DIRECT FIR LINEAR EQUALIZATION OF DOUBLY SELECTIVE CHANNELS BASED ON SUPERIMPOSED TRAINING

Jitendra K. Tugnait   *Shuangchi He*

Department of Electrical & Computer Engineering
Auburn University, Auburn, Alabama 36849, USA

ABSTRACT

Design of doubly-selective linear equalizers for single user frequency-selective time-varying communications channels is considered using superimposed training and without first estimating the underlying channel response. Both the time-varying channel as well as the linear equalizers are assumed to be described by a complex exponential basis expansion model (CE-BEM). A periodic (non-random) training sequence is arithmetically added (superimposed) at a low power to the information sequence at the transmitter before modulation and transmission. There is no loss in information rate. Knowledge of the superimposed training is exploited to design the FIR linear equalizer. An illustrative simulation example is presented.

1. INTRODUCTION

Consider a time-varying SIMO (single-input multiple-output) FIR (finite impulse response) linear channel with N outputs. Let $\{s(n)\}$ denote a scalar sequence which is input to the SIMO time-varying channel with discrete-time impulse response $\{\mathbf{h}(n;l)\}$ (N-vector channel response at time n to a unit input at time $n-l$). The vector channel may be the result of multiple receive antennas and/or oversampling at the receiver. Then the symbol-rate, channel output vector is given by

$$\mathbf{x}(n) := \sum_{l=0}^{L} \mathbf{h}(n;l)s(n-l). \tag{1}$$

In a complex exponential basis expansion representation it is assumed that

$$\mathbf{h}(n;l) = \sum_{q=-K/2}^{K/2} \mathbf{h}_q(l)e^{j\omega_q n} \tag{2}$$

where N-column vectors $\mathbf{h}_q(l)$ are time-invariant. Such models have been used in [1] and [2], among others; see Sec. 2 for details. Eqn. (2) is a basis expansion of $\mathbf{h}(n;l)$ in the time variable n onto complex exponentials with frequencies $\{\omega_q\}$. The noisy measurements of $\mathbf{x}(n)$ are given by ($n = 0, 1, \cdots, T-1$)

$$\mathbf{y}(n) = \mathbf{x}(n) + \mathbf{v}(n) \tag{3}$$

A main objective in communications is to recover $s(n)$ given noisy $\{\mathbf{x}(n)\}$. In several approaches this requires knowledge of the channel impulse response. Recently a superimposed training based approach has been explored to this end where one takes

$$s(n) = b(n) + c(n), \tag{4}$$

where $\{b(n)\}$ is the information sequence and $\{c(n)\}$ is a training (pilot) sequence added (superimposed) at a low power to the information sequence at the transmitter before modulation and transmission. There is no loss in information rate but some useful power is wasted in superimposed training which could have otherwise been allocated to the information sequence. Periodic superimposed training for channel estimation via first-order statistics for SISO systems have been discussed in [4], [7], and [8] for time-invariant channels, and in [6] for both time-invariant and time-varying (CE-BEM based) channels. Other works on time-varying channel estimation and data detection using superimposed training include [3] where the estimated channel is used to detect information symbols.

Given the knowledge of the time-varying channel described by CE-BEM, design of (serial) time-varying FIR equalizers has been discussed in [1]. Direct design of time-invariant FIR equalizers based on superimposed training, for time-invariant channels, has been investigated in [5]. In this paper we investigate direct design of time-varying FIR linear equalizers for doubly selective channels using superimposed training and without first estimating the underlying channel response. We exploit the prior results of [1] and [5].

Objectives and Contributions: The main problem considered here is: how to design an equalizer to estimate $\{b(n)\}$ when one knows only $\{c(n)\}$ but not (obviously) $\{b(n)\}$ and one does not also have (frame) synchronization with $\{c(n)\}$ at the receiver. We will design an equalizer to estimate $\{c(n)\}$ with a delay d. We will then show that this equalizer is a scaled version of the corresponding equalizer designed to estimate $\{b(n)\}$ with a delay d.

Notation: Superscripts H, $*$ and T denote the complex conjugate transpose, the complex conjugation and the transpose operations, respectively. $\delta(\tau)$ is the Kronecker delta and I_N is the $N \times N$ identity matrix.

