A JOINT CARRIER OFFSET AND CHANNEL ESTIMATION METHOD FOR SYNCHRONOUS CDMA SYSTEM

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ABSTRACT

This paper addresses the problem of blindly estimating the carrier offset and channel response in the uplink of multi-user CDMA systems having large variation in carrier offset. Two blind subspace-based estimation methods, namely Exact Determinant Minimization Method (EDMM) and Approximate Determinant Minimization Method (ADMM) are proposed, which exploit the orthogonality between the signal and noise subspaces. The performance of the proposed methods is compared with the available solution using Generalized Eigenvalue Problem Method (GEVPM). Simulation results are given to show that the proposed methods give better performance than the GEVPM and have a wider acquisition range for the carrier offset.

1. INTRODUCTION

DS-CDMA is one of the popular multiple-access techniques for many wireless applications. In the uplink scenario, the transmitted signal from each user propagates through a different multipath channel which distorts the signal. The multiuser detector is challenged with the problem of detecting the desired user's signal from the received signal. Another problem is that the received signal even after down-conversion has a residual carrier offset. Each user's carrier offset is independent of the others due to independent transmitters. The carrier offset of each user has to be corrected individually for proper detection. To improve the system performance the detector should have partial or complete knowledge of the channel and the carrier offset. Thus the estimation of the channel response and carrier offset has a significant effect on the performance of the multiuser detector. The very high computational complexity of the maximum likelihood (ML) method [1] for the joint estimation of the channel and carrier offset makes this method impractical though it gives the optimum estimates of the parameters. Li and Liu [2] proposed a subspace-based algorithm to jointly estimate the channel and carrier offset. This method decoupled the estimation of the channel and the carrier offset but the algorithm involved a complicated inverse polynomial matrix manipulation. Attallah and Fu [3] proposed another subspace-based algorithm which converted the joint channel and carrier offset estimation problem into a Generalized Eigenvalue problem, which has a similar performance, but lower computational complexity when compared to the algorithm in [2]. However, the performance of the algorithm in [3] degrades when the carrier offset is relatively large. In this paper, we propose two blind subspace-based methods, namely the Exact Determinant Minimization Method (EDMM) and the Approximate determinant Minimization Method (ADMM). Only the spreading codes of the users are assumed to be known at the receiver. These methods exploit the orthogonality between the signal and the noise subspaces. We will show through simulations that the proposed methods perform better than the GEVPM and provide room for the carrier offset to vary over a wide range.

2. SYSTEM MODEL

The received base-band discrete time signal for a Q-user quasi-synchronous CDMA system can be expressed as [3]

$$\mathbf{y}(i) = \sum_{k=1}^{Q} s_k(i) e^{j(iM+L-2)\phi_k} \mathbf{Z}_k \mathbf{C}_k \mathbf{h}_k + \mathbf{n}(i) \tag{1}$$

with,

$$\begin{aligned}
\mathbf{h}_k &= [h_k(0)\ h_k(1) \cdots h_k(L-1)]^T \\
\mathbf{Z}_k &= diag(\ e^{j\phi_k},\ e^{j2\phi_k}, \cdots,\ e^{jK\phi_k}) \\
\mathbf{C}_k &= \begin{bmatrix} c_k(L-1) & c_k(L-2) & \cdots & c_k(0) \\ c_k(L) & c_k(L-1) & \cdots & c_k(1) \\ \cdots & \cdots & \ddots & \cdots \\ c_k(M-1) & c_k(M-2) & \cdots & c_k(K-1) \end{bmatrix}
\end{aligned}$$

where $s_k(i),\ \phi_k,\ \{c_k(l)\}_{l=0}^{M-1}$ and $\mathbf{h}_k$ are the i^{th} data symbol, carrier offset, spreading code and the FIR channel vector of the k^{th} user respectively. The column vector $\mathbf{n}(i)$ is a white Gaussian noise vector with two-sided spectral density of $N_0/2$. The number of paths in the channel is L and the spreading gain is M. In a quasi-synchronous CDMA system we have $K = M-L+1$ ISI free samples where we assume $L \ll M$. $\mathbf{y}(i)$ is the $K \times 1$ vector representing the ISI free samples.

