# Efficient Chaotic Spreading Codes for DS-UWB Communication System

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***Abstract*— Ultra wideband (UWB) technology is characterized by transmitting extremely short duration radio pulses. To improve its multiple access (MA) capability, UWB technology can be combined with traditional spread spectrum (SS) techniques. Existing spreading codes are only optimal in additive white Gaussian noise (AWGN) and their performance degrades in multipath fading and narrow band interference. In this paper, we propose the use of spreading codes obtained using a novel design methodology based on genetic programming (GP) and DNA computation for DS-UWB communications. The spreading codes obtained by this novel design methodology performs better than traditional spreading codes in both AWGN and multipath fading. In addition, spreading codes with desired spectral characteristics could be designed to minimize the mutual interference between the DS-UWB and co-existing narrowband systems. The proposed design methodology is attractive for spreading code in terms of performance and flexibility to design spreading codes for a specific design criteria.**

## I. Introduction

With surging increase in demand for personal wireless radio communications within the past decade, there is a growing need for technological innovations to satisfy these demands. Recent advancements in high-speed switching technology created a resurgence in UWB techniques for high-speed wireless communication applications [1]. UWB uses ultra-short pulses compressed in the time and therefore spreads the energy over a very wide bandwidth of very low energy levels. Because of this very large bandwidth, UWB system provides better resistance to narrowband interference and noise immunity compared to conventional spread spectrum systems [2].

Direct-sequence (DS) SS technique is a well known and powerful MA technology that could be combined with UWB modulation. In DS-UWB, multiple pulses per bit period are transmitted using bipolar modulation for each pulse, based upon a certain spreading code [3]. This method has many attractive properties, including low peak-to-average power ratio and robustness to multiple access interference (MAI) [4]. In addition to the pulse shape, the correlation properties of the spreading codes has a major impact on the performance of the resulting UWB MA system. Existing spreading codes based on Gold coder and shift register are only optimal in AWGN noise and their performance degrades in multipath and non-Gaussian environments [5], [6].

Signals generated from chaotic dynamical systems possess numerous properties that makes them an excellent candidate for communication systems design, secure communication, system identification and information theory [7], [8]. A chaotic sequence generator visits an infinite number of states in a deterministic manner therefore outputs a sequence that never repeats itself. This property results in the generation of very large sequence sets with good correlation properties. Moreover, the same chaotic system is capable of generating different codes of various lengths. The traditional analytical techniques for chaotic sequence design uses the ergodic properties of chaotic maps [9]. Since, these techniques are based on asymptotic performance measures, the predicted performance bounds are attained only when the length of chaotic sequences are very long.

In this paper, a design methodology based on DNA computation [10] to quickly identify the initial conditions that generate optimal spreading codes from any given chaotic map is proposed. In addition, nonlinear chaotic maps with a ability to generate these spreading codes is designed using GP [11], [12]. The combination of GP and DNA computation leads to an efficient GP-DNA algorithm [13], [14], that can design optimal codes by specifying the nonlinear chaotic map and the optimal initial conditions for the map.

## II. Performance criteria for spreading code design

We assumed antipodal signaling scheme for the DS-UWB modulation. Second derivative of the Gaussian pulse was used as basic UWB pulse shape. The pulse waveform, $p(t)$ can be expressed as,

$$p(t) = \left[1 - (2\pi t f_0)^2\right] e^{-\pi t f_0{}^2} \tag{1}$$

The pulse waveform $p(t)$ is assumed to be non-zero only during the interval $0 \leq t \leq T_M$, and zero outside the interval. $T_M$ is the minimum resolvability of the pulse waveform. The Fourier transform of the pulse waveform can be given by,

$$P(f) = \frac{2}{\sqrt{\pi f_0^2}} \left(\frac{f}{f_0}\right)^2 e^{\left(\frac{f}{f_0}\right)^2} \tag{2}$$

We consider a DS-UWB system with $K$ users and each user has a specific spreading code with length $N_c$ chips per message symbol period $T_b$. The processing gain of such a system is

