BLIND ADAPTIVE EQUALIZATION METHOD WITHOUT CHANNEL ORDER ESTIMATION

I. Kacha [1,2], *K. Abed-Meraim* [1], *A. Belouchrani* [2]

[1] Télécom Paris (ENST) - Département TSI
37-39 rue Dareau, 75014 Paris, France
[2] École Nationale Polytechnique (ENP) - Département d'Électronique
10 avenue Hassan Badi El-Harrach, 16200 Alger, Algérie
e-mail:{kacha, abed}@tsi.enst.fr, adel.belouchrani@enp.edu.dz

ABSTRACT

In this paper, we propose a new blind minimum mean square error (MMSE) equalization algorithm of noisy single-input multiple-outputs finite impulse response (SIMO-FIR) systems, relying only on second order statistics. This algorithm offers an important advantage, a total independence of the channel order. Exploiting the fact that the equalizer filter belongs both, to the signal subspace and to the kernel of truncated data covariance matrix, the algorithm achieves blindly a direct estimation of the zero-delay MMSE equalizer parameters. The proposed approach has several features that are studied in this work. More precisely, we develop a two-step procedure to further improve the performance gain and to control the equalization delay. We present an efficient adaptive implementation of our equalizer, which reduces the computational complexity from $O(n^3)$ to $O(n^2p)$, where n is the data vector length and p is the number of sensors. Simulation results are provided to illustrate the effectiveness of the proposed blind equalization algorithm.

1. INTRODUCTION

Since the pioneer work of Tong *et al* [1], active research in blind Identification/Equalization area has led to a variety of second-order statistics-based algorithms (see the book [2], as well as the references therein). Many efficient solutions (e.g. [3]) suffer from the lack of robustness against channel order overestimation errors. A lot of research effort has been done to either develop efficient techniques for channel order estimation [4, 5] or to develop blind identification/equalization methods robust to channel order estimation errors. Several *robust* techniques have been proposed so far [6, 7, 8], but all of them depend explicitly or implicitly on the channel order and hence have only a relative robustness. In this paper, we describe a new technique for direct design of SIMO blind MMSE equalizer, completely independent of the channel order. We show first that the zero-delay equalizer filter belongs simultaneously, to the signal subspace and to the kernel of truncated data covariance matrix. This property is used, under some weak conditions, to estimate the parameters of the equalizer by maximizing a certain quadratic form subject to a properly chosen constraint. We present an efficient adaptive implementation of the novel algorithm, having only $O(n^2p)$ complexity. A two-step estimation procedure is included (in batch or adaptive way), which allows us to compensate for the performance loss of the equalizer compared to the non-blind one and to choose a non-zero equalization delay.

2. DATA MODEL

Consider a SIMO system of p outputs, given by

$$\mathbf{x}(t) = \sum_{k=0}^{L} \mathbf{h}(k)s(t-k) + \mathbf{b}(t), \tag{1}$$

where $\mathrm{h}(z) = \sum_{k=0}^{L} \mathbf{h}(k)z^{-k}$ is an unknown causal FIR $p \times 1$ transfer function. We assume **(A1)** $\mathrm{h}(z) \neq 0, \forall z$. **(A2)** The input (non-observable) signal $s(t)$ is a scalar iid zero-mean process of power σ_s^2. **(A3)** $\mathbf{b}(t)$ is an additive spatially and temporally white noise of power $\sigma_b^2\mathbf{I}_p$ and independent of the transmitted sequence $\{s(t)\}$. By stacking N successive samples of the received signal $\mathbf{x}(t)$ into a single vector, we obtain the n-dimensional $(n = Np)$ vector

$$\begin{aligned} \mathbf{x}_N(t) &= [\mathbf{x}(t)^T \ \mathbf{x}(t-1)^T \ \ldots \mathbf{x}(t-N+1)^T]^T \\ &= \mathbf{H}_N\mathbf{s}_m(t) + \mathbf{b}_N(t), \end{aligned} \tag{2}$$

where $\mathbf{s}_m(t) = [s(t) \ \ldots \ s(t-m+1)]^T$, $(m = N+L)$, $\mathbf{b}_N(t) = [\mathbf{b}(t)^T \ \ldots \ \mathbf{b}(t-N+1)^T]^T$ and $\mathbf{H}_N$ is the channel convolution matrix of dimension $n \times m$, given by

$$\mathbf{H}_N = \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(L) & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}. \tag{3}$$

It is shown in [9], that if $N > L$ and under assumption **(A1)**, matrix $\mathbf{H}_N$ is full column rank.

