TAP SELECTION BASED MULTIPATH CHANNEL EQUALIZATION FOR UWB SYSTEMS

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ABSTRACT

In this paper, tap selection based Minimum Mean Square Error (MMSE) equalization technique is discussed for Ultra Wideband (UWB) systems in the presence of Inter Symbol Interference (ISI) and Multiple Access Interference (MAI). A novel tap selection technique based on Matching Pursuit (MP) algorithm is proposed. Given finite training sample support, Diagonal Loading (DL) technique is incorporated into MP algorithm to insure the robustness of tap selection. Moreover, a fast heuristic MP based tap selection technique is developed to facilitate the tradeoff between equalization performance and computational complexity. Simulation results show that the proposed tap selection based equalizer outperforms the RAKE receiver and the conventional MMSE equalizer significantly given the same amount of training symbols.

1. INTRODUCTION

Impulse Radio (IR) technique [1] [2], which uses very short duration baseband pulses, is one of the most promising approaches for UWB systems. Compared with multicarrier Orthogonal Frequency Division Multiplexing (OFDM) based UWB method [3], pulse based UWB has the advantages such as very low power consumption, simple transceiver design and robustness due to its resolvable multipath. For UWB multipath channel with long delay spread, channel response may span over multiple symbol frames, an MMSE [4] equalizer with large number of taps is superior for ISI and MAI mitigation than a conventional RAKE receiver. But this requires large amount of training symbols and involves large computational complexity. A key challenge is to develop a high performance equalizer given limited training sample support with manageable complexity.

Non-uniformly spaced equalizer has been discussed for sparse multipath channels [5][6] with reduced complexity. In these works, various intuitive tap selection techniques are discussed assuming known Channel Impulse Response (CIR). These prior works motivate us to consider tap selection based technique for UWB channel equalization. One issue is that the multipath channel needs to be estimated before tap selection can be made on the estimated CIR. Accurate channel estimation in the presence of unknown MAI is a hard task. Channel estimation error will affect tap selection and equalization performance.

In this paper, a novel tap selection method based on Matching Pursuit (MP) [7] algorithm is proposed. This algorithm is directly implemented on the training samples, without the need of channel estimation.

2. SYSTEM MODEL AND LEAST SQUARES ESTIMATION

Let the transmitted signal for transmitter k be $s^{(k)}(t) = \sum_{n=-\infty}^{\infty} x^{(k)}(n)\, w(t - nT_f)$, where $x^{(k)}(n) \in \{\pm 1\}$ is the bipolar data bit stream, $w(t)$ is the pulse waveform and T_f is the symbol duration. Applying a tapped-delay-line channel model $c^{(k)}(t) = \sum_{p=1}^{n_L} \alpha_p^{(k)}\, \delta(t - (p-1)\Delta\tau - \tau_k)$, where $\Delta\tau$ is the sampling duration, τ_k is the channel delay and n_L is the channel length in samples. For simplicity, choose T_f and let $n_\tau = T_f/\Delta\tau$ be an integer. The number of symbols affected by ISI is given as $n_{ISI} = n_L/n_\tau$, where assuming n_L is an integer multiples of n_τ and this may be done by truncating the very low power tail of CIR. In the presence of ISI and MAI, an UWB system model can be represented as follows,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{H}^{(MAI)}\mathbf{x}^{(MAI)} + \mathbf{m} \tag{1}$$

where $\mathbf{H}$ is the channel transmission matrix in block Toeplitz form that is constructed from the sampled generalized CIR ($h(t) = c(t) * w(t)$), $\mathbf{y}$ is the received sampling signal vector and $\mathbf{m}$ is the corresponding AWGN vector with covariance of $\sigma_m^2\mathbf{I}$. The superscript (MAI) denotes the terms corresponding to the MAI transmitters. For simplicity, the self spreading multipath channels of UWB are utilized for multiple access without adding extra spreading sequence. In addition, strict channel synchronization is not necessary since the tap selection technique will automatically choose the right paths for the desired symbol equalization.

In practice, the receiver does not have the knowledge of the MAI transmitters. Blind estimation of multipath channels for $\mathbf{H}$ and $\mathbf{H}^{(MAI)}$ is a complicated task. On the other hand, with training symbol sequence $\mathbf{x}_{tr} = [x(1), \cdots, x(n_{tr})]^T$ for the desired transmitter, the system model for Least Squares (LS) estimation of equalization filter $\mathbf{f}$ can be formulated as

$$\mathbf{x}_{tr} = \mathbf{Y}_{tr}\mathbf{f} \tag{2}$$

where the block Toeplitz matrix $\mathbf{Y}_{tr}$ consists of samples of received signal and is represented by,

$$\mathbf{Y}_{tr} = \begin{pmatrix} y^{(1)}(1) & \cdots & y^{(n_\tau)}(1) & y^{(1)}(2) & \cdots & y^{(n_\tau)}(n_{ISI}) \\ \vdots & & \vdots & \vdots & & \vdots \\ y^{(1)}(n_{tr}) & \cdots & y^{(n_\tau)}(n_{tr}) & y^{(1)}(n_{tr}+1) & \cdots & y^{(n_\tau)}(n_{tr}+n_{ISI}-1) \end{pmatrix} \tag{3}$$

where $\{y^{(p)}(k),\ p = 1, \ldots, n_\tau\}$ denotes n_τ received signal samples for the *k-th* symbol period, the matrix $\mathbf{Y}_{tr}$ is $n_{tr} \times n_L$ and n_{tr} is the amount of training symbols. Assuming $n_{tr} \geq 2n_L$ in order to obtain a reasonably good result, the LS estimation [8] of the filter $\hat{\mathbf{f}}$ is given by,

$$\hat{\mathbf{f}} = (\mathbf{Y}_{tr}^T\mathbf{Y}_{tr})^{-1}\mathbf{Y}_{tr}^T\mathbf{x}_{tr} \tag{4}$$

