UNDER-DETERMINED BLIND IDENTIFICATION OF CYCLO-STATIONARY SIGNALS WITH UNKNOWN CYCLIC FREQUENCIES

Saloua Rhioui, Nadège Thirion-Moreau, Eric Moreau

STD, ISITV, Université de Toulon
Avenue George Pompidou, BP 56
F-83162 La Valette du Var, Cedex, France
{ rhioui, thirion, moreau }@univ-tln.fr

ABSTRACT

In this communication, we propose a new method to blindly identify the mixing matrix of a possibly under-determined mixture of cyclostationary source signals. It is based on the use of a linear operator applied on the observations correlation matrix. Exploiting the properties of the above transformed matrix, a set of cyclic frequencies is first estimated. Then it is used to construct different estimations of the mixing matrix column vectors. Finally using a classification procedure, the mixing matrix is then estimated.

I. INTRODUCTION

We consider the source separation problem which finds numerous applications in diverse fields of engineering and applied sciences as e.g. data communications, seismic exploration, array processing, speech processing etc... It can be simply formulated as follows. Several linear mixtures of different signals called sources are observed. The purpose is then to recover the unknown original sources without knowing the mixing system. Hence this must be realized from the only observations and this is the reason why this problem is often qualified as "blind" or "unsupervised".

Among the great number of approaches that have been proposed in the recent literature, we are primary concerned with a particular class of source signals that is the class of cyclostationary source signals, see e.g. [1][2][3][4].

It has to be noticed that our developments are based on the works in [4]. We use the same properties of a transformed correlation matrix but in a rather different manner. Indeed, in [4], the number of observations is assumed to be greater than or equal to the number of source signals and the algorithm is based on the optimization of a given contrast function.

It is pointed out in [1] that the contrast function to be maximized cannot in general be estimated consistently if the cyclic frequencies of the second order statistics of the observations are unknown. However, it was showed in [2][3] that if the second order statistics of various sources signals do not share the same cyclic frequencies, then their knowledge is not required.

The main purpose of this communication is to show that the mixing matrix can be estimated in a wide context. Indeed we consider the case where the number of observations is greater than or equal to two and where the (second order) cyclic frequencies of source signals are unknown. Notice that in our approach, it is assumed that it exists at least one different cyclic frequency for all source signals. Thus the under determined case can be considered. This is carried out in fully exploiting the particular structure of the correlation matrix after the application of a linear transform thanks to an automatic hierarchical ascendent classification method [5][6]. Computer simulations illustrate the effectiveness of the proposed approach.

II. PROBLEM FORMULATION AND MATRIX DECOMPOSITION

The classical linear memoryless mixture model is considered. It reads

$$\mathbf{x}(t) = \mathbf{M}\mathbf{s}(t) \tag{1}$$

where $\mathbf{x}(t)$ is the $(M, 1)$ vector of observation signals, $\mathbf{s}(t)$ the $(N, 1)$ vector of source signals and $\mathbf{M}$ the (M, N) mixing matrix assumed full rank. We assume that M and N belongs to $\mathbb{N} \setminus \{0, 1\}$.

The source signals are assumed cyclostationary. Hence their autocorrelation functions $R_{s_i}(t, \tau) = \mathsf{E}\{s_i(t)s_i^*(t - \tau)\}$, $i = 1, \ldots, N$ are thus periodic in t with periods T_i, $i = 1, \ldots, N$ respectively. $\mathsf{E}\{.\}$ stands for the mathematical expectation operator.