2. DOUBLY SELECTIVE CHANNEL MODEL

Consider a time-varying channel with continuous-time, baseband received signal $x(t)$ and transmitted signal $s(t)$ (with symbol interval T_s sec.) related by impulse response $h(t;\tau)$ (response at time t to a unit impulse at time $t-\tau$). Let τ_d denote the (multipath) delay-spread of the channel and let f_d denote the Doppler spread of the channel. If $x(t)$ is sampled once every T_s sec. (symbol rate), then by [2] (see also [1]), for $t = nT_s + t_0 \in [t_0, t_0 + TT_s)$, the sampled signal $x(n) := x(t)|_{t=nT_s+t_0}$ has the representation

$$x(n) = \sum_{l=0}^{L} h(n;l)s(n-l) \tag{5}$$

where

$$h(n;l) = \sum_{q=-K/2}^{K/2} h_q(l)e^{j\omega_q n}, \quad L := \lfloor \tau_d/T_s \rfloor, \tag{6}$$

$$w_q = \frac{2\pi q}{T}, \quad K := 2\lceil f_d T T_s \rceil. \tag{7}$$

The above representation is valid over a duration of TT_s sec. (T samples). Eqn. (1) arises if we follow (5) and consider an SIMO model arising due to multiple antennas at the receiver.

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3. TIME-VARYING FIR EQUALIZERS

We will restrict ourselves to serial linear equalizers instead of block linear equalizers, since as shown in [1], the latter are computationally prohibitive (compared with the former). We look for a time-varying linear equalizer $\mathbf{g}(n;l)$ ($l = 0, 1, \cdots, L_e$) over the same time-block as the received data with channel model (2). We note that for an arbitrary time-varying impulse response $\tilde{\mathbf{g}}(n;l)$, the following is always true

$$\tilde{\mathbf{g}}(n;l) = \sum_{q=-(T-1)/2}^{(T-1)/2} \tilde{\mathbf{g}}_q(l) e^{j\omega_q n}, \quad n = 0, 1, \cdots, T-1. \quad (8)$$

We would like to use a more parsimonious (but approximate) representation for $\tilde{\mathbf{g}}(n;l)$, denoted by $\mathbf{g}(n;l)$, given by

$$\mathbf{g}(n;l) = \sum_{q=-Q/2}^{Q/2} \mathbf{g}_q(l) e^{j\omega_q n}, \quad n = 0, 1, \cdots, T-1, \quad (9)$$

where $Q \ll (T-1)$. In order to estimate the input sequence $\{s(n)\}$ (see (1)), we may seek a linear time-varying FIR estimator to yield an estimate with equalization delay d

$$\hat{s}(n-d) = \sum_{i=0}^{L_e} \mathbf{g}^H(n;i)\mathbf{y}(n-i). \quad (10)$$

Existence of a zero-forcing linear equalizer has been discussed in [1]. Their conclusion is that if N is at least 2, then with probability one, one has a zero-forcing solution for sufficiently large L_e and Q. For linear MMSE solution, existence is not an issue, although MMSE equalizer performance can be expected to be "good" if zero-forcing equalizers exist [1]. In this paper we will seek a least squares solution $\mathbf{g}(n;l)$ to minimize a cost such as

$$\frac{1}{T}\sum_{n=0}^{T-1} |s(n-d) - \hat{s}(n-d)|^2 . \quad (11)$$

The main problem considered here is: how to design an equalizer to estimate $\{b(n)\}$ when one knows only $\{c(n)\}$ but not (obviously) $\{b(n)\}$ and one does not also have (frame) synchronization with $\{c(n)\}$ at the receiver. Here we will follow the time-invariant results of [5].

4. LINEAR LEAST-SQUARES FIR CE-BEM EQUALIZERS

We first state the underlying model assumptions.

(H1) The information sequence $\{b(n)\}$ is zero-mean, i.i.d. (independent and identically distributed), with $E\{|b(n)|^2\} = \sigma_b^2$.

(H2) The measurement noise $\{\mathbf{v}(n)\}$ is **zero-mean** ($E\{\mathbf{v}(n)\} = \mathbf{0}$), white, independent of $\{b(n)\}$, with $E\{[\mathbf{v}(n+\tau)][\mathbf{v}(n)]^H\} = \sigma_v^2 I_N \delta(\tau)$.

