EXTENDED DIFFERENTIAL UNITARY SPACE-TIME MODULATION: A NON-COHERENT SCHEME WITH ERROR PENALTY LESS THAN 3DB

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ABSTRACT

In this paper we propose an extended differential unitary space-time modulation (xDUSTM) scheme that can offer improved error performance over the differential unitary space-time modulation (DUSTM) scheme. DUSTM is well suited to rapidly time-varying unknown channels. It has a simple structure, but incurs an error performance penalty of about 3dB compared to its coherent counterpart. The xDUSTM scheme considers moderately fast time-varying channels, and is designated to exploit such a characteristic for performance improvement. In xDUSTM, a problem that needs to be addressed is the complexity of its non-coherent maximum-likelihood (ML) receiver. We show that by choosing the orthogonal space-time block code (OSTBC) designs, the ML problem can be reduced to a Boolean quadratic program for which highly effective algorithms are available. Simulation results illustrate that the error performance penalty in xDUSTM can be reduced to 1dB.

1. INTRODUCTION

In wireless communication over a fast time-varying fading channel, it is not always affordable to employ coherent demodulation (or detection) methods because pilot signals have to be regularly re-transmitted to keep track of the channel state information (CSI). In those situations it may be appropriate to consider non-coherent communications [1,2], which assumes no knowledge of CSI at the receiver and transmitter. In single-antenna systems, a well-known non-coherent scheme is the differential phase shift keying (DPSK). The DPSK scheme enjoys its simplicity in the encoding and detection processes, with a reasonable error performance penalty of (approximately) 3dB compared to its coherent counterpart.

In multi-antenna communications, a scheme suitable for rapidly time-varying channels is the differential unitary space-time modulation (DUSTM) [3, 4]. Being a multiple-input-multiple-output (MIMO) generalization of DPSK, DUSTM is based on the reasonable assumption that the channel coefficients can only remain constant over 2 consecutive space-time code blocks. The potential of DUSTM in non-coherent MIMO communications has attracted considerable attention, leading to a variety of space-time unitary code designs for DUSTM. Some representative designs include (but are not restricted to) the diagonal group codes [3, 4], the Cayley codes [5], and the orthogonal space-time block codes (OSTBCs) [6–8]. Each design has its own tradeoff in data rate and diversity performance. Some discussions comparing the various designs can be found in [5, 8]. An important issue in DUSTM is the receiver complexity. Specifically, the non-coherent maximum-likelihood (ML) detector for DUSTM can be complex to use if the size of the space-time code constellation is large. Some ML approximations, which are often code dependent [5, 9], may be required to alleviate the complexity. From a receiver viewpoint, a good unitary code is that based on the OSTBC designs. The OSTBC designs are simple in that the code matrix is a linear combination of some basis matrices. Using this linear property, it is shown that the non-coherent ML detector can be reduced to a simple linear process without approximation [7, 8].

While DUSTM is suitable for rapidly time-varying channels, it incurs an error performance penalty of 3dB compared to its coherent counterpart (which is like the case in DPSK). In this paper we are interested in moderately fast fading channels, where the channel coefficients can remain constant over a frame of multiple code blocks. In practice there are many situations where the channel is static for a moderate duration of, say, 10 to 20 symbols. We propose an *extended DUSTM* (xDUSTM) scheme that exploits this frame static channel property to provide improved error performance. In this new scheme, the differential transmission process is done on a frame-by-frame basis. At the receiver side, the non-coherent detection of the codes is performed for each frame too. Section 3 will describe the details of xDUSTM, and will give arguments on why this scheme is expected to provide enhanced performance. The difficulty in using xDUSTM lies in the implementation of the non-coherent ML detector, which is much more complex than that of DUSTM. To handle this problem, we suggest to employ the OSTBC designs with a quaternary phase shift keying (QPSK) symbol constellation. It is shown that the associated ML detector can be simplified to a Boolean quadratic program (BQP). The BQP is still hard to solve from a complexity theory viewpoint, but in practice there are very effective algorithms for the BQP, such as the sphere decoder [10] and semidefinite relaxation (SDR) [11,12]. Some numerical results showing the fidelity of the sphere decoder and SDR algorithm can be found in [10, 12]. By using either one of these algorithms, the ML implementation can be handled in a computationally efficient manner. Our simulation results, given in Section 4 will show that the error performance of the OSTBC xDUSTM method improves as the frame size increases. In particular, for a frame of 18 symbol intervals, the error performance penalty of OSTBC xDUSTM can be reduced to 1dB.