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$N_c = T_b/T_c$. A typical transmitted waveform of the $k^{th}$ user can be expressed as,

$$u_k(t) = \sqrt{P_k}\sum_{i=-\infty}^{\infty}\sum_{n=0}^{N_c-1} b_i^k c_n^k p\left(t - iT_b - nT_c\right) \quad (3)$$

where $p(\bullet)$ represents the transmitted Gaussian waveform ,$b_i^k$ is the equi-probable binary information symbols of the $k^{th}$ user, and $c_n^k$ is the spreading code. The complex base band impulse response, which includes the frequency selective multipath propagation of the passband-fading channel by ,

$$h(t) = \sum_{l=0}^{L-1} h_l^k \delta(t - lT_M), \quad (4)$$

where $L$ is the number of multipath components present due to the frequency selectivity of the propagation channel, $h_l^k$ are the received amplitudes of the paths of the user $k$. For a DS-UWB system with $K$ active users,we can write the received waveform for the desired user as follows:

$$r(t) = \sum_{l=0}^{L-1} h_l^0 u_0(t - lT_M) + M(t) + n(t) \quad (5)$$

where $M(t)$ represents the MAI and n(t) represents the received AWGN with two-sided power spectral desnisty $N_0/2$. In the above equation, the MAI term can be expressed as,

$$M(t) = \textstyle\sum_{k=1}^{K-1}\sqrt{P_k}\sum_{i=-\infty}^{\infty}\sum_{n=0}^{N_c-1} \sum_{l=0}^{L-1} b_i^k c_n^k h_l^k p\left(t - iT_b - nT_c - lT_M - \tau_k T_M\right) \quad (6)$$

The receiver consists of a matched filter, matched to the pulse waveform $p(t)$, followed by a RAKE receiver which combines the energy from the multipaths similar to that of traditional SS receiver. The matched filter performance critically depends on the correlation between the user spreading codes and has to be minimized to improve the system performance. In addition, multipath interference also affects the MAI performance because the user spreading codes are no longer orthogonal to each other.

In DS-UWB system, it is desired to select a code set that yields small error probabilities in the desired range of SNR values. For DS-UWB systems, the MAI decreases as the cross correlation function reduces between any two codes. In addition, a very good auto correlation ensures a better reception in multipath fading channels and better synchronization. In this code design approach, it is assumed that all users are in exact synchronism in the sense that not only their carrier frequencies and phases the same, but also their expanded data symbols are aligned in time. It should be noted that a similar design approach can be performed for the case, where, the users are not synchronized. In (6), the MAI experienced by user $k$, has zero mean and variance,

$$\sigma^2(k) = \sum_{\substack{k=0\\k\neq j}}^{K-1} \left|\left\langle c_n^k, c_n^j\right\rangle\right|^2 - N_c^2, \quad (7)$$

where, $\langle\bullet\rangle$ is the periodic correlation function. In general, the MAI experienced by the user $j$ is assumed as a Gaussian random variable for large number of users $i.e.$ $K \approx N_c$. It follows that the code design problem for DS-UWB, when a conventional matched filter is used and when the system is judged by the worst case MAI, $\sigma_{WC}^2$, experienced by any user, can be phrased as follows:

Problem 1: Choose the binary spreading code ($c_n \in \{\pm 1\}$) of length $N_c$ to minimize,

$$\sigma_{WC}^2 = max_j\ \sigma^2(j) = max_j \sum_{k=0}^{K-1} \left|\left\langle c_n^k, c_n^j\right\rangle\right|^2 - N_c^2. \quad (8)$$

The optimally fair solution to Problem 1 will result from a solution, when it exists, to the following problem.

Problem 2: Choose the binary spreading code ($c_n \in \{\pm 1\}$) of length $N_c$ to minimize,

$$\sigma_{TOT}^2 = \sum_{j=1}^{K}\sum_{k=0}^{K-1} \left|\left\langle c_n^k, c_n^j\right\rangle\right|^2 - KN_c^2. \quad (9)$$

over all choices of such codes and then (if possible) to satisfy the further condition that, for $1 \leq j \leq K$,

$$\sigma^2(j) = \sum_{j=1}^{K} \left|\left\langle c_n^k, c_n^j\right\rangle\right|^2 - N_c^2 = \frac{1}{K}\sigma_{TOT}^2. \quad (10)$$

*Condition for no MAI*: The binary length $N_c$ spreading codes ($c_n^k\ \forall k = 0, 1, \ldots, K-1$) give $\sigma_{TOT}^2 = 0$ (and hence also $\sigma^2(j) = 0 \forall k = 0, 1, \ldots, K-1$) if and only if the spreading codes are orthogonal. That is,

$$\left\langle c_n^k, c_n^j\right\rangle = 0\ \forall j \neq k. \quad (11)$$

It follows immediately that $\sigma_{TOT}^2 = 0$ is possible only when $K \leq N_c$, since there can be at most $N_c$ orthogonal non-zero codes of length $N_c$. The fact that traditional codes do not exist for all lengths and there is no mathematical way of constructing codes with optimal aperiodic correlation properties necessitates the design of new spreading codes with good aperiodic correlation properties.

