ANALYSIS OF MULTI-STAGE RECEIVERS UNDER FINITE SAMPLE-SUPPORT

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ABSTRACT

We derive the output SINR of a multi-stage CDMA receiver based on the Krylov-subspace spanned by the covariance matrix of the received signal, under the practical assumption of a finite number of samples available at the receiver. The evaluation of the SINR is addressed as an approximation problem in the asymptotic regime defined when both the sample-size and the observation dimension grow together without bound at the same rate. Limiting SINR values in this double-limit context are then more representative of the reality because, as it happens in realistic scenarios, both quantities are considered to be of the same order of magnitude. Our results are based on the asymptotic spectrum of the powers of certain random matrix models, which can be conviniently studied using the combinatorial approach to the theory of free probability.

1. INTRODUCTION

Many current mobile radio systems are supported on air-interfaces based on the code-division multiple access (CDMA) scheme, typically in its direct-sequence implementation. In the last two decades, a huge amount of publications has been dedicated to the theory of multiuser detection dealing with the reception of signals over these channels. In particular, much attention has been given to the linear minimum mean-square-error (MMSE) and decorrelator receivers, as practical alternatives to the highly-complex optimal (finite-alphabet-constrained) maximum likelihood (ML) detector. However, even though they have been proved to perform considerably more efficiently than the conventional single-user matched-filter (MF) receiver, their use tends to be avoided in realistic scenarios. First, the complexity associated with these detection schemes is still prohibitive in situations with a large number of users. On the other hand, the solution following the multiuser detection approach is based on the knowledge of the spreading sequence of all users as well as information about their channels and the background noise level. In some scenarios, like for example in the forward link, it is unrealistic and certainly impractical to consider tracking all this amount of information. In this situations, the detection task is better approached from a point of view of multiple access interference (MAI) suppression [1, 2]. The linear interference suppression filter minimizing the MSE is equivalent to the solution maximizing the output signal-to-interference-plus-noise-ratio (SINR), which in the literature is often regarded as the minimum-variance distorsionless-response receiver (MVDR). This receiver is implemented by simply inverting the covariance matrix of the received signal. In practice, the sample matrix inversion (SMI) algorithm relying on the sample covariance matrix is used.

In the case of MAI filtering operating in a high-dimensional code-space (e.g. long spreading codes), the weight vector dimensionality may be too large for the receiver to be effectively estimated. Apart from the computational complexity issue (mainly due to the inversion of a large covariance matrix), estimating the covariance matrix of high-dimensional signals would require a very large number of samples, which cannot be usually afforded due to the time-variability of the environment. To overcome this problem, reduced-rank filtering methods have been proposed that reduce the number of filter coefficients to be effectively estimated. Thus, under a finite sample-support situation, reduced-rank filters have better convergence performance than the full-rank solution.

The issue of reduced-rank filter design for small-sample-support adaptation was first recognized and discussed in [3], where the authors suggested the conditional optimization (one-by-one) of the reduced-rank filter taps for superior small-sample-support performance. In this paper, we take a first step to the design of interference suppresion schemes for CDMA systems by analyzing the large-system output SINR of a generic reduced-rank LMMSE receiver operating in a finite sample-size situation.

2. SIGNAL MODEL AND REDUCED-RANK LINEAR CDMA INTERFERENCE SUPPRESION

Consider a DS-CDMA system with K users and processing gain N. Let us define the signal model applying to the samples at the output of the receiver matched filter at instant n as

$$\mathbf{r}(n) = \mathbf{S}\mathbf{A}\mathbf{b}(n) + \mathbf{n}(n) = a_1 \mathbf{s}_1 b_1(n) + \mathbf{A}_{\sim 1}\mathbf{S}_{\sim 1}\mathbf{b}_{\sim 1}(n) + \mathbf{n}(n), \tag{1}$$

where $\mathbf{S} = [\mathbf{s}_1 \mid \mathbf{S}_{\sim 1}] \in \mathbb{C}^{N \times K}$ is a spreading matrix with its columns being the signature sequence of the different users, $\mathbf{b}(i) = \left[b_1(n) \mid \mathbf{b}_{\sim 1}(n)^T \right]^T \in \mathbb{C}^K$ is a vector whose entries correspond to the transmitted symbol of each user, and $\mathbf{n}(n)$ models the background noise at chip-sampling-rate. Different receive amplitudes are taken into consideration through the diagonal matrix $\mathbf{A}$, which models the atennuation effect due to the fading channel. In the sequel and without loss of generality, the user 1 is regarded as the desired user and $p_1 = |a_1|^2$ is its received power.

