# CONSTRAINED LEAST SQUARES ESTIMATION FOR POSITION TRACKING

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## ABSTRACT

This paper describes an effective method for mobile location tracking and velocity estimation in a network-based wireless localization system. We propose a constrained least squares estimation algorithm for real-time tracking of the location and dynamic motion of a mobile user using TOA measurements. The tracking problem is formulated in a state-space framework and the constraints on system states are considered explicitly. Simulation results show that the proposed tracking algorithm can improve the accuracy through eliminating spurious position estimates.

## 1. INTRODUCTION

Over the past few years, mobile positioning systems have received great attention in both research and industry communities [1-3]. The applications of mobile positioning in wireless systems become absolutely essential as they play a key role in providing location based billing, intelligent transportation information and wireless emergency services (E-911). As a matter of fact, it will be useful to be able to trace the mobility of a user forgetting about the privacy issue. The problem itself also generates many technical challenges. If the location as well as the movement of a mobile is known, additional services can be offered to subscribers. Especially in hierarchical cellular mobile communication networks, based on the mobile speed and moving direction, the information of mobility becomes of great assistance in efficient network resource management and in emergency situation, allowing real-time monitoring. These requirements have become one of the major driving forces of research activities on position tracking technologies.

Mobile positioning involves a variety of technologies. The existing global positioning system (GPS) can provide a fairly reliable solution for localization. However, employing the GPS for mobile positioning would mean addition of hardware in the mobile station (MS), which is not a cost effective approach. Recently, many methods have been focused on utilizing the base station (BS) to locate the MS. The common location approaches are based on time-of-arrival (TOA), received signal strength (RSS), time-difference-of-arrival (TDOA), or angel-of-arrival (AOA).

To trace mobile locations, a variety of methods using signal strength, TOA or TDOA method were studied [4-8]. [4] proposed a tracking technique based on piecewise linear optimization to reduce the variation of location estimation. [5] developed a sequential-monte carlo method based on the auxiliary particle filter to perform speaker tracking using TDOA measurements. But the mobile velocity was not included in these tracking results. In [6-7], methods of position location and velocity estimation with signal strength measurement and TOA measurements were suggested. By using a linear recursive model of mobility and by smoothing via Kalman filter, an accurate estimated track can be achieved. A Kalman tracking method based on TDOA measurements for UMTS mobile location has been demonstrated in [8]. Moreover, in [9] and [10] adaptive schemes based on pattern recognition, hidden Markov models and neural networks methods have been employed to estimate the position of mobiles. However, all these methods ignore the constraints on the movement of mobile users. In practical situation, there exist many constraints on the movement, such as, the limitation on speed and geographical blockages. Typically, when a person is walking, their speed is limited to below 2m/s. So, the range and path of a mobile user are always confused by the environment, especially when the user is indoor or there are many obstacles around. Therefore, considering the constraints on movement can eliminate spurious position estimates and improve the accuracy of estimation. In this paper we propose a constrained estimation algorithm in the state-pace framework to deal with these system constraints.

The rest of the paper is organized as follows. Section 2 formulates a dynamic nonlinear mobility model on which our mobility tracing algorithm is based. Section 3 presents the constrained least squares estimation method. Section 4 presents numerical results demonstrating the accuracy of the proposed method. Finally, conclusions are summarized in section 5.

## 2. DYNAMIC MOBILITY STATE MODEL

### 2.1. Source model

The problem considered here is that of tracking the position of a particular mobile user in the XY-plane. It should be stated, however, that the methodology can be easily extended to perform a 3D tracking.

In order to develop time-varying movement patterns, we model a moving object as a dynamic linear system. In the following, $'$ denotes the transpose of a vector or a matrix. The mobile object's state at time $t$ is defined by a vector

$$s(t) = [x(t), y(t), \dot{x}(t), \dot{y}(t)]' \qquad (1)$$

where $x(t)$, $y(t)$ denote the $x$ and $y$ coordinates of a mobile's position and $\dot{x}(t)$, $\dot{y}(t)$ the $x$ and $y$ coordinates of the velocity vector at time $t$. Observations are taken at discrete time points $t_k = t_0 + \Delta t \cdot k$, $k \in N_0$. The integer $k$ denotes the discrete-time index, $\Delta t$ denotes the sampling period. The mobile position $x(t_k)$ satisfies the discrete linear recursion

$$s(t_{k+1}) = Fs(t_k) + Gw(t_k), \quad k \in N_0 \qquad (2)$$

where $F$ and $G$ are the following matrices:

$$F = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}$$

$w(t_k) = [w_1(t_k), w_2(t_k)]'$, $k \in N_0$ are the noise related to mobile movement with state noise covariance matrix $Q = \sigma^2 I_2$. $I_l$ denotes the identity matrix of order $l$.