2. REVIEW OF DUSTM

This section reviews several key ideas of DUSTM.

We consider a scenario where space-time block codes (STBCs) are transmitted over a frequent-flat, time-varying MIMO channel. Let M_t and M_r be the numbers of transmitter and receiver antennas, respectively. Moreover, let T denote the time length of the STBCs. A usual assumption in such a scenario is that the channel coherence interval[1] is larger than T, such that the channel coefficients are (almost) constant during the transmission of a single STBC. The received signal model is given by

$$\mathbf{Y}_p = \mathbf{H}_p\mathbf{C}_p + \mathbf{V}_p \tag{1}$$

$\mathbf{C}_p \in \mathbb{C}^{M_t\times T}$	STBC transmitted at pth time block
$\mathbf{Y}_p \in \mathbb{C}^{M_r\times T}$	received signal matrix at pth time block
$\mathbf{H}_p \in \mathbb{C}^{M_r\times M_t}$	MIMO channel at pth time block
$\mathbf{V}_p \in \mathbb{C}^{M_r\times T}$	independent and identically distributed (i.i.d.) circular Gaussian noise matrix with zero mean and variance $\mathcal{N}_o$.

In DUSTM, a key assumption is that the channel coherence interval is larger than $2T$ such that

$$\mathbf{H}_{p-1} \simeq \mathbf{H}_p \tag{2}$$

for any p. Moreover, $\mathbf{C}_p$ are assumed to be square; i.e., $T = M_t$. At each time block index p, a block of information bits is mapped to a signal matrix $\mathbf{U}_p \in \mathcal{U} \subset \mathbb{C}^{T\times T}$, where $\mathcal{U}$ denotes the constellation set of $\mathbf{U}_p$. The codewords in $\mathcal{U}$ are restricted to be unitary; i.e., $\mathbf{U}^H\mathbf{U} = \mathbf{I}$ for all $\mathbf{U} \in \mathcal{U}$. The signal matrices $\mathbf{U}_p$ are differentially encoded, via the following fundamental transmission equation:

$$\mathbf{C}_p = \mathbf{C}_{p-1}\mathbf{U}_p, \quad p = 1, 2, \ldots, \tag{3}$$

with $\mathbf{C}_0$ initialized to be an arbitrary unitary matrix, assumed to be unknown to the receiver.

For simplicity but without loss of generality, let us consider $\mathbf{Y}_0$ and $\mathbf{Y}_1$ only. Let

$$\tilde{\mathbf{H}} = \mathbf{H}_0\mathbf{C}_0. \tag{4}$$

Note that $\tilde{\mathbf{H}} = \mathbf{H}_1\mathbf{C}_0$, owing to the quasi-static channel property in (2). The following received signal model is obtained:

$$\mathbf{Y}_0 = \tilde{\mathbf{H}} + \mathbf{V}_0 \tag{5a}$$

$$\mathbf{Y}_1 = \tilde{\mathbf{H}}\mathbf{U}_1 + \mathbf{V}_1 \tag{5b}$$

It is interesting to point out that the formulation in (5) is equivalent to that of a pilot-assisted system, where a pilot code $\mathbf{I}$ is transmitted in (5a). If $\tilde{\mathbf{H}}$ is treated as a deterministic unknown, then the ML detection of $\mathbf{U}_1$ for the model in (5) is a joint optimization problem given by

$$\{\hat{\mathbf{H}}, \hat{\mathbf{U}}_1\} = \arg \min_{\substack{\tilde{\mathbf{H}}\in\mathbb{C}^{M_r\times M_t},\\ \mathbf{U}_1\in\mathcal{U}}} \|\mathbf{Y}_0 - \tilde{\mathbf{H}}\|_F^2 + \|\mathbf{Y}_1 - \tilde{\mathbf{H}}\mathbf{U}_1\|_F^2 \tag{6}$$