(H3) The superimposed training sequence $c(n) = c(n+mP)$ $\forall m, n$ is a non-random periodic sequence with period P. Let $\sigma_c^2 := (1/P)\sum_{n=1}^{P} |c(n)|^2$.

(H4) Record length T and period P satisfy $TP^{-1} > K$. Moreover, $P > L + L_e$.

4.1. Equalizer for Training Estimation

The periodic training sequence can be written as

$$c(n) = \sum_{m=0}^{P-1} c_m e^{j\alpha_m n} \quad (12)$$

where $\alpha_m := \frac{2\pi m}{P}$. To design the time-varying linear equalizer to estimate a delayed version of the training sequence $c(n-d)$ ($0 \le d \le L_e$):

$$\hat{c}(n-d) = \sum_{i=0}^{L_e} \mathbf{g}_d^H(n;i)\mathbf{y}(n-i) \quad (13)$$

where we assume that

$$\mathbf{g}_d(n;i) = \sum_{q=-Q/2}^{Q/2} \mathbf{g}_q(i) e^{j\omega_q n}. \quad (14)$$

Choose $\mathbf{g}_q(i)$'s to minimize the time-averaged cost

$$J_c := \frac{1}{T}\sum_{n=0}^{T-1} |c(n-d) - \hat{c}(n-d)|^2 \quad (15)$$

$$= \frac{1}{T}\sum_{n=0}^{T-1} \left| c(n-d) - \sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2} \mathbf{g}_q^H(i) e^{-j\omega_q n}\mathbf{y}(n-i) \right|^2 . \quad (16)$$

By taking the derivative and setting it to be zero, we have

$$0 = \frac{\partial J_c}{\partial \mathbf{g}_{q_1}^*(i_1)} = -\frac{1}{T}\sum_{n=0}^{T-1} e^{-j\omega_{q_1} n}\mathbf{y}(n-i_1)$$

$$\times \left[c^*(n-d) - \sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2} e^{j\omega_q n}\mathbf{y}^H(n-i)\mathbf{g}_q(i) \right] \quad (17)$$

for $i_1 = 0, 1, \cdots, L_e$ and $q_1 = -Q/2, 1-Q/2, \cdots, Q/2$. This leads to

$$\sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2} \left[\frac{1}{T}\sum_{n=0}^{T-1} e^{j(\omega_q - \omega_{q_1})n}\mathbf{y}(n-i_1)\mathbf{y}^H(n-i) \right] \mathbf{g}_q(i) =$$

$$\frac{1}{T}\sum_{n=0}^{T-1} c^*(n-d) e^{-j\omega_{q_1} n}\mathbf{y}(n-i_1) =: \mathbf{R}_c(q_1, i_1). \quad (18)$$

4.2. Equalizer for Data Estimation

To design the time-varying linear equalizer to estimate the information sequence $b(n-d)$ ($0 \le d \le L_e$),

$$\hat{b}(n-d) = \sum_{i=0}^{L_e} \bar{\mathbf{g}}_d^H(n;i)\mathbf{y}(n-i) \quad (19)$$

where we assume that

$$\bar{\mathbf{g}}_d(n;i) = \sum_{q=-Q/2}^{Q/2} \bar{\mathbf{g}}_q(i) e^{j\omega_q n}. \quad (20)$$

Choose $\bar{\mathbf{g}}_q(s)$'s to minimize

$$J_b := \frac{1}{T}\sum_{n=0}^{T-1}\left|b(n-d)-\hat{b}(n-d)\right|^2. \tag{21}$$

Mimicking the results for the superimposed training sequence-based equalization, we have

$$\sum_{i=0}^{L_e}\sum_{q=-Q/2}^{Q/2}\left[\frac{1}{T}\sum_{n=0}^{T-1}e^{j(\omega_q-\omega_{q_1})n}\mathbf{y}(n-i_1)\mathbf{y}^H(n-i)\right]\bar{\mathbf{g}}_q(i)$$

$$=\frac{1}{T}\sum_{n=0}^{T-1}b^*(n-d)e^{-j\omega_{q_1}n}\mathbf{y}(n-i_1)=:\mathbf{R}_b(q_1,i_1). \tag{22}$$

4.3. When are the two equalizers equal?

Comparing (18) and (22), we see that (ignoring the equalizer coefficients) the left-sides of the two are identical whereas the right-sides are different. We now seek to establish that for large T, $\mathbf{R}_c(q_1,i_1)=\beta\mathbf{R}_b(q_1,i_1)\ \forall\, q_1,i_1$, for some scalar β, so that $\mathbf{g}_q(i)=\beta\bar{\mathbf{g}}_q(i)\ \forall\, i$.