We focus here on reduced-rank methods that transform the received signal into a vector lying on a lower dimensional space

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without requiring eigendecomposition or matrix inversion. Specifically, let $\mathbf{S}_D \in \mathbb{C}^{N \times D}$ be a matrix whose columns form a basis for a D-dimensional subspace, where $D < N$. The transformed lower-dimensional received signal is then

$$\tilde{\mathbf{r}}(n) = \mathbf{S}_D^H \mathbf{r}(n). \tag{2}$$

The objective is now to obtain a filter $\tilde{\mathbf{w}}$ that minimizes the reduced-rank MSE, i.e.

$$\mathcal{M} = \mathbb{E}\left\{ \left| b_1(n) - \tilde{\mathbf{w}}^H \tilde{\mathbf{r}}(n) \right|^2 \right\}. \tag{3}$$

The solution is given by

$$\tilde{\mathbf{w}} = \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{s}}_1 = \left(\mathbf{S}_D^H \mathbf{R} \mathbf{S}_D \right)^{-1} \mathbf{S}_D^H \mathbf{s}_1, \tag{4}$$

where $\tilde{\mathbf{R}} = \mathbf{S}_D^H \mathbf{R} \mathbf{S}_D$ and $\tilde{\mathbf{s}}_1 = \mathbf{S}_D^H \mathbf{s}_1$. Finally, the reduced-rank D-dimensional approximation of the N-tap linear MMSE filter is

$$\mathbf{w} = \mathbf{S}_D \left(\mathbf{S}_D^H \mathbf{R} \mathbf{S}_D \right)^{-1} \mathbf{S}_D^H \mathbf{s}_1. \tag{5}$$

In the past years, a number of reduced-rank methods implementing the filter $\mathbf{w}$ have been proposed in the literature [4, 5]. A slightly different approach to the reduced-rank filtering problem was also addressed in [6], where the size of the filter-basis is allowed to grow to infinity if non-orthogonal vectors are considered. They all allow for a reduction of the computational complexity by approximating the optimal estimate in a number of stages or iterations $D << N$. It has been shown that reduced-rank linear filters based on the previously introduced transformation can achieve near-optimum (LMMSE) performance for relatively small values of D, independently of the scaling of N and K, which are assumed to be of the same order of magnitude. Furthermore, the receiver can be regarded as an approximation of the Wiener filter obtained by forcing the filter to lie on the D-dimensional Krylov subspace generated by the covariance matrix of the received signal and the matched filter of the desired user [7]. Hence, the linear interference suppresion filter can be obtained as

$$\mathbf{w} = \sum_{i=0}^{D-1} \omega_i \mathbf{R}^i \mathbf{s}_1. \tag{6}$$

If the true covariance matrix is available, it results from the Cayley-Hamilton theorem [8] that the optimal solution will only be achieved when the weighted sum of matrix powers perfectly matches the inverse of the covariance matrix (i.e. $D = N$), or, equivalently, when the reduced-rank approximation approaches the full-rank (optimal) Wiener solution. If an estimate of the true covariance matrix is used in the computations, as it usually happens to be the case in practical implementations, the optimality statement above is no longer true: the optimal filter in the sequence will generally be found after $D < N$ stages or iterations. In other words, if a sample covariance matrix is available as an estimate of the theoretical covariance matrix, one can do better than directly computing the minimum variance receiver (or individual LMMSE solution) by just searching for an optimal filter in a D-dimensional ($D < N$) space as defined above.