where $\hat{\mathbf{H}}$ and $\hat{\mathbf{U}}_1$ denote the ML solution of $\tilde{\mathbf{H}}$ and $\mathbf{U}_1$, respectively, and $\|.\|_F$ is the Frobenius norm. By exploiting the unitarity of $\mathbf{U}_p$, it is shown [13] that (6) can be reduced to

$$\hat{\mathbf{U}}_1 = \arg \max_{\mathbf{U}_1\in\mathcal{U}} \|[\, \mathbf{Y}_0 \; \mathbf{Y}_1 \,][\, \mathbf{I} \; \mathbf{U}_1 \,]^H\|_F^2 \tag{7}$$

[1] Here, the channel coherence interval is defined to be the maximum number of consecutive time samples that the fading coefficients remain (approximately) static.

where $\hat{\mathbf{U}}_1$ can be obtained alone without $\hat{\mathbf{H}}$. Problem (7) can be further simplified to

$$\hat{\mathbf{U}}_1 = \arg \max_{\mathbf{U}_1\in\mathcal{U}} \mathrm{Re}\{\mathrm{tr}\{\mathbf{Y}_0\mathbf{U}_1\mathbf{Y}_1^H\}\} \tag{8}$$

The ML detector illustrated above is known as the deterministic ML detector in the blind detection context. It is worthwhile to mention that in the DUSTM literature, there are several other alternate ML formulations following different assumptions; e.g., the 'virtually coherent' ML detector [7, 8] which treat $\tilde{\mathbf{H}}$ as if it were the true channel, and the stochastic ML detector [1, 3, 4] which uses the assumption of i.i.d. zero-mean circular Gaussian $\tilde{\mathbf{H}}$. Interestingly all these ML detectors are equivalent in that they solve (7) or (8).

As mentioned in the last section, there are various designs for the unitary signal constellation. This work focuses on the orthogonal space-time block code (OSTBC) designs, with an emphasis on the QPSK symbol constellation. Given a block of information bits $\mathbf{s} = [\, s_1, \ldots, s_K \,]^T \in \{\pm 1\}^K$ where K is the number of bits per block, a QPSK OSTBC function $\mathbf{Q} : \mathbb{R}^K \to \mathbb{C}^{M_t\times T}$ is a linear combination of matrices

$$\mathbf{Q}(\mathbf{s}) = \frac{1}{\sqrt{K}}\left(\sum_{k=1}^{K/2}\mathbf{A}_k s_k + j\sum_{k=1}^{K/2}\mathbf{B} s_{k+K/2}\right), \tag{9}$$

where $\mathbf{A}_k, \mathbf{B}_k \in \mathbb{R}^{M_t\times T}$ are known matrices. In the above formulation s_k and $s_{k+K/2}$ serves as real and imaginary parts of a QPSK symbol, respectively. In the OSTBC designs $\mathbf{A}_k$ and $\mathbf{B}_k$ are carefully chosen such that $\mathbf{Q}(\mathbf{s})\mathbf{Q}^H(\mathbf{s}) = (\|\mathbf{s}\|^2/K)\mathbf{I} = \mathbf{I}$; see [7, 8] and the references therein. It is convenient to rewrite (9) as:

$$\mathbf{Q}(\mathbf{s}) = \sum_{k=1}^{K}\mathbf{X}_k s_k \tag{10}$$

where $\mathbf{X}_k \in \mathbb{C}^{M_t\times T}$. In DUSTM via OSTBCs, each signal matrix $\mathbf{U}_p$ is a square OSTBC

$$\mathbf{U}_p = \mathbf{Q}(\mathbf{s}_p) \tag{11}$$

where $\mathbf{s}_p = [\, s_{1p}, \ldots, s_{Kp} \,]^T \in \{\pm 1\}^K$ is the vector of transmitted bits at the p time block.