We have

$$\mathbf{R}_c(q_1,i_1)=\frac{1}{T}\sum_{n=0}^{T-1}c^*(n-d)e^{-j\omega_{q_1}n}$$

$$\times\left\{\sum_{l=0}^{L}\mathbf{h}(n-i_1;l)s(n-i_1-l)+\mathbf{v}(n-i_1)\right\}$$

$$=\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{m_1=0}^{P-1}\sum_{m_2=0}^{P-1}c_{m_1}^*c_{m_2}e^{j\alpha_{m_1}d}e^{-j\alpha_{m_2}(i_1+l)}$$

$$\times e^{-j\omega_k i_1}\mathbf{h}_k(l)A_0+\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{m_1=0}^{P-1}c_{m_1}^*e^{j\alpha_{m_1}d}e^{-j\omega_k i_1}$$

$$\times\mathbf{h}_k(l)A_1+\sum_{m_1=0}^{P-1}c_{m_1}^*e^{j\alpha_{m_1}d}\mathbf{A}_2 \tag{23}$$

where

$$A_0:=\frac{1}{T}\sum_{n=0}^{T-1}e^{j(-\alpha_{m_1}+\alpha_{m_2}-\omega_{q_1}+\omega_k)n}$$

$$A_1:=\frac{1}{T}\sum_{n=0}^{T-1}e^{j(-\alpha_{m_1}-\omega_{q_1}+\omega_k)n}b(n-i_1-l)$$

$$\mathbf{A}_2:=\frac{1}{T}\sum_{n=0}^{T-1}e^{-j(\alpha_{m_1}+\omega_{q_1})n}\mathbf{v}(n-i_1).$$

Under the condition $TP^{-1}>K$ (then $(\alpha_m+\omega_q)=(\alpha_n+\omega_k)$ iff $m=n$ and $q=k$), we have

$$A_0=\delta(m_1-m_2)\delta(q_1-k). \tag{24}$$

Furthermore we have

$$E\left\{|A_1|^2\right\}$$

$$=\frac{1}{T^2}\sum_{n_1=0}^{T-1}\sum_{n_2=0}^{T-1}e^{j(-\alpha_{m_1}-\omega_{q_1}+\omega_k)(n_1-n_2)}\sigma_b^2\delta(n_1-n_2)=\frac{\sigma_b^2}{T}. \tag{25}$$

Similarly, it follows that

$$E\left\{\|\mathbf{A}_2\|^2\right\}=\frac{N\sigma_v^2}{T}. \tag{26}$$

In the mean-square sense (and thus in probability), we then have the following two limits

$$\lim_{T\to\infty}A_1\overset{\text{m.s.}}{=}0 \text{ and } \lim_{T\to\infty}\mathbf{A}_2\overset{\text{m.s.}}{=}\mathbf{0}. \tag{27}$$

Thus for "large" T, we have (after some manipulations)

$$\lim_{T\to\infty}\mathbf{R}_c(q_1,i_1)\overset{\text{m.s.}}{=}$$

$$=\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{m=0}^{P-1}|c_m|^2e^{j\alpha_m(d-i_1-l)}e^{-j\omega_k i_1}\mathbf{h}_k(l)\delta(q_1-k). \tag{28}$$

If the training sequence $\{c(n)\}$ is periodic white (i.e. $P^{-1}\sum_{n=0}^{P-1}c(n)c^*(n-l)=\sigma_c^2\delta(l\bmod P)$), then

$$\sum_{m=0}^{P-1}|c_m|^2e^{j\alpha_m(d-i_1-l)}=\sigma_c^2\delta((d-i_1-l)\bmod P). \tag{29}$$

This fact then leads to

$$\lim_{T\to\infty}\mathbf{R}_c(q_1,i_1)\overset{\text{m.s.}}{=}$$

$$\begin{cases}\sigma_c^2e^{-j\omega_{q_1}i_1}\mathbf{h}_{q_1}((d-i_1)\bmod P) & \text{if } |q_1|\le K/2\\ 0 & \text{otherwise}\end{cases} \tag{30}$$

for $i_1=0,1,\cdots,L_e$ and $q_1=-Q/2,1-Q/2,\cdots,Q/2$.