## III. GP-DNA BASED SPREADING CODE DESIGN

### A. DNA computation for initial condition optimization

In this section, a DNA optimization approach based on chaotic dynamics is discussed [15]. In [13], a chaotic system is used as the core engine of the search process. Instead of using deterministic bit stream matching algorithms, a nonlinear chaotic computing model adapts itself to find optimal solutions autonomously.

In the developed DNA computing model, a large amount of information generated by the chaotic dynamics is exploited to represent potential candidate solutions for the optimization problem in analogy to biological DNA synthesis. This information is then processed by using a simple feedback scheme that tracks the dynamics of the system and acts on its inputs in order to select the desired solution among the generated candidates. DNA computation involves the manipulation of sequences of nucleotides. A DNA consists of four nucleotides A, T, G and C that are used to form strings to encode information. These letters form the basic alphabet set $\Sigma = \{A, G, C, T\}$ and is used to form strings to encode the information. A multi set of such strings can be used to represent candidate solutions of an optimization problem. The actual or optimal solution (or one of the solutions) must be selected among the many candidates. In Fig. 1, $Lor$ is the Lorenz system that

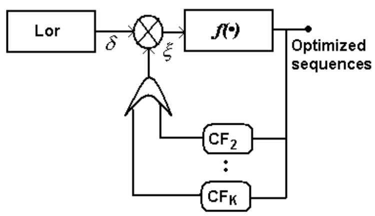


Fig. 1. Schematic diagram of DNA computation based on chaos

generates the perturbations which are used to generate the GP designed chaotic map $f(\bullet)$ to produce optimal spreading codes. Initially, $c_1(t)$ is obtained by optimizing its auto correlation function which forms the basis code. The control feedback $CF_2$ evaluates the correlation properties of $c_2(t)$ with $c_1(t)$. It outputs a logical 0 only when the new spreading codes is optimized (*i.e.* orthogonal). Similarly, the $k^{th}$ control feedback $CF_k$ evaluates the correlation properties of the spreading code $c_n^k$ with previously optimized spreading codes. Since each of the spreading codes need to be optimized with respect to all other codes, an OR operation is performed to the individual $CF$ outputs. The resulting signal is then multiplied by a perturbation value $\delta$ generated by a independent Lorenz system, to obtain the actual perturbation value $\xi$. The perturbation value is then added to the current initial condition of the chaotic map to form the new initial condition.

### B. Chaotic map design using GP

In GP, the individual functions (chaotic maps represented in functional domain) are represented as trees, in which the leaf nodes are variables from the initial terminal set, and the internal nodes are functional operators from the functional set F. Since the objective of GP design is to search for chaotic maps that can provide maximum number of spreading codes of the given length that satisfy a given objective function, the fitness of the individuals in any population of GP is assigned depending on the number of optimal codes (rank of the individual) that can be obtained. A more detailed description of GP based chaotic map design is reported in [11]. The GP design methodology can be breifly summarized as follows:

- GP holds a population of candidate solutions.
- It assigns the fitness based on the obtained performance measure.
- Evolves the functions across many generations until the functional form is achieved or the maximum number of generations exceeded.

Initially, the GP generates a pool of potential candidates (i.e. chaotic maps) generated in a random fashion. The functions is usually selecetd from a functional set and generated in a tree structure. The candidate solutions are evaluated based on performance measure and are arranged in a particular order (i.e. best to worst). These functions are allowed to crossover, mutate,reproduce according to predefined probability. (i.e. $P_{cross} = 0.9$, $P_{rep} = 0.05$ and $P_{mut} = 0.05$). The new population of candidate solutions are again subject to the performance criteria. These processes is performed until the required performance measure is achieved or maximum number of generations is exceeded.

## IV. Performance analysis

In this section, the performance of GP-DNA designed codes is compared with traditional spreading codes in AWGN and multipath fading channels. In addition to the Gold codes, spreading codes generated by chaotic Markov map was also used for performance evaluation. The receiver has perfect synchronization and perfect knowledge of the channel status. The GP-DNA was applied to design discrete maps for generating codes of length $N_c = 31$ and $K = 31$ users. The evolution of best fitness across generations is shown in Fig. 2. The map

