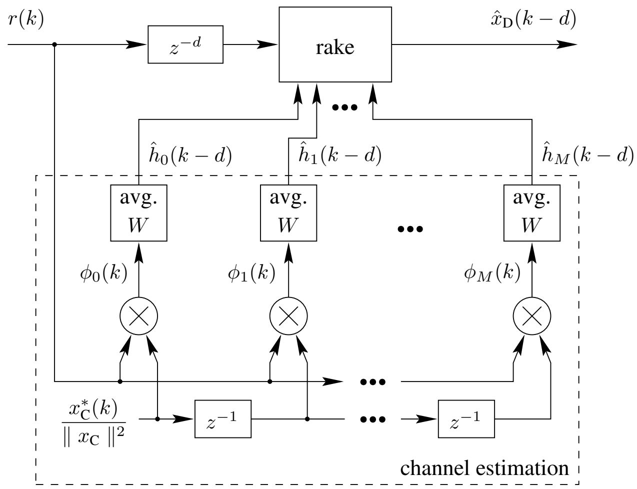

Fig. 2. The CDMA receiver under study.

trol transmitted sequence $x_{\mathrm{C}}(k)$ is available to the receiver. If $x_{\mathrm{C}}(k)$ is white, the exponentially weighted least–squares (LS) estimate of the i–th channel impulse response tap can be simplified as

$$\hat{h}_i = \frac{1-\lambda}{1-\lambda^k} \sum_{n=1}^{k} \lambda^{k-n} \phi_i(k) \tag{2}$$

$$\phi_i(k) = \frac{x_{\mathrm{C}}^*(k-i)}{\| x_{\mathrm{C}} \|^2}\, r(k) \tag{3}$$

where λ is the exponential factor. In recursive form

$$\hat{h}_i(k) = (1-Q(k))\hat{h}_i(k-1) + Q(k)\phi_i(k) \tag{4}$$

where $Q(k) = \frac{Q(k-1)}{Q(k-1)+\lambda}$ and $Q(1) = 1$.

3. ESTIMATION NOISE

By substituting r from eq. (1) into eq. (3) we obtain

$$\phi_i(k) = h_i + v_i(k) \tag{5}$$

$$v_i(k) = u_i(k) + w_i(k) \tag{6}$$

where

$$u_i(k) = \frac{1}{\| x_{\mathrm{C}} \|^2} \left[\sum_{j=0, j\neq i}^{N} x_{\mathrm{C}}^*(k-i) x_{\mathrm{C}}(k-j) h_j + \sum_{j=0}^{N} x_{\mathrm{C}}^*(k-i) x_{\mathrm{D}}(k-j) h_j \right] \tag{7}$$

$$w_i(k) = \frac{x_{\mathrm{C}}^*(k-i)}{\| x_{\mathrm{C}} \|^2} n(k) \tag{8}$$

Specifically, h_i is the desired estimate target, $u_i(k)$ is the inter–chip interference (ICI), while $w_i(k)$ is the N+MAI term.

For a large number of channel taps, $u_i(k)$ converges to a Gaussian limit. If the transmitted sequences are i.i.d., then $u_i(k)$ is zero–mean with a variance

$$\sigma_{u_i}^2 = \frac{\beta_{\mathrm{D}}^2 + \beta_{\mathrm{C}}^2}{\beta_{\mathrm{C}}^2} \| \mathbf{h} \|^2 - h_i^2 \tag{9}$$

It can further be shown that $w_i(k)$ is a zero–mean complex Gaussian i.i.d. sequence with variance $\sigma_{w_i}^2 = \frac{\sigma_{\mathrm{N}}^2}{\beta_{\mathrm{C}}^2}$. Note that $u_i(k)$ and $w_i(k)$ are uncorrelated; therefore $v_i(k)$ is zero–mean Gaussian with variance

$$\sigma_{v_i}^2 = \frac{\| \mathbf{h} \|^2 (\beta_{\mathrm{D}}^2 + \beta_{\mathrm{C}}^2) + \sigma_{\mathrm{N}}^2}{\beta_{\mathrm{C}}^2} - h_i^2 \tag{10}$$