2.1. Cost Function:

The K samples over the N symbol duration are stacked into a $K \times N$ matrix as shown below.

$$\mathbf{Y} = [\mathbf{y}(0), \mathbf{y}(1), \cdots, \mathbf{y}(N-1)] = \mathbf{W} \times \mathbf{S} + \mathbf{B} \quad (2)$$

where $\mathbf{B} = [\mathbf{n}(0), \mathbf{n}(1), \cdots, \mathbf{n}(N-1)]$, $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_Q]$, $\mathbf{w}_k = \mathbf{Z}_k \mathbf{C}_k \mathbf{h}_k$, and

$$\mathbf{S} = \begin{bmatrix} s_1(0)e^{j\theta_1(0)} & \cdots & s_1(N-1)e^{j\theta_1(N-1)} \\ \vdots & & \vdots \\ s_Q(0)e^{j\theta_Q(0)} & \cdots & s_Q(N-1)e^{j\theta_Q(N-1)} \end{bmatrix} \quad (3)$$

where $\theta_k(i) = (iM+L-2)\phi_k$ for $i = 0, 1, \ldots, N-1$. Using the orthogonality between the signal and the noise subspaces it can be shown [2] that

$$\mathbf{U}_o^H \mathbf{Z}_k \mathbf{C}_k \mathbf{h}_k = \mathbf{0}, \quad k = 1, \cdots, Q \quad (4)$$

where $\mathbf{U}_o$ is the noise subspace of dimension $K \times (K-Q)$ where $(K-Q) > L$. In general, the equality in (4) is not valid in the presence of noise. As a result, (4) is replaced with the l_2-norm of the projection of $\mathbf{w}_k$ onto the noise subspace $\mathbf{U}_o$. The cost function for the joint estimation of the channel and the carrier offset is given by

$$J(\phi_k, \mathbf{h}_k) = \|\mathbf{U}_o^H \mathbf{Z}_k \mathbf{C}_k \mathbf{h}_k\|^2 \quad (5)$$

3. ESTIMATION METHODS

3.1. Generalized Eigenvalue Problem Method

The joint channel and carrier offset estimation problem is converted into a generalized eigenvalue problem by using the Taylor's series expansion for the complex exponential in the cost function [3]. It can be easily shown that

$$\mathbf{Z}_k = e^{j\frac{(K+1)\phi_k}{2}} \times \tilde{\mathbf{Z}}_k \quad (6)$$

where

$$\tilde{\mathbf{Z}}_k \; = \; diag(e^{-j\frac{(K-1)\phi_k}{2}}, e^{-j\frac{(K-3)\phi_k}{2}}, \cdots, e^{j\frac{(K-1)\phi_k}{2}})$$

Using Taylor's expansion series

$$\tilde{\mathbf{Z}}_k \; = \; \sum_{n=0}^{\infty} \frac{(j\phi_k)^n}{2^n n!} \tilde{\mathbf{D}}^n \quad (7)$$

where $\tilde{\mathbf{D}} = diag(-(K-1), -(K-3), \cdots, (K-1))$. To avoid the singularity of $\tilde{\mathbf{D}}$ matrix, K is chosen to be even. $\tilde{\mathbf{Z}}_k$ is approximated using Taylor series of order r (expansion with $r+1$ terms) given by $\tilde{\mathbf{Z}}_k \approx \sum_{n=0}^{r} \frac{(j\phi_k)^n}{2^n n!} \tilde{\mathbf{D}}^n$. This approximation is substituted into (5). Thus the cost function is expressed as