3. ALGORITHM DERIVATION

3.1. MMSE equalizer

Consider a τ-delay MMSE equalizer $(\tau \in \{0, 1, \cdots, m-1\})$. Under the above data model, one can show that the $n-$dimensional equalizer vector $\mathbf{v}_\tau$ corresponding to the desired solution is given by

$$\mathbf{v}_\tau = \arg\min_{\mathbf{v}} E(\|s(t-\tau) - \mathbf{v}^H\mathbf{x}_N(t)\|^2) = \mathbf{C}_N^{-1}\mathbf{g}_\tau, \tag{4}$$

where

$$\mathbf{C}_N \overset{\text{def}}{=} E[\mathbf{x}_N(t)\mathbf{x}_N(t)^H] = \sigma_s^2\mathbf{H}_N\mathbf{H}_N^H + \sigma_b^2\mathbf{I}_n, \tag{5}$$

is the data covariance matrix and $\mathbf{g}_\tau$ is an $n \times 1$ vector given by

$$\mathbf{g}_\tau \stackrel{\text{def}}{=} E(\mathbf{x}_N(t)s(t-\tau)^*) = \sigma_s^2 \mathbf{H}_N(:, \tau+1), \qquad (6)$$

where $\mathbf{H}_N(:, \tau+1)$ denotes the $(\tau+1)$-th column vector of $\mathbf{H}_N$[1]. Clearly, the MMSE filter $\mathbf{v}_\tau$ belongs to the signal subspace (i.e. $range(\mathbf{H}_N)$).

3.2. Blind equalization

Our objective here is to derive a blind estimate of the zero-delay MMSE equalizer $\mathbf{v}_0$. From equations (4) and (6), $\mathbf{v}_0$ is given implicitly by

$$\mathbf{C}_N \mathbf{v}_0 = \sigma_s^2 \begin{bmatrix} \mathbf{h}(0) \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}. \qquad (7)$$

If we truncate the first p rows of system (7), we obtain

$$\bar{\mathbf{C}} \mathbf{v}_0 = \mathbf{0}, \qquad (8)$$

where $\bar{\mathbf{C}}$ is an $(n-p) \times n$ sub-matrix of $\mathbf{C}_N$ given by its last $n-p$ rows, i.e.: $\bar{\mathbf{C}} = \mathbf{C}_N(p+1:n, :)$. It is obvious that $\mathbf{v}_0$ belongs to the right null space of $\bar{\mathbf{C}}$. Next, we establish a link between the structure of the right null space of $\bar{\mathbf{C}}$ and the zero-delay MMSE equalizer.

Lemma 1 *Under the above data assumptions and for $N > L+1$, the right null space of $\bar{\mathbf{C}}$: $null_r(\bar{\mathbf{C}}) = \{\mathbf{z} \in \mathbb{C}^n : \bar{\mathbf{C}}\mathbf{z} = \mathbf{0}\}$, is a $p-$dimensional subspace, where only one direction of it belongs to the signal subspace.*