The ML receiver structure of OSTBC DUSTM is attractive compared to that of the other nonlinear DUSTM methods. For a generic unitary constellation, we need to perform an exhaustive search over $\mathcal{U}$ to find the optimal decision in (8). To reduce computational complexity, some approximations may be required [5, 9]. For OSTBC DUSTM, it can be shown [7, 8] from (8) and (10) that the ML detection of $\mathbf{s}_1$ is the simple process $\hat{s}_{k1} = \mathrm{sign}(\mathrm{Re}\{\mathrm{tr}\{\mathbf{Y}_0\mathbf{X}_k\mathbf{Y}_1^H\}\})$ for $k = 1, \ldots, K$.

The above review shows that DUSTM is simple to use and well suited to rapidly time-varying channels. As a tradeoff for these advantages, DUSTM suffers from an error performance penalty of 3dB compared to its coherent counterpart [4]. In the next section, we propose an extended DUSTM scheme that can reduce this penalty effectively.

3. EXTENDED DUSTM

The extended DUSTM (xDUSTM) scheme proposed here is based on the assumption of moderately fast fading channels. Specifically we assume that the the channel coherence interval is no smaller than $(P+1)T$ for some integer $P \geq 1$, so that

$$\mathbf{H}_{p-1} \simeq \mathbf{H}_p \simeq \mathbf{H}_{p+1} \simeq \ldots \simeq \mathbf{H}_{p+P} \tag{12}$$

for $p = 1, 2, \ldots$. Our aim is to exploit the quasi-static channel property in (12) to improve error performance. The differential encoding process of xDUSTM is a 'length extended' version of DUSTM, and it is given as follows: For $\ell = 1, 2, \ldots$,

$$\mathbf{C}_{1+(\ell-1)P} = \mathbf{C}_{(\ell-1)P}\mathbf{U}_{1+(\ell-1)P}, \tag{13a}$$
$$\mathbf{C}_{2+(\ell-1)P} = \mathbf{C}_{(\ell-1)P}\mathbf{U}_{2+(\ell-1)P}, \tag{13b}$$
$$\vdots$$
$$\mathbf{C}_{\ell P} = \mathbf{C}_{(\ell-1)P}\mathbf{U}_{\ell P}, \tag{13c}$$

with $\mathbf{C}_0$ initialized to be any unitary matrix unknown to the receiver. We call ℓ the frame index, and P the frame size. Comparing the fundamental transmission equations of the two schemes [c.f., (3) and (13)], we see that the xDUSTM scheme performs differential encoding on a frame-by-frame basis.

Again, for simplicity but without loss of generality, consider the received signal frame $[\,\mathbf{Y}_0,\ \mathbf{Y}_1, \ldots, \mathbf{Y}_P\,]$. Let $\tilde{\mathbf{H}} = \mathbf{H}_0\mathbf{C}_0$, which equals $\tilde{\mathbf{H}} = \mathbf{H}_p\mathbf{C}_0$ for $p = 1, \ldots, P$ owing to (12). Then the following signal model can be obtained:

$$\mathbf{Y}_0 = \tilde{\mathbf{H}} + \mathbf{V}_0 \tag{14a}$$
$$\mathbf{Y}_p = \tilde{\mathbf{H}}\mathbf{U}_p + \mathbf{V}_p, \qquad p = 1, \ldots, P \tag{14b}$$

Following the deterministic blind ML treatment described in Section 2, the ML detector of $\mathbf{U}_1, \ldots, \mathbf{U}_P$ for the model in (14) is

$$\{\hat{\mathbf{H}}, \hat{\mathbf{U}}_1, \ldots, \hat{\mathbf{U}}_P\} = \arg \min_{\substack{\tilde{\mathbf{H}} \in \mathbb{C}^{M_r \times M_t}, \\ \mathbf{U}_p \in \mathcal{U}, \\ p=1,\ldots,P}} \|\mathbf{Y}_0 - \tilde{\mathbf{H}}\|_F^2 + \sum_{p=1}^{P} \|\mathbf{Y}_p - \tilde{\mathbf{H}}\mathbf{U}_p\|_F^2 \tag{15}$$

Problem (15) is technically equivalent to a semi-blind ML detection problem [12, 14], where there are one pilot code block $\mathbf{I}$ followed by a frame of signals $\mathbf{U}_1, \ldots, \mathbf{U}_P$. In the semi-blind detection context, generally it is found that error performance improves as P increases [12, 14]. Hence, we expect that the xDUSTM scheme for $P > 1$ should offer better error performance compared to the conventional DUSTM. This will be shown to be true in the simulation results in Section 4.

