ROBUST MULTIUSER DETECTION FOR IMPULSE RADIO IN NON-GAUSSIAN UWB CHANNELS

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ABSTRACT

The frequency-selective ultra-wideband (UWB) channels are subject to various types of electromagnetic noise, which is non-Gaussian (impulsive). The conventional linear receivers based on the additive white Gaussian noise assumption exhibit performance degradation. In this paper, a robust receiver using the M-estimation technique is introduced for multiple access UWB channels with impulsive noise, eliminating multiuser interference and impulsive noise at the same time, provided that accurate channel information about the users is available. Both the robust receiver and its simplified version are shown to outperform the linear receiver based on the Gaussian noise assumption.

1. INTRODUCTION

Ultra-wideband (UWB) systems using trains of very short duration pulses have been referred to as “impulse radio” [1], where information is transmitted by either changing the amplitude or the position of the pulses. Multiple users employing different time-hopping (TH) sequences are allowed to share the same channel. Catastrophic collisions between users are avoided by shifting the starting time of each pulse according to the TH sequence. In addition, each information symbol is transmitted using a number of pulses to obtain time diversity at the receiver.

The channel between the transmitter and the receiver is one of the most important factors affecting receiver design. For UWB communication, the multipath characteristics of the wideband indoor channel, lying in the 2-8 GHz band, have been investigated and used in the IEEE 802.15.3a standard [2]. The main observations are that the total energy is distributed over a large number of paths, which are resolved in time, and the multipath delay spread spans several nanoseconds. Other relevant field measurements [3] indicate that the indoor environments are subject to impulsive (non-Gaussian) noise produced by photocopiers, printers, etc. in the office. Thus, the traditional approach of modeling the ambient noise as a Gaussian random process is not realistic. Moreover, the performances of linear receivers based on the Gaussian assumption degrade significantly in the presence of noise with large amplitudes. They should be robustified with respect to the deviations of the distribution of the noise process from Gaussianity. For multiple access UWB channels, this can be accomplished by resorting to M-estimates proposed by Huber in [4]. M-estimates have been successfully applied to direct sequence/code-division multiple access (DS/CDMA) systems in [5], [6], where the structured interference and impulsive noise are eliminated simultaneously.

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In this paper, a multiuser detector for the extremely frequency-selective UWB channels with impulsive noise is proposed. A robust version of the multipath-combining decorrelating (mD) detector in [7] is formed using M-estimation. The proposed robust decorrelator is simplified by selecting a subset of the paths corresponding to times when the signal from the desired user is strong. The rest of the paper is organized as follows. In Section II, the UWB system model is presented. The multiuser detector based on M-estimates is introduced in Section III. The performances of the robust and linear detectors are compared in Section IV, and concluding remarks are made in Section V.

2. UWB SYSTEM MODEL

2.1. The Continuous-Time Model

The users of the impulse radio system employ TH for multiple access and binary phase shift keying (BPSK) for data modulation. The signal transmitted by the kth user is

$$s_{\rm tr}^{(k)}(t) = \sum_{j=-\infty}^{\infty} b_{\lfloor j/N_s \rfloor}^{(k)} w_{\rm tr}(t - jT_f - c_j^{(k)} T_c),$$

where each pulse, $w_{\rm tr}(t)$, is sent during a frame of T_f seconds and the exact position of the pulses is determined by the TH sequence $\{c_j^{(k)}\}_{j=-\infty}^{\infty}$ specific to user k. The starting time of the jth pulse is shifted by $c_j^{(k)} T_c$, where $c_j^{(k)} \in \{0, 1, \ldots, N_h - 1\}$. Here, N_h is the number and T_c is the duration of the bins to which the pulses may hop. Typically, $N_h T_c \le T_f$ and $T_c = T_p$ with T_p being the pulse duration. For each bit, $b_i^{(k)}$, N_s pulses are allocated so that a kind of repetition coding is obtained.