Proof: First, notice that the $(n-p) \times (n-p)$ matrix $\mathbf{C}_{N-1}$ is a full rank sub-matrix of $\bar{\mathbf{C}}$. It follows that $dim(null_r(\bar{\mathbf{C}})) = p$. Let $\mathbf{W} \in \mathbb{C}^{n \times m}$ be an orthonormal basis of the signal subspace. Since $range(\mathbf{H}_N) = range(\mathbf{W}) = range(\mathbf{C}_N\mathbf{W})$, there exists a non-singular $m \times m$ matrix $\mathbf{R}$ such that $\mathbf{C}_N\mathbf{W} = \mathbf{H}_N\mathbf{R}$. Therefore, $\bar{\mathbf{C}}\mathbf{W} = [\mathbf{0}_{(n-p)\times 1} \;\; \mathbf{H}_{N-1}]\mathbf{R}$. As $\mathbf{H}_{N-1}$ is full column rank, it implies that $dim(null_r(\bar{\mathbf{C}}\mathbf{W})) = 1$, what carries out to conclude that only one direction of $null_r(\bar{\mathbf{C}})$ belongs to the signal subspace. ■

To extract the direction of $null_r(\bar{\mathbf{C}})$ which belongs to the signal subspace (the direction of $\mathbf{v}_0$), we use the technique proposed in [10]. The selection of the filter $\mathbf{v}$ corresponding to the equalizer $\mathbf{v}_0$ is obtained by choosing the one with maximum power, i.e.

$$\max_{\mathbf{v} \in null_r(\bar{\mathbf{C}})} E(\|\mathbf{v}^H \mathbf{x}_N(t)\|^2) \Leftrightarrow \max_{\|\tilde{\mathbf{v}}\|=1} (\tilde{\mathbf{v}}^H \mathbf{B} \tilde{\mathbf{v}}), \qquad (9)$$

where $\mathbf{B} \stackrel{\text{def}}{=} \mathbf{A}^H \mathbf{C}_N \mathbf{A}$, matrix $\mathbf{A} \in \mathbb{C}^{n \times p}$ is an orthonormal basis of $null_r(\bar{\mathbf{C}})$ and $\tilde{\mathbf{v}}$ is a p-dimensional vector such as $\mathbf{v} = \mathbf{A}\tilde{\mathbf{v}}$. Indeed, under the unit-norm constraint, the noise contribution in (9) is of constant power and thus the maximization in (9) concerns only the signal term. Besides, the other $p-1$ directions of $null_r(\bar{\mathbf{C}})$, that are complementary to $\mathbf{v}_0$, are almost in the noise subspace (at least for high signal-to-noise ratio (SNR)). Therefore, the maximization in (9) leads essentially to the selection of the desired direction of vector $\mathbf{v}_0$. A batch-processing implementation of the algorithm is summarized in table 1.

[1] We use here some informal MATLAB notations.

$\mathbf{C}_N$	$= \frac{1}{K}\sum_{t=0}^{K-1} \mathbf{x}_N(t)\mathbf{x}_N(t)^H$, ($K$: sample size)
$(\mathbf{U}, \Sigma, \mathbf{V})$	$= svd(\mathbf{C}_N(p+1:n, :))$
$\mathbf{A}$	$= \mathbf{V}(:, n-p+1:n)$
$\mathbf{B}$	$= \mathbf{A}^H \mathbf{C}_N \mathbf{A}$
$\tilde{\mathbf{v}}$	= the dominant eigenvector of $\mathbf{B}$
$\mathbf{v}$	$= \mathbf{A}\tilde{\mathbf{v}}$

Table 1. Blind MMSE equalization algorithm.

3.3. Selection of the equalizer delay

It is shown in [7] that the blind estimation of the zero-delay MMSE filter results in a performance loss compared to the non-blind one. To compensate for this performance loss and also to have a controlled non-zero delay which helps to improve the performance of the equalizer, we propose here a two-step approach to estimate the blind MMSE equalizer. In the first step, we estimate $\mathbf{v}_0$ according to the previous algorithm, while, in the second step, we refine this estimation by exploiting the a priori knowledge of the finite alphabet to which belongs the symbols $s(t)$. This is done by performing a hard decision on the symbols[2] that are then used to re-estimate $\mathbf{v}_\tau$ according to equations (4) and (6). The two-step blind MMSE equalization algorithm is summarized in Table 1 and Table 2.