Turning to (22), we have

$$\mathbf{R}_b(q_1,i_1)=\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\sum_{m=0}^{P-1}c_m\mathbf{h}_k(l)e^{-j\alpha_m(i_1+l)}e^{-j\omega_k i_1}A_3$$

$$+\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\mathbf{h}_k(l)e^{-j\omega_k i_1}A_4+\mathbf{A}_5 \tag{31}$$

where

$$A_3:=\frac{1}{T}\sum_{n=0}^{T-1}e^{j(\alpha_m-\omega_{q_1}+\omega_k)n}b^*(n-d)$$

$$A_4:=\frac{1}{T}\sum_{n=0}^{T-1}e^{j(\omega_k-\omega_{q_1})n}b(n-i_1-l)b^*(n-d)$$

$$\mathbf{A}_5:=\frac{1}{T}\sum_{n=0}^{T-1}e^{-j\omega_{q_1}n}\mathbf{v}(n-i_1)b^*(n-d).$$

We can show (as before) that

$$\lim_{T\to\infty}A_3\overset{\text{m.s.}}{=}0 \text{ and } \lim_{T\to\infty}\mathbf{A}_5\overset{\text{m.s.}}{=}\mathbf{0}. \tag{32}$$

Consider

$$A_6:=\frac{1}{T}\sum_{n=0}^{T-1}e^{j(\omega_k-\omega_{q_1})n}\left[b(n-i_1-l)b^*(n-d)\right.$$

$$-\sigma_b^2\delta(d-i_1-l)\Big]. \tag{33}$$

It then follows (after some manipulations) that

$$E\left\{|A_6|^2\right\} = \frac{1}{T}\left[E\left\{|b(n)|^4\right\} - \sigma_b^4\right]\delta(d-i_1-l). \tag{34}$$

Therefore, we have $\lim_{T\to\infty} A_6 \overset{\text{m.s.}}{=} 0$, and consequently

$$\lim_{T\to\infty} A_4 \overset{\text{m.s.}}{=} \frac{1}{T}\sum_{n=0}^{T-1} e^{j(\omega_k-\omega_{q_1})n}\sigma_b^2\delta(d-i_1-l)$$

$$= \sigma_b^2\delta(d-i_1-l)\delta(q_1-k). \tag{35}$$

Hence, for "large" T, we have $\lim_{T\to\infty}\mathbf{R}_b(q_1,i_1) \overset{\text{m.s.}}{=}$

$$\sum_{k=-K/2}^{K/2}\sum_{l=0}^{L}\mathbf{h}_k(l)e^{-j\omega_k i_1}\sigma_b^2\delta(d-i_1-l)\delta(q_1-k). \tag{36}$$

For $i_1 = 0, 1, \cdots, L_e$ and $q_1 = -Q/2, 1-Q/2, \cdots, Q/2$ but $|q_1| \le K/2$, we therefore have

$$\lim_{T\to\infty}\mathbf{R}_b(q_1,i_1) \overset{\text{m.s.}}{=} \mathbf{h}_{q_1}(d-i_1)e^{-j\omega_{q_1}i_1}\sigma_b^2. \tag{37}$$

If $P > L + L_e$, then (30) equals (37) (within a scale factor). Therefore, for "large" T, $\mathbf{R}_c(q_1,i_1) = \beta\mathbf{R}_b(q_1,i_1)$ $\forall\, q_1, i_1$ with $\beta = \sigma_c^2/\sigma_b^2$; hence $\mathbf{g}_q(i) = \beta\bar{\mathbf{g}}_q(i)\ \forall\, i$.