The signal transmitted by each user goes through a frequency selective UWB channel. The received signal from the kth user, which propagates over the channel $h_k(t) = \sum_{\ell=0}^{L_k-1} \alpha_{k,\ell} \delta(t - \tau_{k,\ell})$ with a gain and delay of $\alpha_{k,\ell}$ and $\tau_{k,\ell}$, respectively, for the ℓth path is

$$s_{\rm rec}^{(k)}(t) = A_k \sum_{j=-\infty}^{\infty} b_{\lfloor j/N_s \rfloor}^{(k)} \sum_{\ell=0}^{L_k-1} \alpha_{k,\ell} w_{\rm rec}(t - T_f - c_j^{(k)} - \tau_{k,\ell})$$

where A_k is the received signal amplitude and $w_{\rm rec}(t)$ is the received pulse shape with unit energy. The channel gains are normalized such that $\sum_{\ell=0}^{L_k-1} \alpha_{k,\ell}^2 = 1, \forall k$, and different paths arrive at integer multiples of the pulse duration. The total received signal is

$$r(t) = \sum_{k=1}^{K} s_{\rm rec}^{(k)}(t) + n(t),$$

where K is the number of users and $n(t)$ is an additive noise component. In this model, for simplicity, all users are assumed to be synchronized.

2.2. The Discrete-Time Model

The received signal, $r(t)$, is passed through a linear filter matched to the received pulse, $w_{\rm rec}(t)$, and the output of this filter is sampled every T_c seconds [8]. A guard time exists between information symbols equal to the length of the channel impulse response so that intersymbol interference (ISI) is avoided [9]. The vector of samples, $\mathbf{r}[i]$, for the ith bit is of dimension $N_h N_s + L - 1$, where $L = \max_k L_k$. It may be represented with the discrete-time model:

$$\mathbf{r}[i] = \bar{\mathbf{S}}[i]\mathbf{H}[i]\mathbf{A}\mathbf{b}[i] + \mathbf{n}[i], \qquad (1)$$

where $\mathbf{H}[i] = \mathrm{diag}(\mathbf{h}_1, \ldots, \mathbf{h}_K)$ with $\mathbf{h}_k = [\alpha_{k,0} \ldots \alpha_{k,L-1}]^T$, the vector containing the channel gains of the kth user during the ith signaling interval, $\mathbf{A} = \mathrm{diag}(A_1, \ldots, A_K)$, and $\mathbf{b}[i] = [b_i^{(1)} \ldots b_i^{(K)}]^T$. The matrix $\bar{\mathbf{S}}[i]$ in (1) contains information about the TH sequences of different users for the ith bit: $\bar{\mathbf{S}}[i] = [\mathbf{S}_1[i] \ldots \mathbf{S}_K[i]]$, where $\mathbf{S}_k[i]$ is a lower triangular matrix having L columns with $\mathbf{s}_k[i]$ on the main diagonal. Each element $\ell = \{1, 2, \ldots, N_s N_h\}$ of the vector $\mathbf{s}_k[i]$ is computed from the TH sequence for the kth user [8]:

$$(\mathbf{s}_k[i])_\ell = \begin{cases} 1, & \text{if} \quad c^{(k)}_{(i-1)N_s + \lfloor \ell/N_h \rfloor} = \ell - \lfloor \frac{\ell}{N_h} \rfloor N_h - 1, \\ 0, & \text{otherwise.} \end{cases}$$

The noise samples $\mathbf{n}[i]$ in (1) are independent and identically distributed random variables with a non-Gaussian probability density function (pdf). The pdf for the noise samples is the mixture of two Gaussians with zero means and different variances, where one is a multiple of the other for the representation of the impulsive component producing large amplitudes:

$$f = (1-\epsilon)\mathcal{N}(0, \sigma^2) + \epsilon\mathcal{N}(0, \kappa\sigma^2), \qquad (2)$$

where $\epsilon \in [0, 1)$, $\sigma^2 = N_0/2$ and $\kappa \geq 1$. The case $\kappa = 1$ corresponds to the additive white Gaussian noise (AWGN) channel. The ϵ-mixture model in (2) is an approximation to Middleton's Class A noise model pdf [10], which consists of an infinite expansion of Gaussian density functions with different variances and identical means. The first two terms of the expansion are usually considered a sufficient representation for the noise process with impulsive components.