Estimate $s(t)$, $t = 0..K-1$, (using $\mathbf{v}$ given by Table 1)	
$\mathbf{g}_\tau$	$= \frac{1}{K}\sum_{t=\tau}^{K+\tau-1} \mathbf{x}_N(t)\hat{s}(t-\tau)^*$
$\mathbf{v}_\tau$	$= \mathbf{C}_N^{-1}\mathbf{g}_\tau$

Table 2. Two-step equalization procedure.

4. ADAPTIVE IMPLEMENTATION

In order to reduce the global computational complexity of the algorithm, from $O(n^3)$ to $O(n^2p)$, an adaptive implementation is proposed. The columns of matrix $\mathbf{A}$ correspond to the p-least eigenvectors of matrix $\mathbf{Q} \stackrel{\text{def}}{=} \bar{\mathbf{C}}^H\bar{\mathbf{C}}$. Thus $\mathbf{A}$ can be obtained as an orthogonal complement of the $(n-p)$-dimensional major subspace of $\mathbf{Q}$. If $\mathbf{A}^\perp \in \mathbb{C}^{n \times (n-p)}$ is an orthonormal basis of the major subspace of $\mathbf{Q}$ then $\mathbf{A}\mathbf{A}^H = \mathbf{I}_n - \mathbf{A}^\perp\mathbf{A}^{\perp H}$.

As matrix $\mathbf{C}_N$ is recursively updated by $\mathbf{C}_N(t) = \beta\mathbf{C}_N(t-1) + \mathbf{x}_N(t)\mathbf{x}_N(t)^H$, where $0 < \beta < 1$ is a forgetting factor, matrix $\bar{\mathbf{C}}$ is then replaced by the recursion $\bar{\mathbf{C}}(t) = \beta\bar{\mathbf{C}}(t-1) + \mathbf{x}_{N-1}(t-1)\mathbf{x}_N(t)^H$. Thus, matrix $\mathbf{Q}(t)$ is recursively given by

$$\mathbf{Q}(t) = \beta^2\mathbf{Q}(t-1) + \mathbf{y}_1(t)\mathbf{y}_1(t)^H + \mathbf{y}_2(t)\mathbf{y}_2(t)^H, \qquad (10)$$

[2] We assume here the use of a differential modulation to get rid of the phase indeterminacy inherent to the blind equalization problem.

where $\mathbf{y}_1(t)$ and $\mathbf{y}_2(t)$ are two n-dimensional vectors given by

$$\begin{aligned}
\mathbf{y}_1(t) &= \sigma(t-1)(\beta\mathbf{x}'(t-1)+\lambda(t-1)\mathbf{x}_N(t)),\\
\mathbf{y}_2(t) &= j\sigma(t-1)(\beta\mathbf{x}'(t-1)-\frac{1}{\lambda(t-1)}\mathbf{x}_N(t)),\\
\lambda(t) &= \frac{1}{2}(\|\mathbf{x}_{N-1}(t)\|^2+\sqrt{4+\|\mathbf{x}_{N-1}(t)\|^4}), \qquad (11)\\
\sigma(t) &= \sqrt{\frac{\lambda(t)}{1+\lambda(t)^2}},\ \ \mathbf{x}'(t) \ = \ \bar{\mathbf{C}}(t)^H\mathbf{x}_{N-1}(t).
\end{aligned}$$

It follows that matrix $\mathbf{A}^{\perp}(t)$ can be considered as an orthonormal basis of a major subspace of the *virtual* covariance matrix $\mathbf{Q}(t)$ of data sequence $\{\mathbf{y}_1(t),\mathbf{y}_2(t)\}_{t\in\mathbb{Z}}$. From data vectors $\mathbf{y}_1(t)$ and $\mathbf{y}_2(t)$, the columns of $\mathbf{A}^{\perp}(t)$ can be extracted recursively when performing twice a fast major subspace estimating and tracking algorithm. In this work, we use the OPAST (Orthogonal Projection Approximation Subspace Tracking) algorithm [11] that is summarized in Table 3, where $\mathbf{y}'(t)$ stands alternatively for $\mathbf{y}_1(t)$ and $\mathbf{y}_2(t)$. The choice of OPAST algorithm is motivated by its simplicity and its remarkable performance compared to other existing subspace tracking algorithms of similar computational complexity. Finally, the extraction of the dominant eigenvector of $\mathbf{B}(t)$ is