Now our goal is to solve the ML problem in (15). Like the treatment described in the last section, Problem (15) can be shown to be equivalent to (see; e.g., [14])

$$\{\hat{\mathbf{U}}_p\}_{p=1}^{P} = \arg \max_{\substack{\mathbf{U}_p \in \mathcal{U}, \\ p=1,\ldots,P}} \left\| \mathbf{Y}_0 + \sum_{p=1}^{P} \mathbf{Y}_p \mathbf{U}_p^H \right\|_F^2. \tag{16}$$

To solve (16) exactly for a generic constellation set $\mathcal{U}$, we may need an exhaustive search which has a complexity proportional to $|\mathcal{U}|^P$ (where $|\mathcal{U}|$ is the cardinality of $\mathcal{U}$). This computational overhead is not affordable even for moderate P. To overcome this hurdle, we consider the QPSK OSTBC constellation; cf., (11) and (10). The objective function of (16) can be decomposed as

$$\left\| \mathbf{Y}_0 + \sum_{p=1}^{P} \mathbf{Y}_p \mathbf{U}_p^H \right\|_F^2 \propto 2\mathrm{Re}\left\{ \mathrm{tr}\left\{ \sum_{p=1}^{P} \mathbf{Y}_0 \mathbf{U}_p \mathbf{Y}_p^H \right\} \right\} + \mathrm{tr}\left\{ \sum_{p=1}^{P} \sum_{q=1}^{P} \mathbf{Y}_p \mathbf{U}_p^H \mathbf{U}_q \mathbf{Y}_q^H \right\} \tag{17}$$

By substituting (11) and (10) into (17), we show that the right hand side of (17) can be simplified to

$$2\mathbf{s}_{1:P}^T \mathbf{f} + \mathbf{s}_{1:P}^T \mathbf{G} \mathbf{s}_{1:P} \tag{18}$$

where $\mathbf{s}_{1:P} = [\,\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_P^T\,]^T \in \{\pm 1\}^{KP}$,

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_P \end{bmatrix} \in \mathbb{R}^{KP}, \tag{19}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \ldots & \mathbf{G}_{1P} \\ \vdots & \ddots & \vdots \\ \mathbf{G}_{P1} & \ldots & \mathbf{G}_{PP} \end{bmatrix} \in \mathbb{R}^{KP \times KP}, \tag{20}$$

$$[\mathbf{f}_p]_k = \mathrm{Re}\{\mathrm{tr}\{\mathbf{Y}_0 \mathbf{X}_k \mathbf{Y}_p^H\}\}, \quad k = 1, \ldots, K \tag{21}$$

$$[\mathbf{G}_{pq}]_{ki} = \mathrm{Re}\{\mathrm{tr}\{\mathbf{Y}_p \mathbf{X}_k^H \mathbf{X}_i \mathbf{Y}_q^H\}\}, \tag{22}$$

for $k, i = 1, \ldots, K$. Therefore, the xDUSTM scheme with QPSK OSTBCs can be reduced to a Boolean quadratic program

$$\hat{\mathbf{s}}_{1:P} = \arg \max_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} 2\mathbf{s}_{1:P}^T \mathbf{f} + \mathbf{s}_{1:P}^T \mathbf{G} \mathbf{s}_{1:P}. \tag{23}$$