In the following, the output SINR of a reduced-rank linear interference suppression receiver considering a collection of a finite number of samples is analyzed. For our study, the set of received signals after chip-matched filtering is modeled according to (1) as

$$\mathbf{X} = a_1 \mathbf{s}_1 \mathbf{b}_1^H + \mathbf{R}_N^{1/2} \mathbf{Y}, \tag{7}$$

where the vector $\mathbf{b}_1$ contains the symbols of the target user collected through N samples and $\mathbf{R}_N$ is the interference-plus-noise covariance matrix. Throughout the paper, the following statistical assumptions regarding the previous signal model are used:

($\mathbf{As1}$) The entries of the vector $\sqrt{N}\mathbf{b}_1$ are i.i.d. circularly symmetric random variables with zero mean, variance 1 and bounded forth-order moment, and symbols transmitted by different users are mutually independent.

($\mathbf{As2}$) The matrix $\mathbf{Y}$ is an $M \times N$ random matrix whose entries are modeled as i.i.d. circularly symmetric Gaussian random variables with zero mean and variance $1/N$.

Note that, as symbols from different users and samples are assumed to be statistically independent, the matrix $\mathbf{R}_N^{1/2}\mathbf{Y}$ can be considered to model statistically independent samples of the interference-plus-noise signal contribution. We are now interested in approximating the output SINR by an equivalent limiting expression involving only the true covariance matrix. The rationale behind this is that certain spectral functions of the covariance matrix can be estimated consistently in the double-limit context defined when both the number of observations and the dimension of the sample are of the same order of magnitude and grow without bound. Since such asymptotic approximations turn out to reliably characterize practical situations, this approach may allow us to take into account the fact that only a finite collection of samples should be considered to be available when addressing the design of practical receiver schemes.

3. ASYMPTOTIC SINR OF MULTI-STAGE RECEIVERS BASED ON KRYLOV SUBSPACES

Most of the work published over the last years on the asymptotic weighting ([9] and references herein) and performance analysis ([7] and [10], where the literature and main results in this area are thoroughly reviewed) of reduced-rank multistage receivers based on the aforementioned Krylov subspace rely on the statistical knowledge of the actual structure of the true covariance matrix. In this section we derive the ultimate performance of a reduced-rank multistage receiver in terms of its output SINR, as a function of quantities that are either known or can be properly approximated in practice (e.g. spectral functions of covariance matrices as regarded above). The results from our asymptotic analysis are derived under the assumption that both the sample-size N and the observation dimension M increase without bound with a fixed ratio between them ($M/N \to c$). In this framework, elements of random matrix theory and free probability are employed in the derivation of the analytical expressions. The rapid convergence associated with these techniques allows for the results to resemble those obtained for finite (non-asymptotic) matrix dimensions. Thus, these results are then more representative of the reality because, as it turns out to be the case in realistic scenarios, both quantities M and N are considered to be of the same order of magnitude In our analysis, the convergence of the output SINR will be derived in the random signature framework, i.e.

($\mathbf{As3}$) The chips in the code sequence vector $\mathbf{s}_1$ are assumed i.i.d. circularly symmetric random variables with zero mean and variance $1/M$. They are also independent of the received noise and transmitted symbols.

($\mathbf{As4}$) The matrix $\mathbf{R}_N$ has uniformly bounded spectral radius for all M and an empirical eigenvalue distribution $H^M(\lambda)$ converging to a limitin (non-random) p.d.f. $H(\lambda)$.

Furthermore, let denote by $m_{\mathbf{C}}^k$ the kth eigenvalue moment of a $M \times M$ matrix $\mathbf{C}$ wrt. its empirical eigenvalue distribution function $F^M(\lambda)$, defined as

$$m_{\mathbf{C}}^k = \int \lambda^k dF^M(\lambda) = \mathrm{Tr}\left[\mathbf{C}^k\right], \tag{8}$$

where $\mathrm{Tr}[\cdot] = \frac{1}{M}\,\mathrm{trace}[\cdot]$. For the subsequent analysis, the sample covariance matrix $\hat{\mathbf{R}}$ will be modeled according to (7) as

$$\hat{\mathbf{R}} = \mathbf{X}\mathbf{X}^H. \tag{9}$$

The output SINR is now defined in terms of the sample covariance matrix available at the receiver as

$$SINR\left(\hat{\mathbf{R}}\right) = \frac{\left|\hat{\mathbf{w}}^H \mathbf{s}_1\right|^2}{\hat{\mathbf{w}}^H \mathbf{R}_N \hat{\mathbf{w}}} = \frac{p_1 \left|\sum_{i=0}^{D-1} \omega_i \mathbf{s}_1^H \hat{\mathbf{R}}^i \mathbf{s}_1\right|^2}{\sum_{i=0}^{D-1}\sum_{j=0}^{D-1} \omega_i \omega_j \mathbf{s}_1^H \hat{\mathbf{R}}^i \mathbf{R}_N \hat{\mathbf{R}}^j \mathbf{s}_1}. \tag{10}$$