$$\begin{aligned}
\mathbf{y}(t) &= \mathbf{A}^{\perp}(t-1)^H\mathbf{y}'(t)\\
\mathbf{q}(t) &= \tfrac{1}{\beta}\mathbf{Z}(t-1)\mathbf{y}(t)\\
\gamma(t) &= \tfrac{1}{1+\mathbf{y}(t)^H\mathbf{q}(t)}\\
\mathbf{p}(t) &= \gamma(t)(\mathbf{y}'(t)-\mathbf{A}^{\perp}(t-1)\mathbf{y}(t))\\
r(t) &= \tfrac{1}{\|\mathbf{q}(t)\|^2}(\tfrac{1}{\sqrt{1+\|\mathbf{p}(t)\|^2\|\mathbf{q}(t)\|^2}}-1)\\
\mathbf{p}'(t) &= r(t)\mathbf{A}^{\perp}(t-1)\mathbf{q}(t)+(1+r(t)\|\mathbf{q}(t)\|^2)\mathbf{p}(t)\\
\mathbf{Z}(t) &= \tfrac{1}{\beta}\mathbf{Z}(t-1)-\gamma(t)\mathbf{q}(t)\mathbf{q}(t)^H\\
\mathbf{A}^{\perp}(t) &= \mathbf{A}^{\perp}(t-1)+\mathbf{p}'(t)\mathbf{q}(t)^H
\end{aligned}$$

Table 3. OPAST algorithm.

obtained by power iteration as

$$\tilde{\mathbf{v}}(t)=\frac{\mathbf{B}(t)\tilde{\mathbf{v}}(t-1)}{\|\mathbf{B}(t)\tilde{\mathbf{v}}(t-1)\|}. \qquad (12)$$

The outline of the adaptive blind MMSE equalization algorithm is given in Table 4. Using matrix inversion lemma, matrix $\mathbf{F}(t)\overset{\text{def}}{=}\mathbf{C}_N(t)^{-1}$ is updated recursively as

$$\begin{aligned}
\mathbf{F}(t) &= \frac{1}{\beta}(\mathbf{F}(t-1)-\mathbf{f}'(t)\mathbf{f}(t)^H),\\
\mathbf{f}(t) &= \mathbf{F}(t-1)\mathbf{x}_N(t), \qquad (13)\\
\rho(t) &= \mathbf{f}(t)^H\mathbf{x}_N(t),\ \ \mathbf{f}'(t) \ = \ \frac{\mathbf{f}(t)}{\beta+\rho(t)}.
\end{aligned}$$

Thus, the adaptive version of the two-step procedure is summarized in Table 5. Note that the whole processing requires $O(n^2p)$ operations per iteration.

Remark: It is possible to further reduce the computational complexity to $O(np)$. Matrix $\mathbf{A}(t)$ can be estimated directly in $O(np)$ operations using a fast minor subspace tracker such as FDPM (Fast Data Projection Method) algorithm [12]. The computation of the vector $\mathbf{x}'(t)=\bar{\mathbf{C}}(t)^H\mathbf{x}_{N-1}(t)$ in (11) can be reduced from $O(n^2)$ to $O(np)$ by means of the technique described in [13], which exploits the shift invariance property of the correlation matrix. Using the projection approximation $\mathbf{C}_N(t)\mathbf{A}(t)\approx\mathbf{C}_N(t)\mathbf{A}(t-1)$,