In all the following we assume that the source signals are uncorrelated and that the cyclic periods are different two by two, *i.e.* $T_i \neq T_j$, $\forall i, j$, $i \neq j$. We define the set $\mathcal{V}_i$ of cyclic frequencies of the i-th source signal as

$$\mathcal{V}_i = \left\{ \nu_{i,k} = \frac{k}{T_i}, k \in \mathbb{Z} \right\} .$$

The correlation matrix $\mathbf{R}_x(t, \tau)$ of $\mathbf{x}(t)$ is defined as

$$\mathbf{R}_x(t, \tau) = \mathsf{E}\{\mathbf{x}(t)\mathbf{x}^H(t - \tau)\} \quad (2)$$

where $(\cdot)^H$ stands for the conjugate transpose operator. Using (1), it is easy to see that the correlation matrix in (2) admits the following decomposition

$$\mathbf{R}_x(t, \tau) = \mathbf{M}\mathbf{R}_s(t, \tau)\mathbf{M}^H \quad (3)$$

where $\mathbf{R}_s(t, \tau)$ is the correlation matrix of the source signals. Let us now define the following linear operator

$$\mathbf{R}_x^{sf}(\nu, \tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \mathbf{R}_x(t, \tau) \exp(-2i\pi\nu t) dt \quad (4)$$

which operates on the matrix argument component wise. Since this operator is linear, using (3) in (4), we directly have

$$\mathbf{R}_x^{sf}(\nu, \tau) = \mathbf{M}\mathbf{R}_s^{sf}(\nu, \tau)\mathbf{M}^H \quad (5)$$

where $\mathbf{R}_s^{sf}(\nu, \tau)$ is defined similarly to $\mathbf{R}_x^{sf}(\nu, \tau)$ in (4).

Since source signals are uncorrelated, the matrix $\mathbf{R}_s(t, \tau)$ is diagonal for all t and τ. Thus it is also the case of matrix $\mathbf{R}_s^{sf}(\nu, \tau)$ for all ν and τ. Now, using the fact that the source signals have distinct cyclic periods then there exists values of ν for which $\mathbf{R}_s^{sf}(\nu, \tau)$ has a particular structure. Indeed, as $R_{s_i}^{sf}(\frac{1}{T_j}, \tau) = 0$ for all i, j such that $i \neq j$ then

$$\mathbf{R}_x^{sf}(\frac{1}{T_i}, \tau) = R_{s_i}^{sf}(\frac{1}{T_i}, \tau)\mathbf{m}_i\mathbf{m}_i^H \quad (6)$$

where $\mathbf{m}_i$ is the i-th column vector of matrix $\mathbf{M}$. That is $\mathbf{R}_x^{sf}(\frac{1}{T_i}, \tau)$ is rank one for all i.

III. PROPOSED APPROACH

Based on (6), for all values of τ such that $R_{s_i}^{sf}(\frac{1}{T_i}, \tau) \neq 0$ the i-th column vector $\mathbf{m}_i$ of matrix $\mathbf{M}$ can be estimated (up to the multiplication by a scalar coefficient) as the eigenvector of $\mathbf{R}_x^{sf}(\frac{1}{T_i}, \tau)$ associated with the largest eigenvalue. As classically considered, we performed it in using a Singular Value Decomposition (SVD) procedure. Notice that the above fact holds for all values of τ (such that $R_{s_i}^{sf}(\frac{1}{T_i}, \tau) \neq 0$) and for all values of ν keeping property (6).

The above procedure requires the knowledge of the corresponding cyclic frequencies. Let us now consider the case where they are unknown. For that we propose to calculate matrices $\mathbf{R}_x^{sf}(\nu, \tau)$ for a sufficient large set of frequency points. Then we retains all matrices that are rank one. Notice that in practice, a threshold is necessary for the rank one decision procedure. Now, for each of these matrices, the eigenvector associated to the largest eigenvalue corresponds to one column of the mixing matrix. Notice again that the considered frequencies set is assumed sufficient to yield all columns of the mixing matrix.

However a problem subsists. Indeed if we know that a selected frequency is assigned to a particular column vector, we do not know which one it is. Moreover, because matrices are "only" estimated, different vectors estimating one column are very likely not collinear. Thus there is a certain dispersion around their theoretical value. Hence we propose the use of a classification procedure in order to solve the above problem. One classical way is to compute the inertia centers of each clusters of points by merging to the nearest point in order to finally obtain one single point. This automatic hierarchical ascendent classification method used in [5] is known under the name of unweighed pair-group method of aggregation using arithmetic averages [6]. Among all the found inertia centers, only the N ones corresponding to the highest weights are selected. The N associated vectors enable to estimate the N columns of the mixing matrix, up to their order which corresponds to the classical permutation indetermination in the blind separation source problem. We can finally remark that such an approach could also make it possible to estimate the number of sources and the cyclic frequencies when they are assumed unknown. The number of source signals would correspond to the number of inertia centers having a sufficient weight. The cyclic frequencies can be estimated from (non zero) rank one matrices $\mathbf{R}_x^{sf}(\nu, \tau)$ for different values of ν.