$$\begin{aligned} J(\phi_k, \mathbf{h}_k) \; &\approx \; \|\mathbf{U}_o^H \sum_{n=0}^{r} \frac{(j\phi_k)^n}{2^n n!} \tilde{\mathbf{D}}^n \mathbf{C}_k \mathbf{h}_k\|^2 \\ &= \; \sum_{n=0}^{r} \sum_{m=0}^{r} (-1)^m (j\phi_k)^{m+n} \mathbf{h}_k^H \mathbf{F}_{k,m}^H \mathbf{F}_{k,n} \mathbf{h}_k \end{aligned} \quad (8)$$

where $\mathbf{F}_{k,n} = \frac{\mathbf{U}_o^H \tilde{\mathbf{D}}^n \mathbf{C}_k}{2^n n!}$. Let $\mathbf{E}_{k,l}$ be defined as

$$\mathbf{E}_{k,l} = (j)^l \sum_{m=0}^{l} (-1)^m \mathbf{F}_{k,m}^H \mathbf{F}_{k,l-m} \quad (9)$$

Substituting (9) into (8), the cost function can be written as

$$J(\phi_k, \mathbf{h}_k) \approx \sum_{l=0}^{2r} \phi_k^l \mathbf{h}_k^H \mathbf{E}_{k,l} \mathbf{h}_k \quad (10)$$

The channel and carrier offset are estimated by minimizing the cost function in (10) w.r.t $\mathbf{h}_k$ and ϕ_k. To do this, the partial derivative of the cost function w.r.t $\mathbf{h}_k$ is equated to zero i.e $\frac{\partial J(\phi_k, \mathbf{h}_k)}{\partial \mathbf{h}_k^*} = \mathbf{0}$. With this condition it can be easily shown that

$$\left[\mathbf{E}_{k,0}\mathbf{h}_k + \phi_k \mathbf{E}_{k,1}\mathbf{h}_k + \cdots + \phi_k^{2r} \mathbf{E}_{k,2r}\mathbf{h}_k\right] = \mathbf{0} \quad (11)$$

The problem is to find ϕ_k and $\mathbf{h}_k$ such that (11) is satisfied. This problem can be converted into a Generalized Eigenvalue Problem that can be easily solved [4]. Towards this end, a set of $2r+1$ column vectors of dimension $(L \times 1)$ are defined as

$$\begin{aligned} \mathbf{x}_0 \; &= \; \mathbf{h}_k \\ \mathbf{x}_i \; &= \; \phi_k \mathbf{x}_{i-1} \quad for \;\; i = 1, 2, \cdots 2r \end{aligned} \quad (12)$$

Substituting (12) into (11), we get

$$[\mathbf{E}_{k,0}\mathbf{x}_0 + \mathbf{E}_{k,1}\mathbf{x}_1 + \cdots + \mathbf{E}_{k,2r}\mathbf{x}_{2r}] \; = \; \mathbf{0} \quad (13)$$

Given that $\mathbf{E}_{k,2r}$ is non-singular, we can form the $2rL \times 2rL$ eigensystem given below.

$$\begin{bmatrix} \mathbf{0}_{L\times L} & \mathbf{I}_{L\times L} & \cdots & \mathbf{0}_{L\times L} \\ \mathbf{0}_{L\times L} & \mathbf{0}_{L\times L} & \cdots & \mathbf{0}_{L\times L} \\ \vdots & \vdots & \vdots & \ddots \\ \mathbf{0}_{L\times L} & \mathbf{0}_{L\times L} & \cdots & \mathbf{I}_{L\times L} \\ -\mathbf{E}_{k,2r}^{-1}\mathbf{E}_{k,0} & -\mathbf{E}_{k,2r}^{-1}\mathbf{E}_{k,1} & \cdots & -\mathbf{E}_{k,2r}^{-1}\mathbf{E}_{k,2r-1} \end{bmatrix} \times \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{2r-2} \\ \mathbf{x}_{2r-1} \end{bmatrix} = \phi_k \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{2r-2} \\ \mathbf{x}_{2r-1} \end{bmatrix} \quad (14)$$

