MULTIPLIER-FREE BANDPASS FILTER-BASED CHANNELIZER

Siwoo Noh *Ohbong Kwon* *Fred Taylor, IEEE Life Fellow*

University of Florida

ABSTRACT

A new technique for implementing channelizers, based on multiplier-free bandpass filters is presented. The technical foundation for the new channelizer is found in the number theoretic properties of ternary valued polynomials and conventional cascaded integrator-comb (CIC) filters. The proposed channelizer possesses many of the attributes of the traditional CIC-based channelizer, plus others which are unique. The result is a new class of robust low-complexity channelizers.

1. INTRODUCTION

The force of the digital revolution has left a number of traditional analog-enabled technologies in its wake. A classic illustration can be found in radio where analog subsystems are continually being replaced with their digital counterparts. More recently, the potential of programmable and robustness of *software-defined radios* (SDR) has become an economic and technological driving force. However, if the SDR inspired *multi-channel* wireless systems are to become a reality, several technology barriers will need to be removed, beginning with the venerable digital channelizer. The conventional channelizer, or digital down converter (DDC), is intrinsically a multirate lowpass filter (see Figure 1). DDCs require the presence of a preprocessing digital mixer and direct digital synthesizer (DDS), sometimes referred to as a numerically controlled oscillator or NCO, to heterodyne a specific subband down to DC. This is normally accomplished using a direct digital synthesizer (DDS) consisting of sine/cosine generator and digital multiplier. Physically, a conventional channelizer resides between the system's input ADC and back-end processor. The input sample rate f_s is assumed to be (much) higher than the output sample rate f_0 ($f_0 << f_s$). If the sample rate conversion factor $R= f_s/ f_0$ is greater than 10, the channels are considered to be narrowband. If the decimation factor is less than 10, the process is considered broadband. The channelizer engine is generally a classic multiplier-free N^{th} order cascaded integrator-comb (CIC) filter has a transfer function [1, 2, 3]:

$$H_{CIC}(z) = \left(\frac{1 - z^{-S}}{1 - z^{-1}}\right)^N \qquad 1.$$

where S=RM, R is the interpolation rate and M is the comb delay (typically M=1). The CIC secret is exact pole-zeros cancellation at z=1 (DC) of multiplicity N. An N^{th} order CIC filter has a magnitude frequency response given by:

$$H(f) = \left(\frac{\sin(\pi f)}{\sin(\pi f / S)}\right)^N \approx \left(S\frac{\sin(\pi f)}{\pi f}\right)^N \qquad 2.$$

for RM >>1. It can therefore be claimed that a CIC section is decidedly lowpass. The decimated output leaving the CIC filter is finally presented to a linear phase FIR which performs final spectral shaping. Implementation of a classic channelizer is normally performed using 2's complement to manage potential run-time register overflow that can occur due the filters potentially high internal gain ($G = S^N$). This venerable design strategy is at the core of Intersil, Texas Instruments, National, and others down conversion products. While other forms of channelizers have been proposed, it is the CIC-based channelizer that persists in the marketplace and will undoubtedly continue to define channelizing art into the foreseeable future, provided a few barriers are removed.

2. DIGITAL RECIEVERS

The conventional channelizer's performance advantage is gained from the fact that a CIC filter is *multiplier-free* and can therefore operate at real-time data rates typically bounded below 250 MHz supporting 1, 2, or 4 channel channelization. They are found in common use in

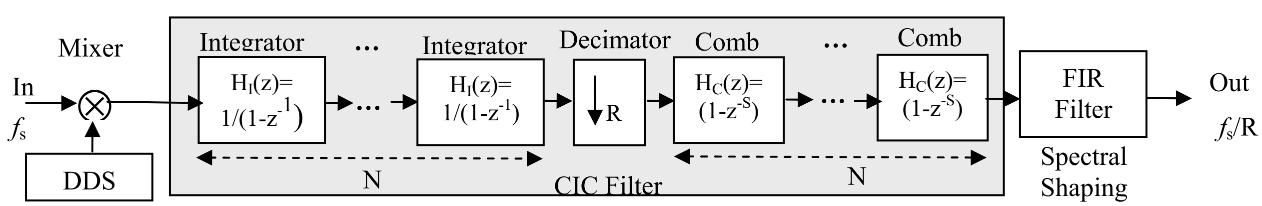


Figure 1: Typical N^{th}-order CIC-based digital down-converter (DDC).