3. ROBUST DETECTION WITH M-ESTIMATES

Before the introduction of the robust multiuser detector based on M-estimates, the matrix notation for the received signal model in (1) is modified. By defining $\theta_k \triangleq A_k b_i^{(k)}$, the matrix $\mathbf{A}$ and the vector $\mathbf{b}[i]$ can be replaced by a single vector $\mathbf{\Theta} = [\theta_1 \ldots \theta_K]^T$. Moreover, $\bar{\mathbf{S}}[i]\mathbf{H}[i]$ is represented with a single matrix $\mathbf{S_c}$, which has as columns $\tilde{\mathbf{s}}_k = \mathbf{S}_k[i]\mathbf{h}_k$, the convolution of $\mathbf{s}_k[i]$ with $\mathbf{h}_k$. Dropping the index for the information bit,

$$\mathbf{r} = \mathbf{S_c}\mathbf{\Theta} + \mathbf{n}. \qquad (3)$$

If the noise samples are from a Gaussian distribution, the solution to (3) is given by the least-squares solution, which is identical in form to the linear decorrelator output [5]:

$$\hat{\mathbf{\Theta}} = (\mathbf{S_c^T S_c})^{-1}\mathbf{S_c}^T\mathbf{r}.$$

With the assumption that the channel gains of all of the users are known, the multipath decorrelator formed, which is the multipath-combining decorrelating (mD) detector in [7], has optimum near-far resistance properties. This receiver consists of a bank of filters matched to $\tilde{\mathbf{s}}_k$, which performs $\tilde{\mathbf{s}}_k^T\mathbf{r}$, followed by the decorrelator, $\tilde{\mathbf{R}}^{-1} = (\mathbf{S_c}^T\mathbf{S_c})^{-1}$. The matched filter for $\tilde{\mathbf{s}}_k$ is a rake receiver with maximal ratio combining of the outputs of the fingers (i.e., the weighting coefficients are the channel gains). Thus, paths are combined before decorrelating the signals from different users.

Due to the large number of paths produced by UWB channels, decorrelating the user signals from different paths before multipath combining, as in [6], is not feasible. Such a detector exhibits performance deterioration more and more rapidly as the number of paths increases due to the fact that multipath decorrelating operation is performed on a larger population, i.e., KL unknowns are to be estimated [6].

The equivalence between the least-squares solution and the linear decorrelator output enables us to form a robust version of this linear multipath decorrelator using the robustified least squares solution in [4]. Instead of minimizing a sum of squares as in the least-squares approach, a sum of less rapidly increasing functions of the residuals is minimized to obtain the robust solution:

$$\hat{\mathbf{\Theta}} = \arg\min_{\mathbf{\Theta}} \sum_{n=1}^{N_s N_h + L - 1} \rho\left((\mathbf{r})_n - \sum_{k=1}^{K}(\mathbf{S_c})_{nk}\theta_k\right),$$

where $(\mathbf{r})_n$ is the nth element of $\mathbf{r}$, and $(\mathbf{S_c})_{nk}$ is the element of $\mathbf{S_c}$ on the nth row and kth column. If ψ is the derivative of ρ, and ρ is convex, the solution to

$$\sum_{n=0}^{N_s N_h + L - 1} \psi\left((\mathbf{r})_n - \sum_{k=1}^{K}(\mathbf{S_c})_{nk}\theta_k\right)(\mathbf{S_c})_{np} = 0 \qquad p = 1, \ldots, K$$

are the M-estimates we are looking for, as well. The choice $\rho(x; \theta) = -\log f(x; \theta)$ gives the ordinary maximum likelihood estimate [4]. The least-squares estimate corresponds to $\rho(x) = x^2$.

The function, $\rho_H(x)$, chosen by Huber minimizes the maximal asymptotic variance of the estimator over the set of ϵ-contaminated Gaussian models [5]. Then, $\psi_H(x)$, is the maximum likelihood estimate of the least favorable distribution in the ϵ-contaminated model set minimizing the Fisher information [4]:

$$\psi_H(x) = \begin{cases} -k, & x < -k\sigma^2, \\ \frac{x}{\sigma^2}, & -k\sigma^2 \leq x \leq k\sigma^2, \\ k, & x > k\sigma^2, \end{cases}$$

with the trimming parameter k obtained from

$$\frac{\phi(k\sigma)}{k\sigma} - \Phi(-k\sigma) = \frac{\epsilon}{2(1-\epsilon)},$$

where ϕ and Φ are the pdf and cumulative distribution function (cdf) of the standard normal random variable, respectively. Using $\psi_H(x)$, large noise amplitudes are clipped.