The Random acceleration from time $t_k$ to $t_{k+1}$ is given by

$$A(t_k) = \left\| (w_1(t_k), w_2(t_k))' \right\| = (w_1^2(t_k) + w_2^2(t_k))^{1/2}$$

It is well know that $A(t_k)$ is Rayleigh distributed with parameter $\sigma^2$ and expectation

$$E(A(t_k)) = \sigma\sqrt{\pi / 2} \qquad (3)$$

This allows for estimating $\sigma^2$ from an estimator $\hat{\alpha}$ of $E(A(t_k))$ by setting $\hat{\sigma}^2 = 2\hat{\alpha}^2 / \pi$ [6].

### 2.2. Measurement model

From the viewpoint of geometric location approach, time of arrival (TOA) measures can be used to estimate the position of the mobile object at any time instant. So, we use the time sequence of TOA measures to perform position tracing and velocity estimation for mobile positioning.

It is assumed that all measurements we utilize for mobile location come from line-of-sight (LOS) propagation. Let $s(t_k) = [x(t_k), y(t_k), \dot{x}(t_k), \dot{y}(t_k)]'$ be the mobile user's position and velocity to be determined at discrete time $t_k$. $[a_i, b_i]$ is the known coordinate of the $i$th base station (BS), $i = 1, 2, \ldots M$, where $M$ is the total number of receiving LOS BSs. The distance between the mobile user and the $i$th BS at time $t_k$, which is denoted by $d_i(t_k)$, is given by

$$d_i(t_k) = \sqrt{(x(t_k) - a_i)^2 + (y(t_k) - b_i)^2}, \quad i = 1, 2, \ldots M$$

Obviously, the distance is a nonlinear function of $s(t_k)$.

At time $t_k$, the measured distance vector relating the true distance $d_i(t_k)$, $i = 1, 2, \ldots M$, and the mobile user's position $s(t_k)$ is modeled as

$$Y(t_k) = G(s(t_k)) + v(t_k) \qquad (4)$$

where

$$G_i(s(t_k)) = \sqrt{(s_1(t_k) - a_i)^2 + (s_2(t_k) - b_i)^2}, \; i = 1, 2, \ldots M$$

$v(t_k)$ is the measurement noise related to base stations with measurement noise covariance $R$.

The sate-space model relating the measurement $Y$ and the state $s$ takes as the following form:

$$\begin{aligned} s(t_{k+1}) &= Fs(t_k) + Gw(t_k) \\ Y(t_k) &= G(s(t_k)) + v(t_k) \end{aligned} \qquad (5)$$

## 3. MOBILITY TRACKING ALGORITHM

If a mobile object moves within a short distance, the route could be approximated by a straight line. Although arbitrary location technique could be used to estimate the mobile location, these estimated locations are always not exactly the mobile position, i.e. the X-Y components of the position coordinates are both noisy [7]. Therefore, it is not proper to derive the fitted straight line using linear regression. In this section, a constrained least squares (LS) estimation method is used to track the position of the mobile user.

$$s^*(t_k) = \arg \min_{s(t_k),\, w(t_k)} \left( \left\| s(t_k) - \hat{s}(t_k) \right\|_{P_{t_k}}^2 + \sum_{i=1}^{M} \| Y_i(t_k) - (G_i(s(t_{k-1}) + w(t_k)) \|_{R^{-1}}^2 + \sum_{i=1}^{2} \| w_i(k) \|_{Q^{-1}}^2 \right) \qquad (6\text{-}1)$$

subject to

$$\begin{aligned} s(t_{k+1}) &= Fs(t_k) + Gw(t_k) \\ Y(t_k) &= G(s(t_k)) + v(t_k) \end{aligned} \qquad (6\text{-}2)$$

$$\begin{aligned} -N \le w(t_k) &\le N \\ -L \le s(t_k) &\le L \qquad k = 1, 2, \ldots \end{aligned} \qquad (6\text{-}3)$$