The following claim gives a recurrence relation that allows us to iteratively obtain the limiting values above as a function of the sample interference-plus-noise covariance matrix, modeled as $\hat{\mathbf{R}}_N = \mathbf{R}_N^{1/2}\mathbf{Y}\mathbf{Y}^H\mathbf{R}_N^{1/2}$.

Proposition 1 *Under the previous statistical assumptions and as both the number of samples M and the observation dimension N increase without bound with a fixed ratio c between them, the scalars $\mathbf{s}_1^H \hat{\mathbf{R}}^i \mathbf{s}_1$ and $\mathbf{s}_1^H \hat{\mathbf{R}}^i \mathbf{R} \hat{\mathbf{R}}^j \mathbf{s}_1$ converge with probability 1 as*

$$\mathbf{s}_1^H \hat{\mathbf{R}}^i \mathbf{s}_1 \asymp \sum_{k=0}^{i} \bar{\gamma}_{i-k}\, m_{\hat{\mathbf{R}}_N}^k \tag{11}$$

$$\begin{aligned}\mathbf{s}_1^H \hat{\mathbf{R}}^i \mathbf{R}_N \hat{\mathbf{R}}^j \mathbf{s}_1 \asymp{} & \sum_{k=0}^{i}\sum_{l=0}^{j} \bar{\gamma}_{i-k}\bar{\gamma}_{j-l}\, \mathrm{Tr}\left[\mathbf{R}_N \hat{\mathbf{R}}_N^{k+l}\right] + \\ & + \sum_{m=0}^{i-1}\sum_{n=0}^{j-1} \bar{\alpha}_{i-m}\bar{\alpha}_{j-n}\, \mathrm{Tr}\left[\mathbf{R}_N \hat{\mathbf{R}}_N^{m+n+1}\right],\end{aligned} \tag{12}$$

where $a \asymp b \iff |a \asymp b| \overset{wp1}{\to} 0$ and the asymptotic coefficients $\bar{\gamma}_p$, $\bar{\alpha}_p$ are defined as

$$\bar{\gamma}_{p+1} \asymp |a_1|^2 \sum_{i=0}^{p} \bar{\gamma}_{p-i} m_{\hat{\mathbf{R}}_N}^i + a_1^* \sum_{j=0}^{p-1} \bar{\alpha}_{p-j} m_{\hat{\mathbf{R}}_N}^{j+1} \tag{13}$$

$$\bar{\alpha}_{p+1} \asymp a_1 \sum_{i=0}^{p} \bar{\gamma}_{p-i} m_{\hat{\mathbf{R}}_N}^i \tag{14}$$

with $\bar{\gamma}_0 = 1$.

Proof. The proof is based on an iterative expansion of $\mathbf{s}_1^H \hat{\mathbf{R}}^i = \mathbf{s}_1^H \left(\mathbf{X}\mathbf{X}^H\right)^i$ to avoid applying the multinomial formula to the powers of $\mathbf{X}\mathbf{X}^H$. See further [11]. ■

Note that in obtaining the asymptotic expression of terms with index $D-1$, all other terms in (10) involving lower powers are also found. We shall stress the interest in computing the asymptotic output SINR in terms of true covariance matrices to reveal the effect of dealing with a finite observation-window. The rest of the paper is dedicated to obtaining the different moments involving $\hat{\mathbf{R}}_N$ in Proposition 1 as a function of the spectrum of $\mathbf{R}_N$. We next introduce a result on the asymptotic moments of sample correlation matrices that allows us to compute the asymptotic equivalents of the moments in (11), (13) and (14).