IV. COMPUTER SIMULATIONS

The above proposed approach is now illustrated thanks to computer simulations. We consider three mixing matrices. The first one

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0.65 \\ 0.4 & 1 \end{bmatrix}$$

corresponds to a square mixture ($M = 2, N = 2$), the second one

$$\mathbf{M}_2 = \begin{bmatrix} 1 & 0.65 & 0.4 \\ 0.4 & 1 & 0.8 \\ 0.7 & 0.5 & 1 \end{bmatrix} .$$

corresponds to a square mixture ($M = 3, N = 3$), and the third one

$$\mathbf{M}_3 = \begin{bmatrix} 1 & 0.65 & 1 \\ 0.4 & 1 & 0.8 \end{bmatrix} .$$

corresponds to an under-determined mixture, ($M = 2, N = 3$). For simplicity, we consider discrete time source signals described by the model

$$s(n) = \sum_{k \in \mathbb{Z}} a(k)h(n - kT)$$

where $a(n)$ is an i.i.d. zero-mean random sequence referred to as the transmitted symbols, T is an integer related to the period symbol and $h(n)$ is a deterministic waveform signal. In all cases, $a(n)$ is chosen to take values in the set $\{-1, 1\}$ with equal probabilities. The waveform $h(n)$ is chosen triangular defined for an even cyclic period as: $h(n) = \frac{2}{T}n$ if $0 \leq n \leq \frac{T}{2}$; $h(n) = -\frac{2}{T}n + 2$ if $\frac{T}{2} + 1 \leq n \leq T - 1$ and $h(n) = 0$ otherwise. For $N = 2$ (mixing matrix $\mathbf{M}_1$) the cyclic period of the two considered source signals are $T_1 = 4$ and $T_2 = 10$. For $N = 3$ (mixing matrices $\mathbf{M}_2$ and $\mathbf{M}_3$), the cyclic period of the three considered source signals are $T_1 = 4$, $T_2 = 10$ and $T_3 = 6$. In all cases, the mean square errors (MSE) of estimated sources and/or of estimated mixing matrix columns have been evaluated over 100 Monte Carlo runs.

A priori known cyclic periods: We estimate the two (resp. three) columns vector of square mixing matrix $\mathbf{M}_1$ (resp. $\mathbf{M}_2$). We use an SVD decomposition of $\mathbf{R}_x^{sf}(\frac{1}{T_1}, \tau)$ and $\mathbf{R}_x^{sf}(\frac{1}{T_2}, \tau)$ (resp. $\mathbf{R}_x^{sf}(\frac{1}{T_1}, \tau)$, $\mathbf{R}_x^{sf}(\frac{1}{T_2}, \tau)$ and $\mathbf{R}_x^{sf}(\frac{1}{T_3}, \tau)$ $\forall \tau$).

In figure (1) (resp. (2)), we both plot the MSE of estimated columns of mixing matrix $\mathbf{M}_1$ (resp. $\mathbf{M}_2$) and of estimated source signals versus the number of used samples for $\tau = 0$ and $\tau = 1$. In the under-determined experiment when the cyclic frequencies are supposed known. By using $\mathbf{R}_x^{sf}(\frac{1}{T_1}, \tau)$, $\mathbf{R}_x^{sf}(\frac{1}{T_2}, \tau)$ and $\mathbf{R}_x^{sf}(\frac{1}{T_3}, \tau)$ we estimate the three column vectors of $\mathbf{M}_3$. In figure (3), we plot the MSE of estimated columns versus the number of used samples for $\tau = 0$ and $\tau = 1$. As classically observed a good estimation performance is obtained when the number of samples is large enough.