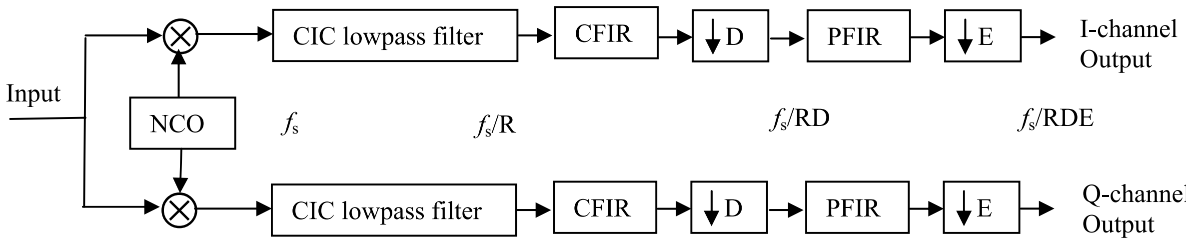


Figure 2: CIC-based Receiver.

communications receiver as motivated in Figure 2. The digital receiver consists of a quadrature NCO, creating I and Q-channels, two N^{th} order decimating CIC filters, two compensation CFIR FIR filters that remove the sin(x)/x roll-off of the CIC filter, and two programmable coefficient PFIR FIR filters that provide spectral housekeeping services. Additional decimation may occur at the output of the CFIR and PFIR filters.

The front-end of the receiver needs to run at the input ADC rate (2 GHz to 80 KHz typical), with back-end processing performed at a decimated rate. This is no challenge to the multiplier-less CIC filter but does have an impact on the input mixer/NCO design. If this condition could be relaxed, then a less complex channelizer could be realized with an attendant area and power advantage. This, in concept, can be achieved if the lowpass filters, found in Figure 2, are replaced by a bandpass filter as shown in Figure 3, allowing the mixer to be relocated at to the output side of the bandpass filter [4]. This method uses the noble identity, from the theory of multirate systems [5], to justify eliminating the need for a mixer as shown in Figure 3. This method, however, only has merit if the compact low-power bandpass filters can run at the ADC speed, suggesting that it too must be multiplier-free.

Another problem associated with the use of classic lowpass CIC channelizer is their susceptibility to large intentional (jamming) or unintentional aliasing. Strong signals that reside outside the subband of interest interval, when channelized and re-sampling, are folded back into output baseband contaminating the desired subband process.

3. MULTIPLIER-FREE BANDPASS FILTERS

Replacing a classic multiplier-free CIC lowpass filter with a bandpass filter can only result in a successful channelizer if run-time performance is not compromised. Throwing vast amount of arithmetic resources at the bandpass filter will only exacerbate already stressed packaging and power dissipation problems. What is needed is a multiplier-free programmable bandpass filter that can selectively and efficiently extract a subband from a wideband signal process. In concept, this could be realized by using a CIC-structure that moves the point of pole-zero from DC to a new location on the periphery of the unit circle ($z=e^{j\theta}$). Here θ denotes the desired bandpass center frequency. Unfortunately, moving the pole to another location on the unit circle would require a filter take the form:

$$H(z) = (1 - z^{RM})^N / (1 + \alpha z^{-1} + \beta z^{-2})^N \qquad 3.$$

If the coefficients α and β are real, this model can extract a high MAC penalty plus introduce stability questions due to imprecise pole-zero cancellation. If, however, bandpass filters could be defined in the context the multiplier-free CIC-like filter, then a potentially viable filter technology will result. In general the pole locations of a constant real valued coefficient filter are defined in terms of the roots of an M^{th}-order polynomial $\Psi_i(z)$, where:

$$\Psi_i(z) = a_0 + a_1 z^{-1} + \ldots.. + a_M z^{-M} \qquad 4.$$

A solution has been proposed that is based on ternary valued coefficients (i.e., $a_i \in \{0, \pm 1\}$) and number theory [6]. The resulting filter, therefore, is multiplier-free and capable of enabling high-speed channelizers based on bandpass filters and low-data rate NCOs. Such a filter can be defined in a number theoretic sense in terms of the polynomial generated by:

$$\Psi_j(z) = N(z)/D(z) = (1 - z^{-j}) / \Pi\Psi_i(z); \quad \forall i < j \qquad 5.$$

where i is relatively prime to j. $\Psi_j(z)$) has poles residing on the periphery of the unit circle in the z-plane at locations $z=e^{j2k\pi/S}$ for some k. The resulting bandpass filter has only ternary valued coefficients and is given by:

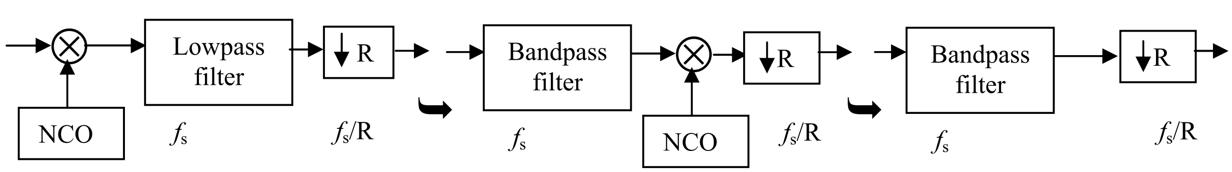


Figure 3: Bandpass filter based channelizer options.

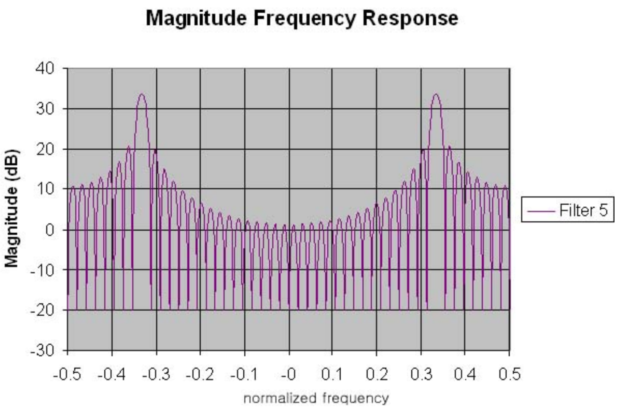


Figure 4: Magnitude frequency response for filter #5 (after Table 1) [7].

$$H_i(z) = (1 - z^{-s})^N \big/ (\Psi_j)^N \qquad 6.$$

where j is an integer multiple of i. Since filter coefficients in both the feedforward and feedback paths are ternary valued, exact pole-zero cancellation can be guaranteed. In addition, Equations 5 and 6 provide a polynomial production rule for ternary valued bandpass filters as illustrated in Table 1.

Table 1: Sample Transfer Functions [7]

No	$\Psi_i(z)$	H(z)	H	Freq. (f_s=48)	G
1	$1-z^{-1}$	$1-z^{-48}/1-z^{-1}$	1	0	1
2	$1+z^{-1}$	$1-z^{-48}/1+z^{-1}$	1	-24	1
3	$1+z^{-2}$	$1-z^{-48}/1+z^{-2}$	2	+/-12	2
4	$1-z^{-1}+z^{-2}$	$1-z^{-48}/1-z^{-1}+z^{-2}$	2	+/-8	$\sqrt{3}$
5	$1+z^{-1}+z^{-2}$	$1-z^{-48}/1+z^{-1}+z^{-2}$	2	+/-16	$\sqrt{3}$
6	$1+z^{-4}$	$1-z^{-48}/1+z^{-4}$	4	+/-6, +/-18	4
7	$1-z^{-2}+z^{-4}$	$1-z^{-48}/1-z^{-2}+z^{-4}$	4	+/-4, +/-20	$2\sqrt{3}$
8	$1+z^{-8}$	$1-z^{-48}/1+z^{-8}$	8	+/-3, +/-9, +/-15, +/-21	8
9	$1-z^{-4}+z^{-8}$	$1-z^{-48}/1-z^{-4}+z^{-8}$	8	+/-2, +/-10, +/-14, +/-22	$2\sqrt{3}$
10	$1-z^{-8}+z^{-16}$	$1-z^{-48}/1-z^{-8}+z^{-16}$	16	+/-1, +/-5, +/-7, +/-11, +/-13, +/-17, +/-19, +/-23	$8\sqrt{3}$

H=number of harmonics; *G*=maximum gain

The data found in Table 1 examines the ternary valued polynomials for selectable center frequencies on $f_s/48$ Hz centers. Up to 24 positive and 24 negative subbands can be extracted by the multiplier-free filters listed in Table 1. The 48 subbands are spread across ten distinct filters with the magnitude frequency response of filter number 5 shown in Figure 4.