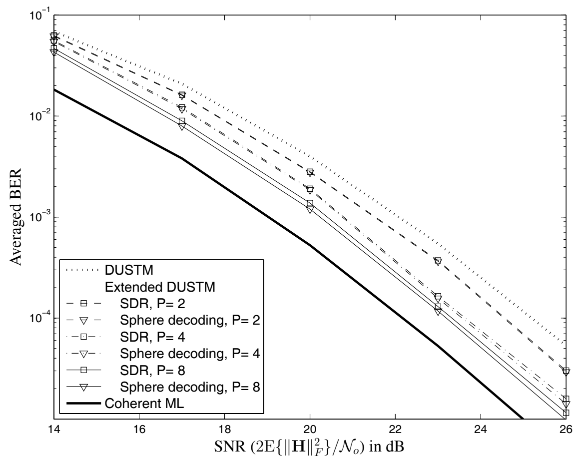
Problem (23) is NP-hard. Fortunately recent advances in MIMO detection have suggested that very effective algorithms are available for (23), at least practically. Specifically one can employ a sphere decoding algorithm [10] to exactly solve (23), the complexity of which can be very attractive in the average sense. Alternatively, suboptimal but computationally efficient algorithms for (23) can be used. One such algorithm that is worth mentioning is semidefinite relaxation (SDR) [11]. It has been illustrated that the SDR algorithm provides near-ML performance with a polynomial-time worst-case complexity (Note that exact solvers such as sphere decoding do not guarantee polynomial-time worst-case complexity). In this work, the sphere decoding and SDR alternatives are both considered. For more details about the comparison of the two alternatives, please read [12]. The application of SDR to (23) is done by following the procedure in [11, 12]. As for sphere decoding, the procedure is to first reformulate (23) as an integer least squares problem, and then apply the sphere decoder to the reformulated problem. The reformulation step can be found in [12].

It is straightforward that for each frame index ℓ, the non-coherent detection of $\mathbf{s}_{1+(\ell-1)P}, \ldots, \mathbf{s}_{\ell P}$ is done by applying the aforementioned process to $[\,\mathbf{Y}_{(\ell-1)P}, \ldots, \mathbf{Y}_{\ell P}\,]$.

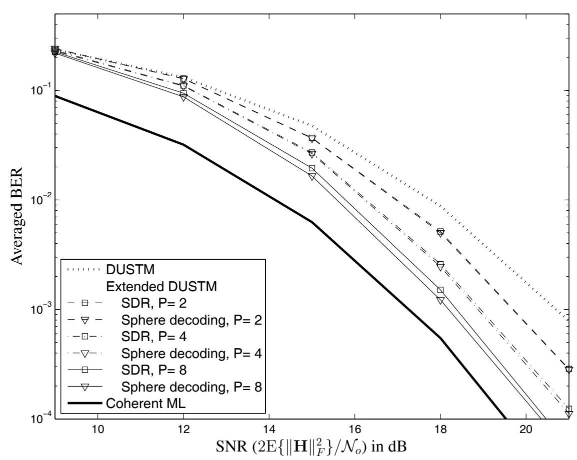
4. SIMULATION RESULTS

In this simulation example, the code function $\mathbf{C}(.)$ is chosen to be the Alamouti space-time code [7,8] for which the number of transmitter antennas is $M_t = 2$. The MIMO channel coefficients are zero-mean, i.i.d. Gaussian distributed. Figs. 1(a) and (b) show the average bit error performance of the OSTBC xDUSTM scheme for various values of the frame size P. The numbers of receiver antennas in Figs. 1(a) and (b) are $M_r = 2$ and $M_r = 4$, respectively. The error rates of the conventional OSTBC DUSTM and coherent OSTBC schemes are also plotted in the same figures. The figure indicates that the error performance of xDUSTM improves with P, which confirms our expectation discussed in the last section. Moreover, for $P = 8$ (which is equivalent to 16 symbols), the error performance of xDUSTM is only about 1dB from that of the coherent OSTBC scheme.

Figs. 1(a) and (b) also illustrate that the two ML implementations, namely the sphere decoder and the SDR algorithm, provide very similar error performance.



(a)



(b)

Fig. 1. BER performance for various frame sizes P. (a) $M_r = 2$; and (b) $M_r = 4$.

5. CONCLUSION AND DISCUSSION

In this paper, an xDUSTM scheme has been proposed to reduce the error performance penalty due to unknown channel state information. It does so by taking advantage of situations where the channel coherence interval can be larger than 2 code blocks. The main problem with using xDUSTM, namely that of the receiver complexity has been addressed. Specifically we suggest to adopt the structurally simple QPSK OSTBC designs at the transmitter side, and to apply either the sphere decoder or SDR algorithm at the receiver side. It is shown by simulations that xDUSTM can offer better error performance than DUSTM, particularly when the channel coherence interval is moderate.

We should add that this work was, in some way, inspired by our semi-blind ML detection idea in [12].

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