A priori unknown cyclic periods: First we select a set of frequencies ν where $\mathbf{R}_x^{sf}(\nu, \tau)$ are rank one. Then thanks to the classification procedure we choose two (resp. three) frequencies points for the square mixture $\mathbf{M}_1$ (resp. the square mixture $\mathbf{M}_2$ and the under-determined mixture $\mathbf{M}_3$) which are inertia centers of each clusters. In figure (4) (resp. (5) and (6)) we have plotted the estimated columns of mixing matrix $\mathbf{M}_1$ (resp. $\mathbf{M}_2$ and $\mathbf{M}_3$) before and after the classification method for $\tau = 0$.

We both plot in figure (7) (resp. (8)), the MSE of estimated columns and of estimated source signals of mixing matrix $\mathbf{M}_1$ (resp. $\mathbf{M}_2$) versus the number of used samples. And we plot in figure (9), the MSE of estimated columns versus the number of used samples in the under-determined mixture case. We also observe that the performances are better when the number of samples is large, that is rather classical. We notice some disturbance both due to the threshold choice in the rank one decision procedure and to the choice of the initial set of frequencies.

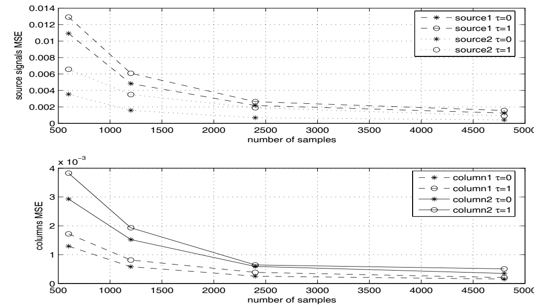

Fig. 1. Sources signal MSE and columns MSE of mixing matrix $\mathbf{M}_1$ versus number of samples for $\tau = 0$ and $\tau = 1$.

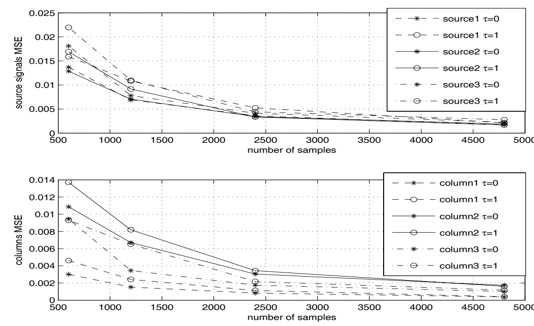

Fig. 2. Sources signal MSE and columns MSE of mixing matrix $\mathbf{M}_2$ versus number of samples for $\tau = 0$ and $\tau = 1$.

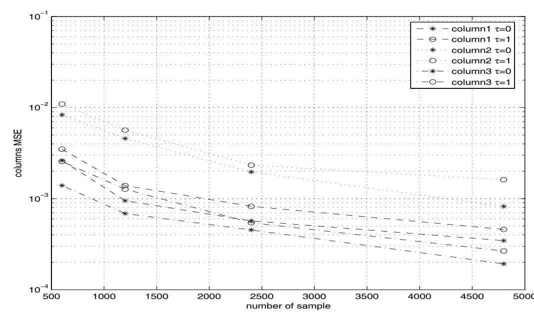

Fig. 3. Columns MSE of mixing matrix $\mathbf{M}_3$ versus number of sample for $\tau = 0$ and for $\tau = 1$.

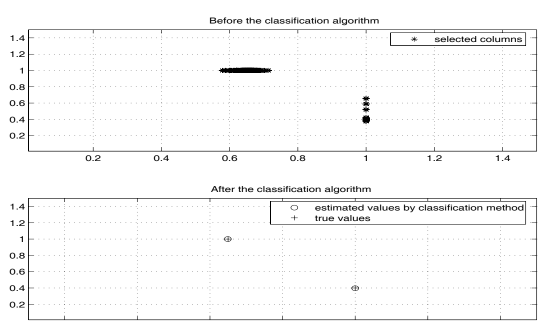

Fig. 4. The estimated square mixing $\mathbf{M}_1$ matrix columns using a classification procedure for $\tau = 0$.