M-estimates with the nonlinearity $\psi_H(x)$ are computed using the modified residuals method in [4]. This iterative algorithm can be summarized as follows with the superscript denoting the mth step:

$$\begin{aligned} \mathbf{z}^m &\triangleq \psi_H(\mathbf{r} - \mathbf{S_c}\mathbf{\Theta}^m) \\ \mathbf{\Theta}^{m+1} &= \mathbf{\Theta}^m + \mu(\mathbf{S_c}^T\mathbf{S_c})^{-1}\mathbf{S_c}^T\mathbf{z}^m, \end{aligned}$$

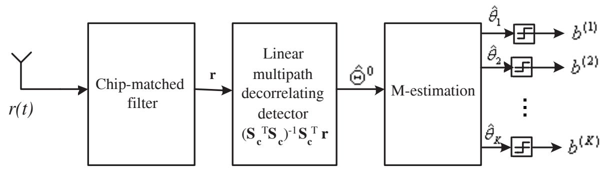

Fig. 1. The robust multipath decorrelator.

where $0 < \mu < 2$ is an arbitrary relaxation factor [4] and the algorithm is initialized with the least-squares solution as in [5]: $\mathbf{\Theta}^0 = \mu(\mathbf{S_c}^T\mathbf{S_c})^{-1}\mathbf{S_c}^T\mathbf{r}$. The algorithm converges, on the average, in ten steps. The bit decisions are made according to $\hat{b}^{(k)} = \text{sgn}(\hat{\theta}_k)$, where $\hat{\theta}_k$ is the kth element of the estimate of $\mathbf{\Theta}$ from the modified residuals algorithm. The robust decorrelator is shown in Fig. 1.

The asymptotic probability of bit error for large processing gain, as $N_sN_h \to \infty$, is calculated using the fact that M-estimates are asymptotically normal [4]. Defining $\mathbf{R} = \mathbf{S_c}^T\mathbf{S_c}$, and

$$\upsilon^2 \triangleq \frac{\int \psi^2(x)f(x)dx}{\left(\int \psi'(x)f(x)dx\right)^2},$$

the asymptotic probability of error for the kth user is given by

$$P_e = \Phi\left(-\frac{A_k}{\upsilon\sqrt{(\mathbf{R}^{-1})_{kk}}}\right). \tag{4}$$

If the performance requirements can be loosened, the multipath decorrelator can be replaced by a simpler version that selects the paths with the largest channel gains for the desired user. The received signal is sampled only at the instants when the signal from the desired user is strong. Since there are N_s pulse transmissions per symbol, if for each pulse P paths are considered to be significant, instead of $N_hN_s + L - 1$, then PN_s or fewer paths are selected depending on collisions between different paths. The received signal model, where k is the desired user, is

$$\mathbf{r}_k = \mathbf{S}_{\mathbf{c},k}\mathbf{\Theta} + \mathbf{n}_k,$$

with $\mathbf{r}_k$, $\mathbf{S}_{\mathbf{c},k}$ and $\mathbf{n}_k$ are obtained from those rows of $\mathbf{r}$, $\mathbf{S_c}$ and $\mathbf{n}$, respectively, in (3) corresponding to the selected paths. Again, $\hat{\mathbf{\Theta}}$ is computed using the modified residuals method, and the sign of the kth row of $\hat{\mathbf{\Theta}}$ is the bit estimate, $\hat{b}^{(k)}$. The asymptotic probability of error is given by (4) with $\mathbf{R} = \mathbf{S}_{\mathbf{c},k}^T\mathbf{S}_{\mathbf{c},k}$.