Eigenvalue decomposition is carried out for the sparse matrix in (14). Unit norm is imposed on the eigenvectors. The minimum eigenvalue is the estimated carrier offset. The channel vector $\mathbf{h}_k$ is obtained by truncating the eigenvector corresponding to the minimum eigenvalue, i.e taking the top L elements from the column vector $[\mathbf{h}_k^T, \mathbf{x}_1^T, \cdots, \mathbf{x}_{2r-1}^T]^T$. The norm of the estimated channel, for the k^{th} user, is normalized to one. Thus in GEVPM both the channel and the carrier offset are simultaneously estimated.

3.2. Exact Determinant Minimization Method

The orthogonality between the noise and signal subspaces leads to

$$\mathbf{A}_k\mathbf{h}_k = \mathbf{0}, \quad for \;\; k = 1, \cdots, Q \tag{15}$$

where

$$\mathbf{A}_k = \mathbf{U}_o^H \mathbf{Z}_k \mathbf{C}_k \tag{16}$$

is a $((K-Q) \times L)$ dimensional matrix and $(K-Q) > L$. From (15) it is clear that $\mathbf{h}_k$ lies in the null space of the matrix $\mathbf{A}_k$.

$$\mathbf{h}_k \in null(\mathbf{A}_k) \tag{17}$$

where $null(\mathbf{A}_k)$ denotes the null space of the matrix $\mathbf{A}_k$. Since $\mathbf{A}_k\mathbf{h}_k = \mathbf{0}$, it is obvious that $\mathbf{A}_k^H\mathbf{A}_k\mathbf{h}_k = \mathbf{0}$ for $k = 1, \cdots, Q$. It is easy to solve for $\mathbf{h}_k$ from the equation $\mathbf{A}_k^H\mathbf{A}_k\mathbf{h}_k = \mathbf{0}$ since $\mathbf{A}_k^H\mathbf{A}_k$ is a $(L \times L)$ dimensional Hermitian matrix and hence its determinant is always real and greater than or equal to zero. The equations $\mathbf{A}_k\mathbf{h}_k = \mathbf{0}$ and $\mathbf{A}_k^H\mathbf{A}_k\mathbf{h}_k = \mathbf{0}$ have the same solution for $\mathbf{h}_k$ [5]. Hence, $\mathbf{h}_k$ can be obtained from the null space of $\mathbf{A}_k^H\mathbf{A}_k$. For $\mathbf{h}_k$ to have a non-trivial solution $\mathbf{h}_k \neq \mathbf{0}$, $\mathbf{A}_k^H\mathbf{A}_k$ should be a singular matrix, that is

$$det(\mathbf{A}_k^H\mathbf{A}_k) = 0 \tag{18}$$

Since the matrix $\mathbf{A}_k$ is a function of the carrier offset ϕ_k, a linear search for ϕ_k in the range $[-\pi, \pi)$ is carried out. The value of ϕ_k which minimizes the determinant of the matrix $\mathbf{A}_k^H\mathbf{A}_k$ is taken as the estimated carrier offset. The cost function after the estimation of ϕ_k is given as

$$J(\mathbf{h}_k) \quad = \quad \|\mathbf{A}_k\mathbf{h}_k\|^2 = \mathbf{h}_k^H\mathbf{A}_k^H\mathbf{A}_k\mathbf{h}_k \tag{19}$$

Thus to minimize the above cost function the channel $\mathbf{h}_k$ is estimated as the eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{A}_k^H\mathbf{A}_k$. Since the EDM method uses no approximations and has a full search, it gives the best performance as we shall see in the simulation results.