Now, from eq. (2) and eq. (6) we obtain

$$\hat{h}_i(k) = h_i + z_i(k) \tag{11}$$

$$z_i(k) = \frac{1-\lambda}{1-\lambda^k} \sum_{n=1}^{k} \lambda^{k-n} v_i(k) \tag{12}$$

where $z_i(k)$ is the zero–mean *estimation noise* due to channel noise, MAI and ICI, with variance

$$\boldsymbol{E}\left\{ z_i^2(k) \right\} = \left(\frac{1-\lambda}{1+\lambda} \right) \left(\frac{1+\lambda^k}{1-\lambda^k} \right) \sigma_{v_i}^2 \tag{13}$$

When $k \to \infty$ we obtain

$$\sigma_{z_i}^2 = \lim_{k\to\infty} \boldsymbol{E}\left\{ z_i^2(k) \right\} = \left(\frac{1-\lambda}{1+\lambda} \right) \sigma_{v_i}^2 \tag{14}$$

If we employ a rectangular window of size W, then eq. (12) and eq. (14) become

$$z_i(k) = \frac{1}{W} \sum_{n=0}^{W-1} v_i(k-n) \tag{15}$$

$$\sigma_{z_i}^2 = \frac{1}{W} \sigma_{v_i}^2 \tag{16}$$

We can now establish a relationship between the exponential λ and the rectangular window W, based on same variances for z_i from eq. (14) and eq. (16)

$$W = \frac{1+\lambda}{1-\lambda} \iff \lambda = \frac{W-1}{W+1} \tag{17}$$

Hereinafter, we assume that λ is calculated according to eq. (17) for a given W. Therefore, for both exponential and rectangular windows, the variance of the tap error is

$$\sigma_{z_i}^2 = \frac{\| \mathbf{h} \|^2 (\beta_{\mathrm{D}}^2 + \beta_{\mathrm{C}}^2) + \sigma_{\mathrm{N}}^2 - \beta_{\mathrm{C}}^2 h_i^2}{\beta_{\mathrm{C}}^2\, W} \tag{18}$$

Since $z_i(k)$ is uncorrelated to $z_j(k)$ for $i \neq j$, the mean–squared error of the channel error vector $\mathbf{z} = \hat{\mathbf{h}} - \mathbf{h}$ due to noise is

$$\mathrm{MSE}_{\mathrm{N}} = \frac{M\, \sigma_{\mathrm{N}}^2 + \left[M\beta_{\mathrm{D}}^2 + (M-1)\beta_{\mathrm{C}}^2 \right] \| \mathbf{h} \|^2}{\beta_{\mathrm{C}}^2\, W} \tag{19}$$

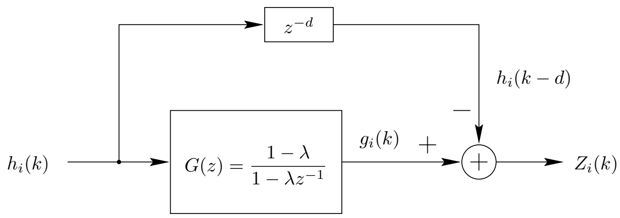

Fig. 3. Error vector due to distortion and lag.

4. ESTIMATION LAG AND DISTORTION

In order to explore the effects of the lag and distortion introduced by the channel estimator in a non–stationary channel environment, we consider the scheme of fig. 3.

The true i–th tap of the non–stationary channel at time k (denoted as $h_i(k)$) is passed through the channel estimation filter (as described in eq. (4) for $k \to \infty$) with a transfer function $G(z)$. The proposed rake receiver operates with a processing lag of d, that is, it must be presented with the tap weight $h_i(k-d)$. Instead, the channel estimation algorithm produces an estimate $g_i(k)$, which is a distorted version of $h_i(k-d)$. The estimation error is then

$$Z_i(k) = (1-\lambda)\sum_{n=0}^{k} \lambda^{k-n} h_i(n) - h_i(k-d) \quad (20)$$