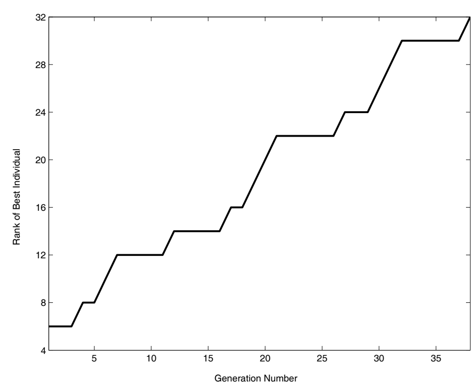


Fig. 2. GP Evolution for the Optimal Sequence Design

designed by GP-DNA at the end of $38^{th}$ generation can be represented as,

$$\begin{aligned} x_n = {} & (mod1(((mod1((x_{n-1}) + (x_{n-1}))) \\ & + (((x_{n-1}).*(x_{n-1})).*((x_{n-1}).*(x_{n-1})))) \\ & + (((mod1(x_{n-1})) + ((x_{n-1}) + (x_{n-1}))) \\ & + (mod1(x_{n-1}))))). \end{aligned} \tag{12}$$

The communication performance of DS-UWB using different spreading codes in AWGN environment is shown in Fig. 3. The Eb/No was fixed at 15 dB and code length at $N_c = 31$. The Gold codes are nearly optimal in AWGN environment and hence there is not much improvement for the GP-DNA codes. The BER performance of the different spreading codes in multipath fading environment is shown in Fig. 4. The receiver has perfect channel estimates and a 6-finger RAKE receiver was employed. In the first case, the number of users was fixed at $K = 7$ with $N_c = 63$. In the second case, the Eb/No

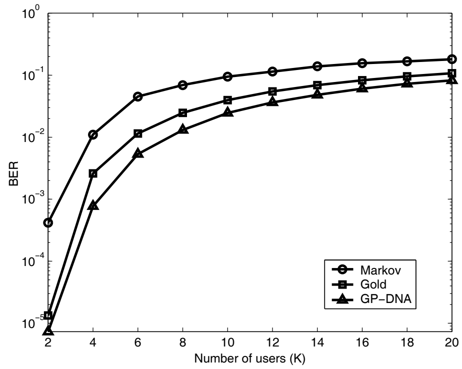



Fig. 3. MA performance in AWGN environment.

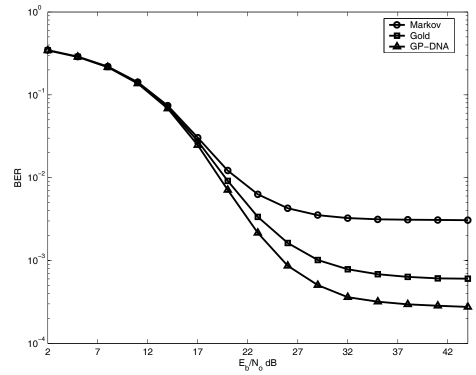



Fig. 5. Communication performance in time varying multipath fading environment.

was fixed at 18 dB. From these figures, we could see that the GP-DNA codes performs much better than the traditional spreading codes. A performance improvement of around 3 dB at a BER of $10^{-3}$ was obtained for GP-DNA codes over the Gold codes. Finally, the performance of the spreading codes

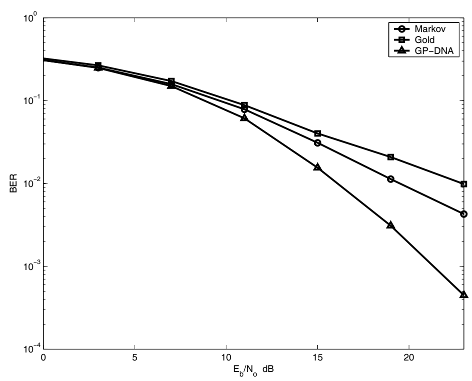



Fig. 4. Communication performance in multipath fading environment.

was evaluated in a time varying multipath fading channel. The number of users was fixed at $K = 7$ with $N_c = 63$. Fig. 5 shows the BER performance as a function of SNR. The multipath channel was assumed to change at a slightly higher rate compared to the channel estimator. All the spreading codes resulted in a BER floor. However, the system employing the GP-DNA had the best BER performance. The GP-DNA codes were optimized to reduce the MAI as well as mulitpath interference and hence it showed a improved performance in these environments.

## V. Conclusion

Multipath interference and MAI can be reduced by optimizing the spreading codes for the DS-UWB communication system. A new design methodology for optimal spreading code design based on GP and DNA computation was proposed. The multipath and MAI was used as the objective functions for the spreading code design. The spreading codes obtained through this method have better performance than traditional spreading codes in multipath and multiuser interference. Performance analysis in multipath and time varying channel illustrates the performance improvement achieved through the new spreading codes. In addition, the design methodology can also be used to design spreading codes based on a specific design criteria.

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