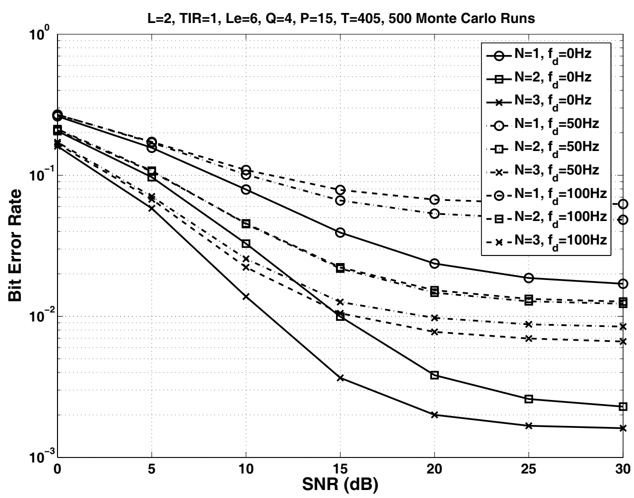

Figure 1. BER (averaged over 500 runs) for different number of receive antennas and varying Doppler spreads. Equalizer length L_e=6, $Q = 4$ and $P = 15$. Each channel tap component follows Jakes' model (not CE-BEM). N= # of receive antennas; solid curves: f_d=0 (time-invariant); dot-dashed: f_d=50 Hz; dashed: f_d=100 Hz.

4.4. Desired Equalizer Design

We execute the following steps:

(i) Pick L_e and d ($= \frac{L_e}{2}$ in Sec. 5). Pick $Q \ge K$, $P > L + L_e$.

(ii) Solve (18), given data $\mathbf{y}(n)$, for $\mathbf{g}_q(i)$ where $0 \le i \le L_e$ and $-\frac{Q}{2} \le q \le \frac{Q}{2}$. Then

$$\mathbf{g}_d(n;i) = \sum_{q=-Q/2}^{Q/2}\mathbf{g}_q(i)e^{j\omega_q n}. \tag{38}$$

(iii) The equalized output is then given by

$$e_1(n) = \sum_{i=0}^{L_e}\mathbf{g}_d^H(n;i)\mathbf{y}(n-i) \approx \alpha_1 c(n-d) + \alpha_2 b(n-d) + \tilde{v}(n) \tag{39}$$

where $\tilde{v}(n)$ is the equalized noise. Estimate α_1 as

$$\hat{\alpha}_1 = \frac{\frac{1}{T}\sum_{n=0}^{T-1}e_1(n)c^*(n-d)}{\frac{1}{T}\sum_{n=0}^{T-1}|c(n-d)|^2} = \frac{\frac{1}{T}\sum_{n=0}^{T-1}e_1(n)c^*(n-d)}{\sigma_c^2}. \tag{40}$$

(iv) Define

$$e_2(n) = e_1(n) - \hat{\alpha}_1 c(n-d) \approx \alpha_2 b(n-d) + \tilde{v}(n). \tag{41}$$

Then we hard-quantize $e_2(n)$ to estimate $b(n-d)$.

5. SIMULATION EXAMPLE

We consider a random frequency-selective Rayleigh fading channel. We took N = 1, 2 or 3 (receiver antennas), and $L = 2$ in (1) with $\mathbf{h}(n;l)$ mutually independent for all l and all components, zero-mean complex-Gaussian with equal variance, following Jakes' model with specified Doppler spread for each tap component. We consider a system with carrier frequency of 2GHz, data rate of 40kB (kB= kilo-Bauds), therefore, $T_s = 25\times10^{-6}$ sec., and a varying Doppler spread f_d. Additive noise was zero-mean complex white Gaussian. The SNR refers to the energy per bit over one-sided noise spectral density with both information and superimposed training sequence counting toward the bit energy. Information sequence was BPSK (binary). We took the superimposed training sequence period $P = 15$ in (**H3**); it is periodically white, as in [4, Eqn. (34)]. The average transmitted power in $c(n)$ was equal to the power in $b(n)$, leading to a training-to-information power ratio (TIR) of 1.0 .

The results averaged over 500 Monte Carlo for a record length of T=405 (=15×27) symbols are shown in Fig. 1 for various Doppler spreads (0, 50 and 100 Hz).

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