We start our estimation process at time $k=0$ with an estimate of the mobile user position and an initial estimated value $\hat{s}(t_0)$. An initial estimation of the mobile user's position $(\hat{x}(t_0), \hat{y}(t_0))$ is provided by the TOA techniques. At each time step $t_k$, the optimal estimated state is computed by solving the following optimization problem, equation (6). We assume that the matrices $Q$, $R$ and $P_{t_k}$ are symmetric positive definite. They can be chosen as the covariance matrices of disturbance $Q$, $R$ and estimate error $s(t_k)-\hat{s}(t_k)$. The pair $(\hat{s}(t_0), P_{t_0})$ summarizes the prior information at time $t_0$ and is part of the data of the state estimation problem. A reasonable choice for the initial guess of $\hat{s}(t_0)$ is

$$\hat{s}(t_0)=[\hat{x}(t_0), \hat{y}(t_0), 0, 0]'$$

where $(\hat{x}(t_0), \hat{y}(t_0))$ is the first estimated position and the velocity is assumed zero initially. Assume the solution to (6) at time $t_k$ is the unique pair $(s_{ls}(t_k), w(t_k))$. At succeeding time steps we can update the prior information $\hat{s}(t_k)$ using the sate-space model (5), and the matrix $P_{t_k}$ can be obtained by solving the extended Kalman filter covariance equation subject to the initial condition $P_{t_0}$ [11].

Note that the equation (6) is a constrained optimization problem. The optimum solutions should satisfy the system dynamic (6-2) and the system constrains (6-3). Equation (6-3) represents the constraints on the system state $s$ and disturbance $w$. The constraints on the state $s$ are imposed so that the estimated position cannot be outside the known range and the velocity of mobile user is restricted. The constraints on $w$ mean the acceleration varies in a limited range. Conceptually these constraints are reasonable because there is a limit to how far a mobile user can move in a given time interval. Therefore, the constraints on mobile user movement can eliminate the spurious or ridiculous estimates and improve the accuracy of the estimation results. As a matter of fact, if we could have access to the geographical on environment information, for instance, possible blocked in front of the moving path, additional constraints can be built in real time. It will improve the localization accuracy.

## 4. SIMULATION EXAMPLE

In our numerical experiments, random mobile trajectories are generated by Matlab using the dynamic state model given in (5). We consider a relevant area of 6×6 km with 3 base stations. This condition can be considered as constraints on mobile position. The mobile moves from the left to the right margin with speed about 15m/ s. The movement trajectory is depicted in Fig. 1. The sampling interval is set to $\Delta t = 3s$ and the total $k=300$ sample points are obtained within the duration of 15 minutes. For the estimation of $Q=\sigma^2 I_2$, we choose an average acceleration as 0.5m/s$^2$ and the estimated $\hat{\sigma}^2 = 0.4$ is calculated from (4). The covariance of measurement noise $R$ is chosen to be 22500. This corresponds to a standard deviation of approximately 150m per measurement. The initial value of $P_{t_0}$ is $P_{t_0} = diag([R, 15^2 I_2])$. $15^2$ (m/s)$^2$ seems to be a reasonable upper bound for the variance of the initial velocity of a mobile.

The static least squares estimation, extended Kalman filter (EKF) described in [6, 8] and the proposed method are used to estimation the mobile position. Fig. 2 shows the estimation results of the static least squares method. The results show strong irregular variations and many estimation vales overrun the constrained condition, which is unreasonably. Fig. 3 shows estimated trajectories obtained by EKF and the proposed algorithm. The solid line is the actual trajectory, dash line is the EKF estimate results and dot line denotes the constrained LS estimate results. The results demonstrate that EKF has a poor performance, particularly when there are sharp or sudden changes in moving direction. On the other hand, the constrained least squares method can guarantee that all the estimated values will locate in the constrained range.