$$\begin{aligned}
\mathbf{y}_1(t) &= \sigma(t-1)(\beta\mathbf{x}'(t-1))+\lambda(t-1)\mathbf{x}_N(t))\\
\mathbf{y}_2(t) &= j\sigma(t-1)(\beta\mathbf{x}'(t-1)-\tfrac{1}{\lambda(t-1)}\mathbf{x}_N(t))\\
(\mathbf{A}'(t),\mathbf{Z}'(t)) &= \text{OPAST}(\mathbf{A}^{\perp}(t-1),\mathbf{Z}(t-1),\mathbf{y}_1(t),\beta)\\
(\mathbf{A}^{\perp}(t),\mathbf{Z}(t)) &= \text{OPAST}(\mathbf{A}'(t),\mathbf{Z}'(t),\mathbf{y}_2(t),\beta)\\
\mathbf{\Pi}(t) &= \mathbf{I}_n-\mathbf{A}^{\perp}(t)\mathbf{A}^{\perp}(t)^H\\
\mathbf{A}(t) &= eigs(\mathbf{\Pi}(t),p)\\
\mathbf{C}_N(t) &= \beta\mathbf{C}_N(t-1)+\mathbf{x}_N(t)\mathbf{x}_N(t)^H\\
\mathbf{B}(t) &= \mathbf{A}(t)^H\mathbf{C}_N(t)\mathbf{A}(t)\\
\tilde{\mathbf{v}}(t) &= \tfrac{\mathbf{B}(t)\tilde{\mathbf{v}}(t-1)}{\|\mathbf{B}(t)\tilde{\mathbf{v}}(t-1)\|}\\
\mathbf{v}(t) &= \mathbf{A}(t)\tilde{\mathbf{v}}(t)\\
\bar{\mathbf{C}}(t) &= \mathbf{C}_N(t)_{(p+1:n,:)}\\
\mathbf{x}'(t) &= \bar{\mathbf{C}}(t)^H\mathbf{x}_{N-1}(t)\\
\lambda(t) &= \tfrac{1}{2}(\|\mathbf{x}_{N-1}(t)\|^2+\sqrt{4+\|\mathbf{x}_{N-1}(t)\|^4})\\
\sigma(t) &= \sqrt{\tfrac{\lambda(t)}{1+\lambda(t)^2}}
\end{aligned}$$

Table 4. Adaptive blind MMSE equalization algorithm.

Table 5. Adaptive two-step equalization procedure.

Estimate $\hat{s}(t)$, (using $\mathbf{v}(t)$ given by Table 4)

$$\begin{aligned}
\mathbf{f}(t) &= \mathbf{F}(t-1)\mathbf{x}_N(t)\\
\rho(t) &= \mathbf{f}(t)^H\mathbf{x}_N(t)\\
\mathbf{f}'(t) &= \tfrac{\mathbf{f}(t)}{\beta+\rho(t)}\\
\mathbf{v}_\tau(t) &= \mathbf{v}_\tau(t-1)-\mathbf{f}'(t)(\mathbf{f}(t)^H\mathbf{g}_\tau(t-1)-\hat{s}(t-\tau)^*)\\
\mathbf{F}(t) &= \tfrac{1}{\beta}(\mathbf{F}(t-1)-\mathbf{f}'(t)\mathbf{f}(t)^H)\\
\mathbf{g}_\tau(t) &= \beta\mathbf{g}_\tau(t-1)+\mathbf{x}_N(t)\hat{s}(t-\tau)^*
\end{aligned}$$

which is valid if matrix $\mathbf{A}(t)$ is slowly varying with time [11], matrix $\mathbf{B}(t)$ can be updated in $O(np)$ operations. Finally the equalizer filter $\mathbf{v}_\tau(t)$ in the two-step procedure can be computed via normalized least mean square (NLMS) algorithm only in $O(np)$ operations. However, the major disadvantage of this approach comes from the fact that fast $O(np)$ minor subspace tracker algorithms suffer from *local minima* problem when the minor subspace's dimension is much lower than that of the total subspace, i.e.: $p\ll n$ (which is exactly, the situation we are facing herein).