In contrast to DS/CDMA with continuous transmission of pulses, there are pulse transmissions only at the instants dictated by the random TH sequence. Therefore, the number of users colliding with the desired user, K_k, given by the nonzero number of columns of $\mathbf{S_{c,k}}$, is usually less than K [8]. To calculate the output of the simplified decorrelator, a matrix of size $K_k \times K_k$ obtained from $\mathbf{S}_{\mathbf{c},k}$ by removing its all-zero columns has to be inverted, instead of a $K \times K$ matrix. This constitutes a substantial amount of reduction in complexity when K is large.

4. EXPERIMENTAL RESULTS

In this section, the performances of the robust and linear receivers are compared in the 4-10 m extreme non-line-of-sight (NLOS) frequency-selective UWB channel (CM4) with an rms delay spread of 25 ns [2]. The additive channel noise has impulsive components.

For the first experiment, the simulated impulse radio system parameters are $T_f = 10$ ns, $T_p = 1$ ns and $R_b = 10$ Mbps, where R_b is the bit rate. Although $N_h = 10$, TH is allowed only in the first half of the frame time to limit inter-frame interference (IFI). The received pulse shape is the second derivative of a Gaussian [11]. The impulsive noise is characterized by $\epsilon = 0.1$ and $\kappa = 100$. The bit-error-rate (BER) curves of the robust and linear receivers with this set-up are shown in Fig. 2 for varying signal-to-noise ratio (SNR) values of the first user and $K = 20$ synchronous equal power users. SNR is defined as $A_k^2N_s/N_0$ for the kth user. The superior performance of the robust decorrelator compared to the linear one is the first observation to make from this figure. Both the robust decorrelator and its simplified versions selecting fewer paths, P (16 and 32 paths in Fig. 2) outperform the linear decorrelator. The BER curve for the linear decorrelator remains well above 1×10^{-2} and there is no marked improvement obtained by increasing P for the simplified versions. With the robust case, however, the choice $P = 32$ produces a curve sufficiently close to that of the robust decorrelator. The asymptotic performance curve in Fig. 2 calculated using the finite-length $\mathbf{S_c}$ matrices upper bounds the performance of the robust decorrelator. The BER curve for the linear decorrelator is well approximated by the asymptotic performance curve obtained via (4).

If the frequency of occurrence of the impulsive noise components is reduced from $\epsilon = 0.1$ to $\epsilon = 0.01$ and the first experiment is repeated, the performances of both the robust and linear decorrelators improve as demonstrated in Fig. 3. While the performance improvement is only 2 dB for the robust detector, it is around 6 dB for the linear one. For $\text{SNR} \geq 8$ dB, the robust decorrelator outperforms the linear decorrelator by more than an order of magnitude despite the 6 dB improvement of the linear decorrelator.

In the second experiment, the effect of changing K on the receiver performances is investigated with a fixed SNR of 10 dB, $\epsilon = 0.1$ and the rest of the parameters remain unchanged from the first experiment. In Fig. 4, the robust decorrelator exhibits performance degradation with increasing K more compared to the linear one, since its performance limiting factor is the decorrelating loss and not the impulsive noise.

The last experiment involves the κ parameter of the impulsive noise, where $K = 20$, SNR = 10 dB and $\epsilon = 0.1$. The parameters of the impulse radio system are, again, $T_f = 10$ ns, $T_p = 1$ ns and $R_b = 10$ Mbps. When $\kappa = 1$ (i.e., AWGN case), the performance of the linear decorrelator is slightly better than that of the robust, as seen in Fig. 5. There is a performance cross-over as κ increases, and while the BER curve of the linear receiver approaches $1/2$, the performance curve of the robust receiver stays relatively constant around 1×10^{-3}, even when $\kappa = 1000$.

5. CONCLUSION

A robust multiuser detector for synchronous impulse radio systems propagating over frequency-selective UWB channels with impulsive noise has been proposed and analyzed. The detector, which is based on M-estimates, is the robust version of the multipath-combining decorrelating (mD) detector in [7], which performs multipath combining prior to decorrelation and requires channel information about the users. The performance limiting factor for this linear decorrelator is the impulsive noise, as the frequency of the impulsive noise components (ϵ) and their intensity (κ) determine its performance. For all SNR values, the robust detector is shown to outperform the linear one, because it can effectively eliminate multiuser interference and impulsive noise simultaneously. The effect of channel estimation errors on the receiver performance will be investigated, as well.