If $\rho_i(\tau)$ is the autocorrelation function of $h_i(k)$, the variance of Z_i is

$$\begin{aligned} \boldsymbol{E}\left\{Z_i(k)^2\right\} &= \boldsymbol{E}\left\{h_i(k-d)^2\right\} + \boldsymbol{E}\left\{g_i(k)^2\right\} - \\ &\quad \boldsymbol{E}\left\{h_i(k-d)g_i^*(k)\right\} - \\ &\quad \boldsymbol{E}\left\{h_i^*(k-d)g_i(k)\right\} \end{aligned} \quad (21)$$

The first term in the right hand side of eq. (21) is

$$\boldsymbol{E}\left\{h_i(k-d)^2\right\} = \rho_i(0) \quad (22)$$

Elaborating for the second term, we obtain

$$\begin{aligned} \boldsymbol{E}\left\{g_i(k)^2\right\} &= (1-\lambda)^2 \boldsymbol{E}\left\{\left(\sum_{n=0}^{k} \lambda^{k-n} h_i(n)\right)^2\right\} \\ &= (1-\lambda)^2 \sum_{n=0}^{k} \lambda^n \sum_{m=1}^{k} \lambda^m \rho_i(n-m) \end{aligned} \quad (23)$$

For the third term in the right hand side of eq. (21) we obtain

$$\boldsymbol{E}\left\{h_i(k-d)g_i^*(k)\right\} = (1-\lambda)\sum_{n=0}^{k} \lambda^n \rho_i(n-d) \quad (24)$$

Accordingly, for the fourth term in the right hand side of eq. (21) we have

$$\boldsymbol{E}\left\{h_i^*(k-d)g_i(k)\right\} = (1-\lambda)\sum_{n=0}^{k} \lambda^n \rho_i(d-n) \quad (25)$$

Substituting the four terms from eq. (22), eq. (23), eq. (24) and eq. (25) into eq. (21) we obtain

$$\begin{aligned} \boldsymbol{E}\left\{Z_i(k)^2\right\} &= \rho_i(0) + \\ &\quad (1-\lambda)^2 \sum_{n=0}^{k} \lambda^n \sum_{m=0}^{k} \lambda^m \rho_i(n-m) - \\ &\quad (1-\lambda)\sum_{n=0}^{k} \lambda^n (\rho_i(n-d) + \rho_i(d-n)) \end{aligned}$$

Summarising, the mean–squared deviation of the error vector $\mathbf{Z}$ due to the filtering distortion will be (assuming Z_i and Z_j are independent for $i \neq j$).

$$\begin{aligned} \mathrm{MSE_D} &= \sum_{i=0}^{M} \lim_{k\to\infty} \left\{\boldsymbol{E}\left\{Z_i(k)^2\right\}\right\} = \sum_{i=0}^{M} \rho_i(0) + \\ &\sum_{i=0}^{M} \lim_{k\to\infty} \left\{(1-\lambda)^2 \sum_{n=0}^{k} \lambda^n \sum_{m=0}^{k} \lambda^m \rho_i(n-m) - \right. \\ &\left. (1-\lambda)\sum_{n=0}^{k} \lambda^n (\rho_i(n-d) + \rho_i(d-n)) \right\} \end{aligned} \quad (26)$$

5. COMBINED EFFECTS OF NOISE, LAG AND DISTORTION

Since the channel estimate noise is uncorrelated to the distortion due to the filtering, we can write the combined MSE of the channel estimation error as

$$\mathrm{MSE_{total}}(\hat{\mathbf{h}} - \mathbf{h}) = \mathrm{MSE_N}(\hat{\mathbf{h}} - \mathbf{h}) + \mathrm{MSE_D}(\hat{\mathbf{h}} - \mathbf{h}) \quad (27)$$

6. EXPERIMENTAL RESULTS

For the following experiments, a *classic* channel Doppler profile was employed, with psd

$$S_{\mu\mu}(f) = \begin{cases} \dfrac{2\sigma_0^2}{\pi f_{\max}\sqrt{1-(f/f_{\max})^2}} & , \quad |f| \le f_{\max} \\ 0 & , \quad |f| > f_{\max} \end{cases} \quad (28)$$

and auto–correlation function $\rho(t) = 2\ \sigma_0^2\ J_0(2\pi\ f_{\max}\ t)$, where $f_{\max}$ is the maximum Doppler frequency, and $J_0(x)$ is the zero–th order Bessel function of the first kind. The amplitude coefficients $\beta_D = 0.6022$ and $\beta_C = 0.7984$, the data spreading factor was 64, the control spreading factor 256, the Signal–to–Noise–plus–Interference Ratio after despreading was SNIR $=$ 6dB, and the transmission channel was a simulated vehicular test environment with high antenna characterised by 6 multipaths with relative delays 0ns, 310ns, 710ns, 1090ns, 1730ns and 2510ns, and average power 0dB, -1dB, -9dB, -10dB, -15dB and -20dB. The chip–rate was 3.84 Mcps. Finally channelisation and long scrambling codes were used, according to [6].