There are several design parameters that can be used to adjust the sensitivity and frequency selectivity of the CIC-enabled multiplier-free bandpass filters. The filter's bandwidth and center frequencies are established by S which defines the number of unit circle zeros of multiplicity N. The depth of the stopband, and steepness of the filter skirt, are primarily influenced by the order parameter N. Increasing any of these parameters will, however, increase the internal worst-case gain given by $G=(S)^N$.

4. CHANNELILZER ARCHITECTURE

The multiplier-free bandpass filter can operate at input ADC rates and thereby overcome the problem associated with multiplier-bound bandpass channelizers. In fact, the bandpass filter described in Equation 6 has the complexity of a basic CIC filter. The problem is that some of the bandpass filters have multiple passbands. This situation can be mitigated by mixing the output of a bandpass filter with a sinusoid having a center frequency located at the center of the subband to be channelized, say ω_0. This is essentially the middle option shown in Figure 3. Decimating the multiplier-less bandpass filter, as shown on the right in Figure 3, is not a viable option since all the active subbands will be aliased down of baseband, rendering the filter useless. Instead, a low complexity replacement for an NCO or DDS sinusoid generator and mixer is proposed which defines a solution which is somewhere between the middle and right side panel in Figure 3. In order to eliminate the need for a multiplier-based NCO/mixer at the output of the bandpass filter, the use of a severely quantized binary-valued modulation process is proposed, where the modulation signal is:

$$s[k] = \mathrm{sign}(\sin[k\omega_0]) = \{1, -1\} \qquad 7.$$

The Fourier transform of $s[k]$ is known to have a strong component at the designated ω_0 as well as attendant harmonics located on $k\omega_0$ centers. The binary–valued mixing signal $s[k]$ is trivially generated for I and Q channels and used to modulate the bandpass filter's output at the bandpass filter's sample rate. It should be appreciated that the mixer multiplier is now, in fact, a simple ± switch (sign change). The fundamental frequency is desired but the harmonics will produce unwanted out-of-band signals at multiples of $k\omega_0$. The removal of the unwanted out-of-band signal components traditionally requires the use of a lowpass filter. This approach is only valid if the lowpass excision filter is low complexity and can operate at the bandpass filter sample rate. Such a filter exists and has existed for decades. It is the CIC filter. The result is the solution proposed in Figure 5. It is worth noting that the reason why the binary-valued modulation signal was not directly applied to the ADC output is due to the fact that sidebands of $s[k]$ could modulate undesired signal components into the passband(s) of the bandpass filter.

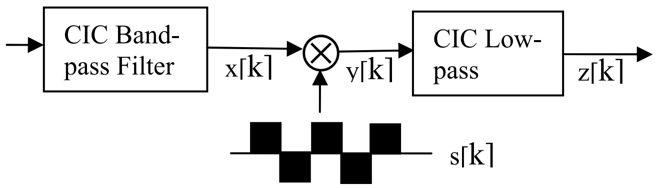
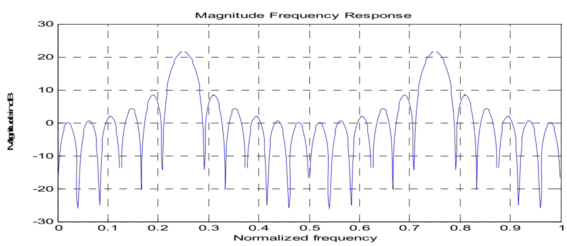


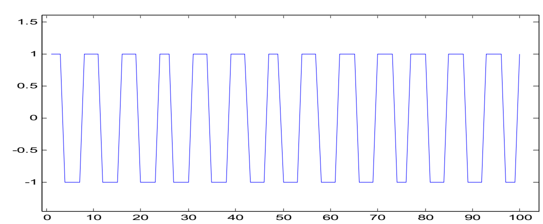
Figure 5: Proposed Multiplier-less Channelizer.