Proposition 2 *(Asymptotic moments of a sample covariance matrix with outer correlations). As both the number of samples M and the observation dimension N increase without bound with $M/N \to c$ between them, the asymptotic kth eigenvalue moment of the sample covariance matrix $\hat{\mathbf{R}}_N$ can be obtained as*

$$m_{\hat{\mathbf{R}}_N}^k = \frac{1}{n+1}\sum_{k=1}^{n}\binom{n+1}{k} c^k \sum_{\substack{i_1+\cdots+i_k=n \\ i_1,\ldots,i_k\in\{1,\ldots,n\}}} m_{\mathbf{R}_N}^{i_1}\cdots m_{\mathbf{R}_N}^{i_k}. \tag{15}$$

Proof. The proof is based on the application of the Lagrange inversion formula to the Stieltjes transform of a sample covariance matrix of the type of $\hat{\mathbf{R}}_N$, which is expressed as a function of $\mathbf{R}_N$. See [11] for a proof. ■

In order to obtain the asymptotic moments in (12), we rely on an alternative interpretation of (15), which builds upon the conection between the theory of lattices of non-crossing partitions (introduced by Kreweras in [12]) and the combinatorial approach to the theory of free probability developed by Speicher (see e.g. [13]). Under this framework, a relation is found between the asymptotic moments of non-commutative random variables (as defined in (8) when $M \to \infty$) and the coefficients of the R-transform in free probability (i.e. the free cumulants of $H(\lambda)$). In order to prove our previous result with elements of this theory, it is first helpful to reformulate the moment calculation problem as

$$\mathrm{Tr}\left[\hat{\mathbf{R}}_N^n\right] = \mathrm{Tr}\left[\left(\mathbf{R}_N^{1/2}\mathbf{Y}\mathbf{Y}^H\mathbf{R}_N^{1/2}\right)^n\right] = \mathrm{Tr}\left[\left(\mathbf{Y}^H\mathbf{R}_N\mathbf{Y}\right)^n\right],$$

where $\mathbf{Y}^H\mathbf{R}_N\mathbf{Y}$ is identified as a non-commutative random variable, whose asymptotic moments are to be obtained. Then, an expression equivalent to (15) is obtained by directly invoking the moment-cumulant formula in [13] and using the fact that, when dealing with the powers of $\mathbf{Y}^H\mathbf{R}_N\mathbf{Y}$, an explicit computation of its free cumulants in terms of the limiting eigenvalue moments of $\mathbf{R}_N$ is possible. We skip further details for the sake of brevity and refer the reader to [11]. We remark that a similar approach was also followed in [10, 3.1.6]. The advantage of using this framework is twofold. First, a recursive formula to obtain $\mathbf{Y}^H\mathbf{R}_N\mathbf{Y}$ (and accordingly $\mathbf{R}_N^{1/2}\mathbf{Y}\mathbf{Y}^H\mathbf{R}_N^{1/2}$) may be readily found. Furthermore, the previous result can be easily extended to the case of expressions like

$$\mathrm{Tr}\left[\mathbf{Y}^H\mathbf{C}_1\mathbf{Y}\cdots\mathbf{Y}^H\mathbf{C}_n\mathbf{Y}\right], \tag{16}$$

where $\mathcal{C}(n) = \{\mathbf{C}_1,\ldots,\mathbf{C}_n\}$ is an arbitrary set of (possibly different) *non-free* random elements (e.g. polynomials in $\mathbf{R}_N$). The following result gives a recursive formula to obtain (16) for a set of matrices $\mathcal{R}(n) = \left\{\mathbf{R}_N^l,\ldots,\mathbf{R}_N\right\}$ in terms of the spectrum of $\mathbf{R}_N$ and the asymptotic moments of $\hat{\mathbf{R}}_N$.

Proposition 3 *Under the statistical assumptions considered above and as M, N grow without bound with a fixed ratio c, the normalized trace of the matrix $\mathbf{Y}^H \mathbf{R}_N^l \mathbf{Y} \left(\mathbf{Y}^H \mathbf{R}_N \mathbf{Y}\right)^{n-1}$ converge with probability 1 to*

$$\sum_{s=1}^{n} m_{\mathbf{R}_N}^{s+l-1} \sum_{\substack{i_1+\cdots+i_s=n-s \\ i_1,\ldots,i_s \in \{0,1,\ldots,n-s\}}} m_{\mathbf{Y}^H \mathbf{R}_N \mathbf{Y}}^{i_1} \cdots m_{\mathbf{Y}^H \mathbf{R}_N \mathbf{Y}}^{i_s}, \tag{17}$$

where $m_{\mathbf{Y}^H \mathbf{R}_N \mathbf{Y}}^{k} = c m_{\mathbf{R}_N^{1/2} \mathbf{Y} \mathbf{Y}^H \mathbf{R}_N^{1/2}}^{k}$.