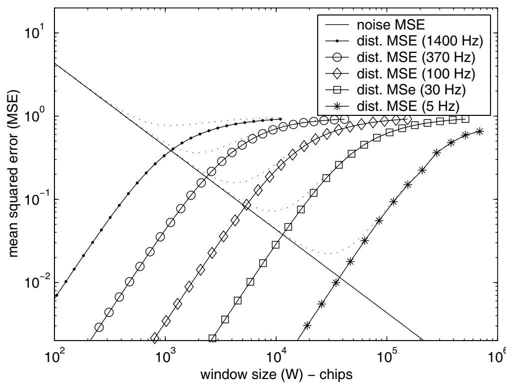

Fig. 4. Theoretical MSE due to noise and distortion. ($d = 0$)

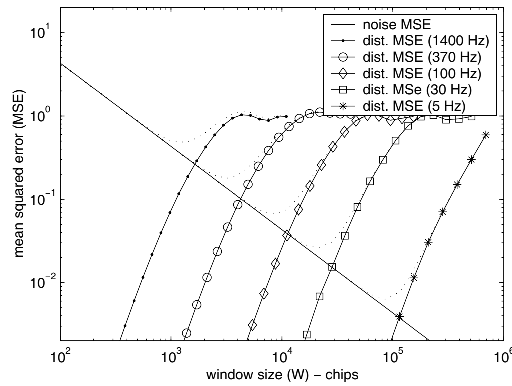

Fig. 5. Theoretical MSE due to noise and distortion. ($d = W/2$)

Figure 4 depicts the noise and distortion MSE of the channel estimate vector for five different Doppler frequencies and a processing delay $d = 0$. The combined effect of both estimation errors is also depicted with dotted curves. As expected from eq. (19), the noise term $\text{MSE}_\text{N}(\hat{\mathbf{h}} - \mathbf{h})$ is decreasing linearly in log–log scale with W and does not depend on the statistics of the channel. On the other hand, the distortion term $\text{MSE}_\text{D}(\hat{\mathbf{h}} - \mathbf{h})$ (as expected from eq. (26)) depends on the channel statistics and grows with W, since an increasing averaging window increases the distortion.

This suggests a trade–off optimisation, where a large averaging window is required to smooth out the effects of the noise, while a small window would track the non–stationary channel better. The optimisation points are obviously located at the minima of the combined MSE curves, and depend on the maximum Doppler frequency of the non–stationary channel.

Figure 5 presents the aforementioned experimental results with $d = W/2$. It is clear, that operating the rake receiver with a processing delay which is half the averaging window size, compensates for the inevitable lag the channel estimator introduces. According to the results, the performance benefit in the total estimation MSE can be as high as 5dB for the low Doppler frequencies.

7. CONCLUSIONS

In this paper, we propose a least squares channel estimator for wideband DS–CDMA rake receivers. Because of the white nature of chip sequences, LS estimation is reduced to a simple averaging scheme, with the averaging window size being the single tuning parameter. Then, we derived an analytical expression for the mean–squared estimation error of the proposed channel estimator. Through analytical inference and experimental results we demonstrated the grounds for a trade–off optimisation on the window–size parameter. Thus, by minimising the total MSE (as given in eq. (27)) with respect to the averaging window size we obtain a *minimum mean squared error* optimised LS channel estimator.

The minimisation of eq. (27) is possible only by means of iterative numerical methods, since the mean–squared distortion error (MSE_D in eq. (26)) is not analytically tracktable. Finally, an estimate for the maximum Doppler frequency of the non–stationary channel is required, if a classic Doppler psd profile is assumed.

8. REFERENCES

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