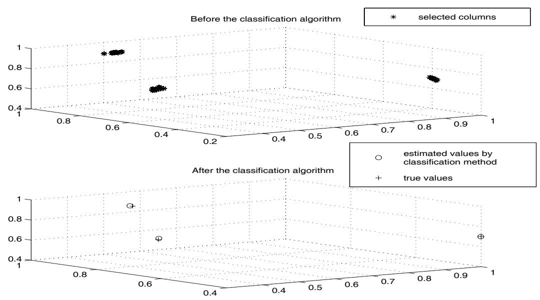

Fig. 5. The estimated square mixing $\mathbf{M}_2$ matrix columns using a classification procedure for $\tau = 0$.

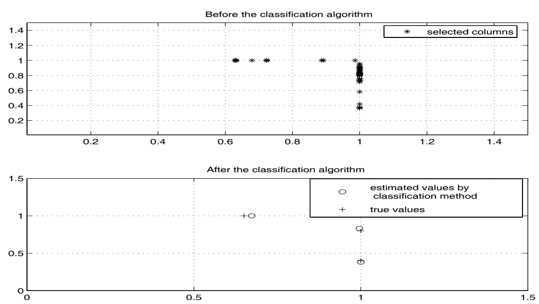

Fig. 6. The estimated mixing $\mathbf{M}_3$ matrix columns in the under-determined case using a classification procedure for $\tau = 0$.

V. REFERENCES

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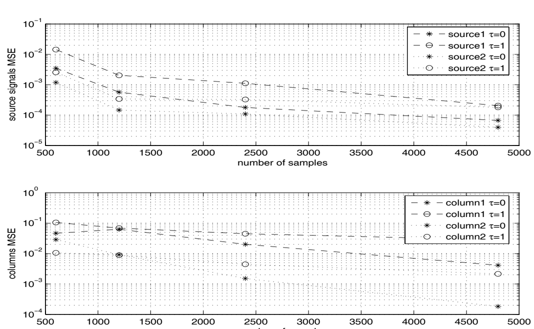

Fig. 7. Sources signal MSE and columns MSE of mixing matrix $\mathbf{M}_1$ versus number of samples for $\tau = 0$ and $\tau = 1$.

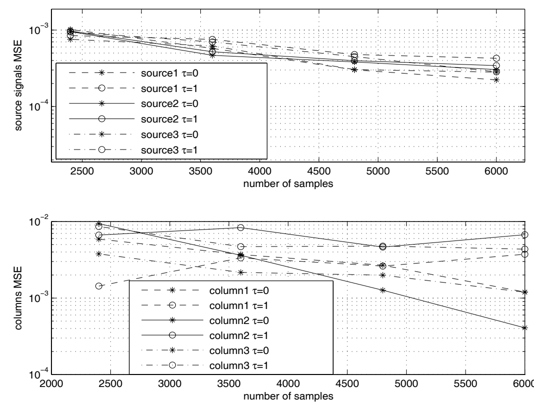

Fig. 8. Sources signal MSE and columns MSE of mixing matrix $\mathbf{M}_2$ versus number of samples for $\tau = 0$ and $\tau = 1$.

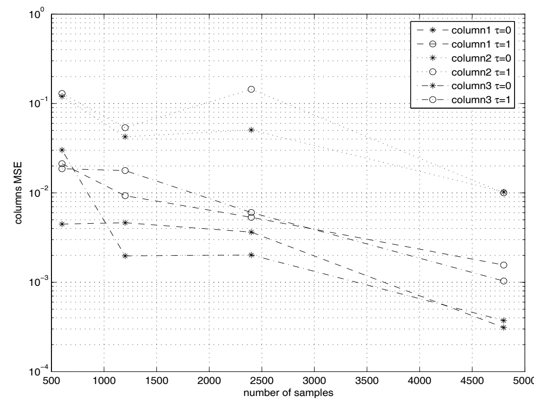

Fig. 9. Columns MSE of mixing matrix $\mathbf{M}_3$ versus number of samples for $\tau = 0$ and $\tau = 1$.