We use root mean squared error (RMSE) as a figure of merit to compare a given trajectory $\{x_n, y_n\}$ and its estimated trajectory $\{\hat{x}_n, \hat{y}_n\}$

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}[(\hat{x}_n - x_n)^2 + (\hat{y}_n - y_n)^2]}$$

The values of RMSE calculated by the static LS estimation, EKF and our proposed constrained LS method are 352.26, 131.16 and 78.52, respectively. It can be seen that the deviation of the estimation results by the constrained LS method is less than that obtained by EKF method. It is obvious that the proposed position method has the best performance and gives the most satisfactory tracking capability. Fig. 4 shows the estimation results of the velocity.

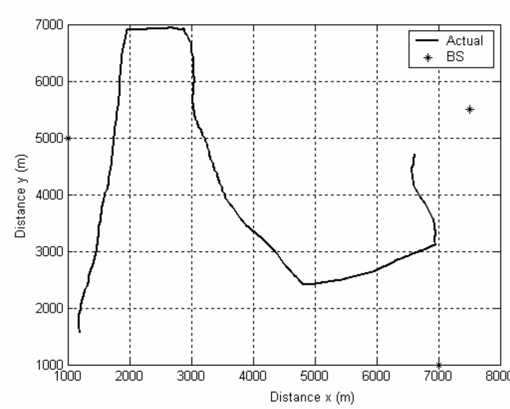


Fig.1. Base stations and the mobile's trace

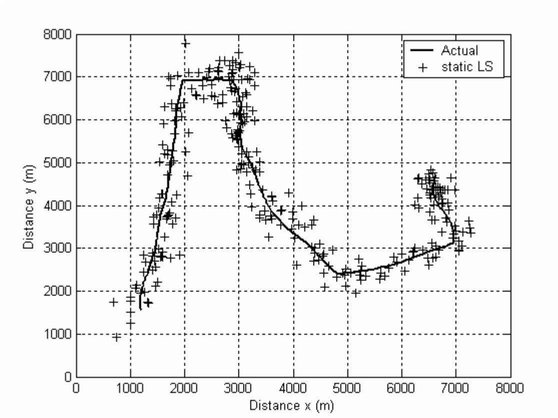


Fig.2. The estimated position obtained by using the static least squares method

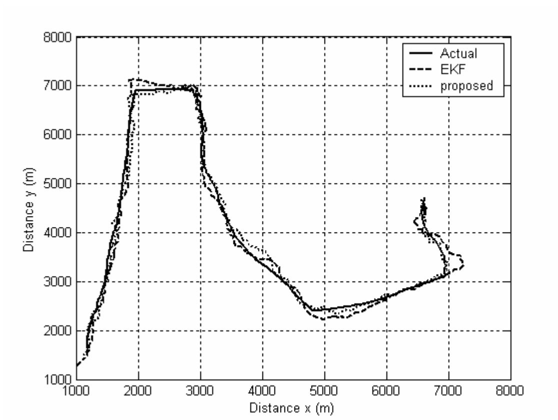


Fig.3. The estimated trajectory by using EKF and our proposed method

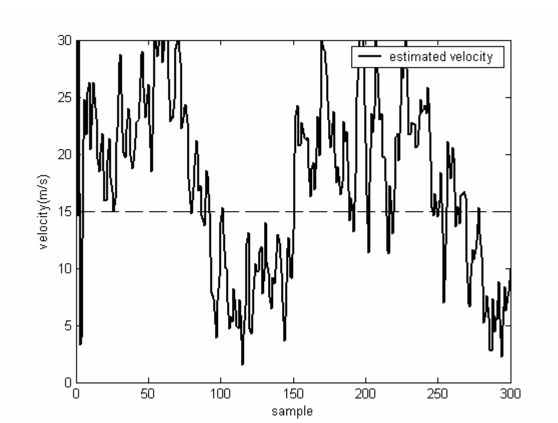


Fig.4. The estimated velocity (m/s) by using the proposed method

## 5. CONCLUSION

In this paper, we describe a real-time constrained least squares estimation algorithm for tracking the position in cellular network using TOA measurements. A dynamic mobile model is used to describe the movement. A series of TOA measurements are used to track the mobile user's position via a constrained least squares method. By solving a constrained optimization problem, we can obtain the object's trace, which satisfies the system constraints. This algorithm can guarantee that the estimated trajectory does not go out of the given range and the estimated velocity and acceleration locate in a reasonable interval. By using the proposed method the spurious estimates can be eliminated and an accurate estimated trace is obtained.

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