5. SIMULATION RESULTS

We provide in this section some simulation examples to illustrate the performance of the proposed blind equalizer. Our tests are based on SIMO channels ($p=3$ and $L=4$). The channel coefficients are chosen randomly at each run according to a complex Gaussian distribution. The input signals are iid unit-power QAM4 sequences. The width of the temporal window is $N=6$. As a performance measure, we estimate the average MSE given by $\text{MSE}=\frac{1}{K}\sum_{t=\tau}^{K+\tau+1}(|s(t-\tau)-\hat{\mathbf{v}}_\tau^H\mathbf{x}_N(t)|^2)$ (resp. $\text{MSE}(t)=|s(t-\tau)-\hat{\mathbf{v}}_\tau(t)^H\mathbf{x}_N(t)|^2)$ in batch processing case (resp. adaptive case), over 100 Monte-Carlo runs. The MSE is compared to the theoretical MSE given by $\text{MSE}_{th}=1-\mathbf{g}_\tau^H\mathbf{C}_N^{-1}\mathbf{g}_\tau$. SNR is defined by $\text{SNR}=-20\log(\sigma_b)$. In Fig.1, we plot the MSE (in dB) against SNR (in dB) for $K=500$. One can observe the performance loss of the zero-delay MMSE filter compared to the optimal one (especially at high SNRs) due, as shown in [7], to the blind estimation procedure. Also, it illustrates the effectiveness of the two-step approach, which allows us to compensate for the performances loss and to choose a non-zero equalization delay. Fig.2 illustrates the convergence rate of the adaptive algorithm with SNR$=15$dB. Fig.3 is dedicated to robustness against overestimation errors. The plots compare the MSE obtained by the algorithms in [7, 8] to those obtained by our algorithm (exact order $L=4$, SNR$=15$dB, $K=500$). Clearly, our method is insensitive to channel order over-estimation errors.

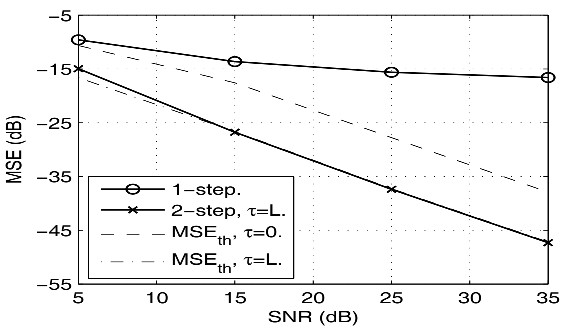

Fig. 1. Performance of the equalizer.

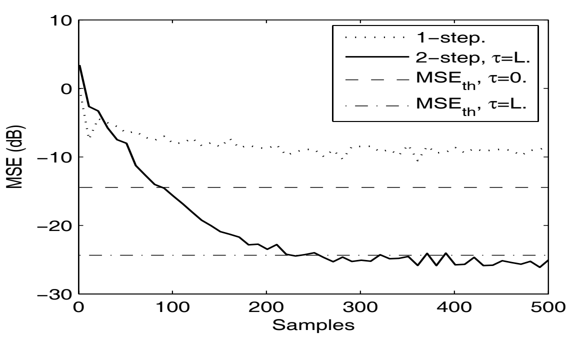

Fig. 2. Convergence of the adaptive equalizer.

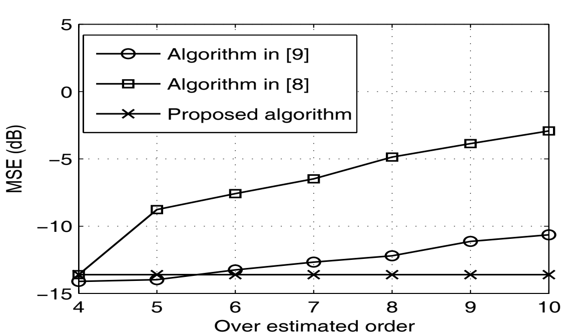

Fig. 3. Robustness against channel order over-estimation errors.

6. CONCLUSION

In this paper we have proposed a blind equalization technique for SIMO-FIR systems that does not need an estimate of the channel order. Batch and adaptive implementation are developed. A two-step approach, using the *a priori* knowledge of the source signal finite alphabet, has been proposed to compensate for the performance loss and to choose a non-zero equalization delay. Note that, the extension of the proposed technique to MIMO case is straightforward.

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