Proof. See further [11]. ■.

Note that the limiting expression of the moments in (15) and (12) are obtained using $l = 1$ and $l = 2$, respectively.

Since $m_{\mathbf{R}_N}^k \to m_{\mathbf{R}}^k$ as M and N go to infinity with the ratio $M/N \to c$ kept fixed (see e.g. Appendix I in [9]), all asymptotic expressions derived above in terms of the limiting eigenvalue moments of the interference-plus-noise covariance matrix $\mathbf{R}_N$ may be equivalently regarded in terms of the covariance matrix of the received signal $\mathbf{R}$. This technicality allows us to operate formally, in the double-limit context at hand, in terms of a covariance matrix $\mathbf{R}$ that, unlike the matrix $\mathbf{R}_N$, can be estimated from the samples obtained at the receiver.

4. NUMERICAL VALIDATION

In this section, we validate the theoretical set-up derived in previous sections for the case of a multistage receiver of rank $D = 3$. Since only the accuracy of our asymptotic SINR evaluation is here of actual importance, all coefficients ω_i are set to 1 in the numerical simulations. We considered a scenario with $K = 30$ users that are received with a power 5 dB above the noise floor in a CDMA system with a spreading length of $N = 512$ chips. Concerning the observation-window, a ratio $c = 0.5$ is assumed. It is important to note that the case of a sample-size smaller than observation dimension (i.e. $c > 1$) could be analyzed using exactly the same tools. The output $SINR\left(\hat{\mathbf{R}}\right)$ converges almost surely to a limiting value $\overline{SINR}(\mathbf{R}_N)$, which is obtained by substituting each term in the (finite) sums in (10) by its asymptotic approximation. Note further that, as we pointed out in the previous section, we accordingly have $\overline{SINR}(\mathbf{R}_N) \to \overline{SINR}(\mathbf{R})$ in the double-limit context considered here. In Figure 1, the simulated output SINR and its limiting value as derived in previous sections are compared.

5. CONCLUSION

We derived an asymptotic approximation of the output SINR of a reduced-rank multi-stage receiver based on the Krylov-subspace defined by the covariance matrix and the signature of the desired user. We approach the problem by first obtaining an asymptotic expression of the output SINR as a function of information available at the receiver, regarding the desired user and certain spectral functions of the covariance matrix $\mathbf{R}$. The receiver design task would consist of finding the rank and coefficients maximizing the output SINR. The asymptotic evaluation presented here allows us to design these coefficients by evaluating the asymptotic output SINR expression upon replacing the functions of the (unknown) true covariance matrix by its (available) doubly-consistent estimators.

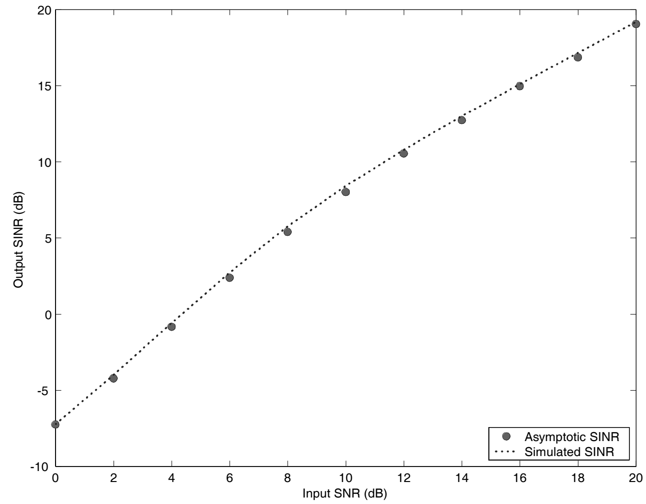

Fig. 1. Simulated and asymptotic output SINR.

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