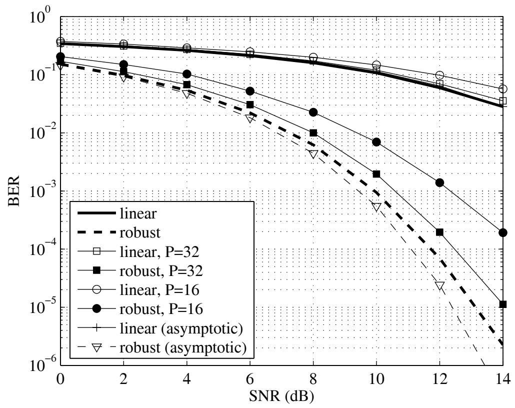

Fig. 2. BER vs. SNR of the robust and linear decorrelators for the first user of a synchronous TH impulse radio system with $T_f = 10$ ns, $T_p = 1$ ns and $R_b = 10$ Mbps. ($\epsilon = 0.1$, $\kappa = 100$, $K = 20$.)

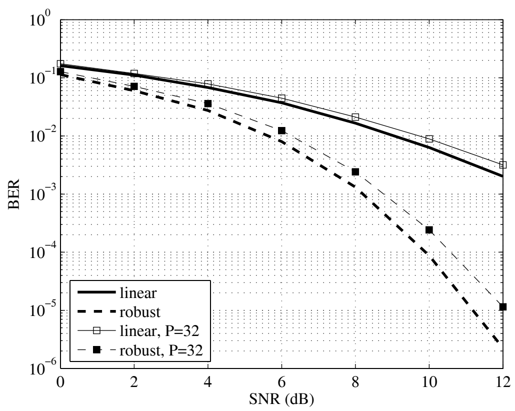

Fig. 3. BER vs. SNR of the robust and linear decorrelators for the first user of a synchronous TH impulse radio system with $T_f = 10$ ns, $T_p = 1$ ns and $R_b = 10$ Mbps. ($\epsilon = 0.01$, $\kappa = 100$, $K = 20$).

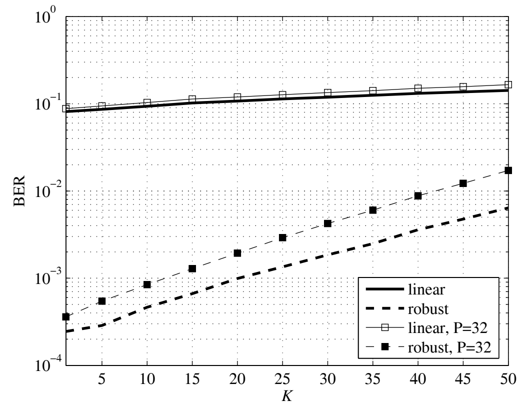

Fig. 4. BER vs. K of the robust and linear decorrelators for the first user of a synchronous TH impulse radio system with $T_f = 10$ ns, $T_p = 1$ ns, $R_b = 10$ Mbps. ($\epsilon = 0.1$, $\kappa = 100$, $\text{SNR} = 10$ dB.)

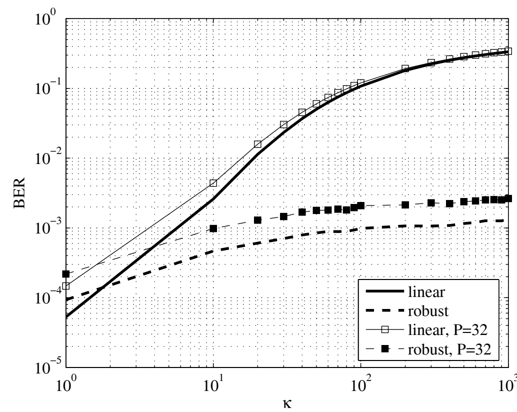

Fig. 5. BER vs. κ of the robust and linear decorrelators for the first user of a synchronous TH impulse radio system with $T_f = 10$ ns, $T_p = 1$ ns, $R_b = 10$ Mbps. ($\epsilon = 0.1$, $K = 20$, $\text{SNR} = 10$ dB.)

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