5. PERFORMANCE EXAMPLE

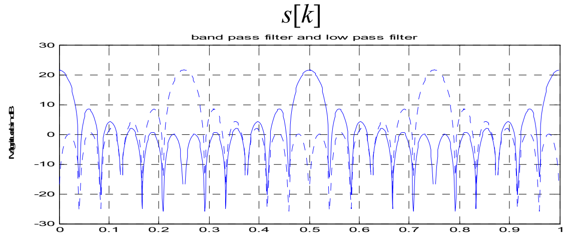
The proposed channelizer was simulated using MATLAB. The results are reported in Figure 6. The multi-channel multiplier-less bandpass filter was chosen to be a 1st order (N=1) two-band filter (Filter 5, Table 1). The output filter is a 4th order (N=4) lowpass CIC filter. The outcome is reference to Figure 5 and is shown as:



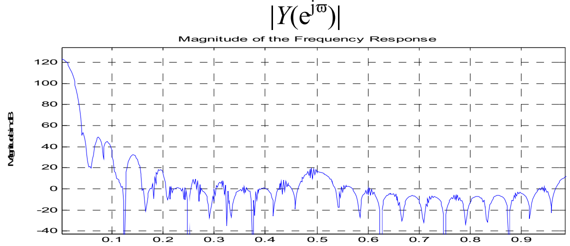
$|X(e^{j\varpi})|$



$s[k]$



$|Y(e^{j\varpi})|$



$|Z(e^{j\varpi})|$

Figure 6: Channelizer simulation.

- Magnitude frequency response ($|X(e^{j\varpi})|$) of a multi-channel multiplier-less bandpass filter.
- Mixing signal s[k] for the first passband in the multi-channel multiplier-less bandpass filter.
- Frequency response of multi-channel multiplier-less bandpass filter and modulator output ($|Y(e^{j\varpi})|$).
- Lowpass CIC filter output ($|Z(e^{j\varpi})|$).

The out-of-band attenuation of the system is more that 70dB down from the passband. The stopband attenuation can be further improved by increasing the order of the bandpass or lowpass multiplier-less filters. The spectral housekeeping filter denoted CFIR and PFIR, found in Figure 2, can also be added to the final design for shape the final outcome.

6. CONCLUSION

A new class of channelizer is presented. It combines a CIC-like bandpass filter with a CIC-lowpass filter and thereby defines a multiplier-free structure. Replacing the standard NCO and mixer with a multiplier-free binary valued modulator provides an end-to-end multiplier-free solution that is both fast and compact. One important point of differentiation between existing CIC channelizers and that proposed is and ability to excise unwanted narrowband interference signals whether intentional of unintentional. By placing interfering signals in the stopbands of a narrow bandpass filter, these signals can be isolated and blocked. In addition, the proposed design eliminates the channelizers dependence on NCOs and digital mixers which provides a number of design advantages.

7. REFERENCES

[1] E. R. Hogenauer, “An Economical Class of Digital Filter for Decimation and Interpolation,” *IEEE Trans. on ASSP*, Apr. 1981.

[2] Chester, D., “Digital IF Filter Technology for 3G Systems,” *IEEE Communications Magazine*, Feb. 1999.

[3] F. Taylor and J. Mellott, *Hands-on Digital Signal Processing*. New York: McGraw-Hill, 1998.

[4] F. Harris, C. Dick, and M. Rice, “Digital Receivers and Transmitters Using Polyphase Filter Banks for Wireless Communications” *IEEE Trans. on Microwave Theo. And Techniques*,” April 2003.

[5] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs, NJ: Prentice-Hall, 1993.

[6] A. Santraine, S. Leprince, and F. Taylor, “Multiplier-Free Bandpass Channelizer for Undersampled Applications,” *IEEE Sig. Proc. Lett.*, Oct., 2004

[7] W. Lawton, “Analysis of Ternary-Valued CIC Filter Based OFDM Channelizers in Modern Wireless Communications Systems,” MS Thesis, University of Florida, 2005