3.3. Approximate Determinant Minimization Method

This method is derived from the observation of (11), that is

$$\left[\mathbf{E}_{k,0} + \phi_k\mathbf{E}_{k,1} + \phi_k^2\mathbf{E}_{k,2} + \cdots + \phi_k^{2r}\mathbf{E}_{k,2r}\right]\mathbf{h}_k \quad = \quad \mathbf{0}$$

Let $\Gamma_k = \mathbf{E}_{k,0} + \phi_k\mathbf{E}_{k,1} + \phi_k^2\mathbf{E}_{k,2} + \cdots + \phi_k^{2r}\mathbf{E}_{k,2r}$. Besides the trivial solution $\mathbf{h}_k = \mathbf{0}$, (11) will have a non-zero solution for $\mathbf{h}_k$ if the matrix Γ_k is singular, that is $det(\Gamma_k) = 0$. Matrix Γ_k is a $L \times L$ Hermitian matrix and hence its determinant is always real and greater than or equal to zero. The algorithm for the ADMM is same as that of EDMM except that the matrix $\mathbf{A}_k^H\mathbf{A}_k$ is replaced with the approximate matrix Γ_k using the Taylor's series. The linear search range for ϕ_k is determined by the maximum variation for the carrier offset. The cost function for EDMM after estimation of ϕ_k becomes $J(\mathbf{h}_k) \approx \mathbf{h}_k^H\Gamma_k\mathbf{h}_k$. The channel $\mathbf{h}_k$ is estimated as the eigenvector corresponding to the minimum eigenvalue of the matrix Γ_k.

4. MMSE DETECTOR

The estimated channel $\mathbf{h}_k$ and the carrier offset ϕ_k, for the k^{th} user, are used to construct a subspace-based MMSE multiuser detector [6]. Since the channel is estimated from the eigenvector of a matrix, the solution may not be unique, because any complex scalar multiple of the eigenvector is also a valid eigenvector. This problem is overcome by using differential encoding and decoding at the transmitter and the receiver respectively. The optimum weight vector used by the detector is given below [6].

$$\mathbf{q}_k \quad = \quad \frac{1}{\mathbf{w}_k^H\mathbf{U}_s\mathbf{\Lambda_s}^{-1}\mathbf{U}_s^H\mathbf{w}_k}\mathbf{U}_s\mathbf{\Lambda_s}^{-1}\mathbf{U}_s^H\mathbf{w}_k$$

$\mathbf{U}_s$ is the signal subspace and $\mathbf{\Lambda_s}$ is a diagonal matrix with the eigenvalues corresponding to the signal subspace as its diagonal elements. The output of the detector for the k^{th} user is given as $\hat{s}_k[i] = \mathbf{q}_k^H\mathbf{y}(i)$. The MMSE detector output is differentially decoded as $\hat{b}_k[i] = sign\{Re(\hat{s}_k[i]\hat{s}_k^*[i-1]e^{-jM\phi_k})\}$.

5. SIMULATION RESULTS

A synchronous CDMA system with 10 users using random spreading codes with a spreading gain of $M = 32$ is simulated. The subspace estimation window is of length $N = 100$, the carrier offset is a uniformly distributed random variable and the length of the Rayleigh fading channel is 3 for all users ($L = 3$). The modulation used is DBPSK. The intended users signal power is 10 dB less than the power of the interferers. For GEVPM and ADMM, Taylor's series of order 3 is used. The carrier offset ϕ_k is searched in steps of 0.001 for both EDMM and ADMM. The search range for EDMM is $[-\pi, \pi)$ and for ADMM the search range is limited by the maximum variation in the carrier offset. Fig.1 and Fig.2 show the performance of the estimation methods for $(-0.1 \leq \phi_k \leq 0.1)$ and $(-0.2 \leq \phi_k \leq 0.2)$ respectively. The performance of EDMM for $(-0.5 \leq \phi_k \leq 0.5)$ is shown in Fig.3. The ideal performance, i.e. the BER with perfect knowledge of the channel and carrier offset is used as a reference for comparing the performance of the estimation methods. For small carrier offsets the GEVPM, EDMM and ADMM have similar performances, this is evident from Fig.1. But as the carrier offset increases GEVPM has an error floor. Since the ADMM uses approximation, a higher order Taylor's series must be used to achieve better performance which is clearly evident from Fig.2. The EDMM has no approximation and hence its performance is close to the ideal performance irrespective of the carrier offset, this is clearly shown in Fig.3.

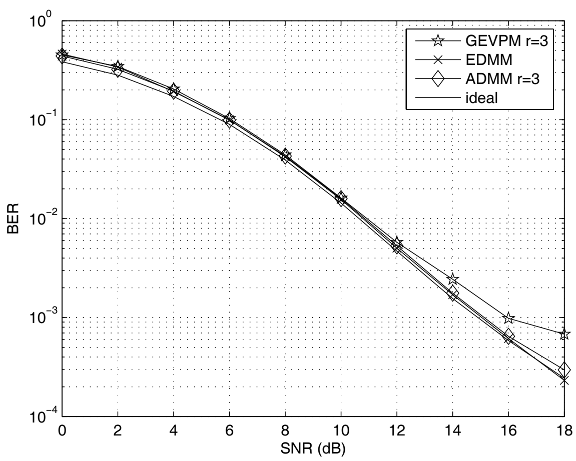

Fig. 1. BER vs SNR for $(-0.1 \leq \phi_k \leq 0.1)$

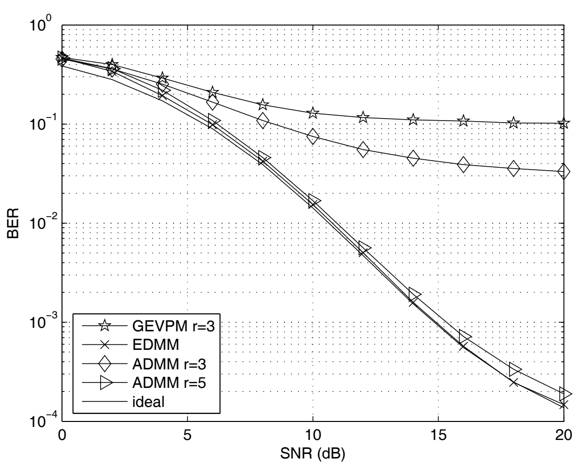

Fig. 2. BER vs SNR for $(-0.2 \leq \phi_k \leq 0.2)$

6. CONCLUSION

In this paper we have proposed two new subspace-based methods for joint estimation of channel response and carrier offset, namely Exact Determinant Minimization Method (EDMM) and Approximate Determinant Minimization Method (ADMM).

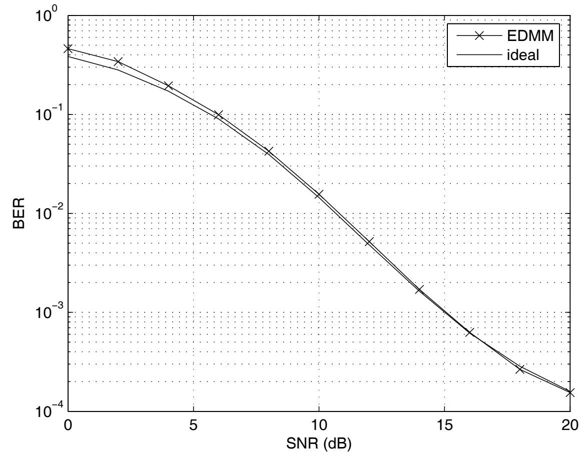

Fig. 3. BER vs SNR for $(-0.5 \leq \phi_k \leq 0.5)$

We have shown through simulations that in a synchronous uplink CDMA scenario, EDMM always gives better performance for large carrier offsets. Higher order Taylor's series must be used in ADMM to achieve better performance for large carrier offsets. We have also shown that EDMM and ADMM outperform GEVPM. Thus EDMM and ADMM are best suited for channel and carrier offset estimation in the uplink of synchronous CDMA systems.

7. REFERENCES

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