Tap selection can be utilized to effectively reduce the requirement of training symbols to $n_{tr} \geq 2n_S$, where n_S is the number of taps to be selected and $n_S \ll n_L$. Let $n_{eff} \approx n_{ISI}(1+n_{MAI})$ be the approximate number of effective transmitters due to ISI and MAI (more accurately, n_{eff} can be estimated by the number of significant eigenvalues associated with the covariance matrix of the received signal samples). It is important to have $n_S \geq n_{eff}$ to insure the order of diversity for effective ISI and MAI mitigation.

3. MATCHING PURSUIT BASED TAP SELECTION TECHNIQUE

Tap selection can be considered as forming an observation sample matrix $\mathbf{Y}_S$ by selecting a subset of columns from $\mathbf{Y}_{tr}$ which corresponds to a subset of taps to minimize the LS estimation error

$$\mathcal{E}_S = \mathbf{x}_{tr}^T\mathbf{x}_{tr} - \mathbf{x}_{tr}^T\mathbf{P}_S\,\mathbf{x}_{tr} \tag{5}$$

or equivalently to maximize the projection power

$$\|\mathbf{P}_S\mathbf{x}_{tr}\|^2 = \mathbf{x}_{tr}^T\mathbf{P}_S\,\mathbf{x}_{tr} \tag{6}$$

where the projection matrix

$$\mathbf{P}_S = \mathbf{Y}_S(\mathbf{Y}_S^T\mathbf{Y}_S)^{-1}\mathbf{Y}_S^T \tag{7}$$

has the property of $\mathbf{P}_S^T\mathbf{P}_S = \mathbf{P}_S = \mathbf{P}_S^T$ [8].

For optimal selection of n_S taps out of total n_L taps, an exhaustive search is needed and this requires $\mathbf{P}_S$ to be evaluated for $\binom{n_L}{n_S}$ possible combination of the tap subsets. This is not feasible for large n_L. The near optimal tap selection can be achieved by applying greedy algorithm to incrementally select the taps one by one. But this may still be too complicated to implement in real time. On the other hand, tap selection can be considered as a subset selection problem [9]. This problem can be solved by MP algorithm [7] which is a fast algorithm that progressively refines the signal approximation with an iterative procedure. Thus an MP algorithm based tap selection method is detailed as follows and is with the computational complexity on the order of $\mathcal{O}(n_S n_{tr} n_L)$.

MP Based Tap Selection Method

1. Initialize by setting $n=0$ and $\mathbf{x}_{res}^{(0)} = \mathbf{x}_{tr}$
2. Set $n = n+1$. Select a new column $\mathbf{y}_s$ by $s = \arg\max_k \|\mathbf{P}_{\mathbf{y}_k}\mathbf{x}_{res}^{(n-1)}\|^2$, where k denotes the unselected column indices for $\mathbf{Y}_{tr}$, the projection matrix for column vector $\mathbf{y}_k$ is defined as $\mathbf{P}_{\mathbf{y}_k} = \frac{\mathbf{y}_k\mathbf{y}_k^T}{\|\mathbf{y}_k\|^2}$ and the projection power of signal $\mathbf{x}_{res}^{(n-1)}$ is given as $\|\mathbf{P}_{\mathbf{y}_k}\mathbf{x}_{res}^{(n-1)}\|^2 = (\mathbf{x}_{res}^{(n-1)})^T\mathbf{P}_{\mathbf{y}_k}\mathbf{x}_{res}^{(n-1)} = \frac{|\mathbf{y}_k^T\mathbf{x}_{res}^{(n-1)}|^2}{\|\mathbf{y}_k\|^2}$.
3. Compute and update the residue $\mathbf{x}_{res}^{(n)} = \mathbf{x}_{res}^{(n-1)} - \mathbf{P}_{\mathbf{y}_s}\mathbf{x}_{res}^{(n-1)}$, continue the iteration loop for tap selection until $n = n_S$

On the other hand, the MMSE detection for system (1) can be written as

$$\hat{x}(n) = \mathbf{f}_{MMSE}^T\mathbf{y} = B_1 x(n) + I_{residual}^{(ISI)} + I_{residual}^{(MAI)} + \mathbf{f}_{MMSE}^T\mathbf{m} \tag{8}$$

where assume that the desired symbol $x(n)$ is the first element in $\mathbf{x}$. $\mathbf{f}_{MMSE}$ denotes the equalization filter. $B_1 = (\mathbf{f}_{MMSE}^T\mathbf{H})_1$. $I_{residual}^{(ISI)}$ and $I_{residual}^{(MAI)}$ represent the residual ISI and MAI respectively.

It is observed that under finite training sample support ($n_{tr} < n_L$), incorrectly estimated $\hat{\mathbf{f}}$ may amplify the noise term in (8) with variance of $\|\hat{\mathbf{f}}\|^2\sigma_m^2$. The term $\|\hat{\mathbf{f}}\|^2$ plays an important role in minimizing the Mean Square Error (MSE). This suggests the use of Quadratic Constraint (QC) ($\|\mathbf{f}\|^2 = g_0$) for minimizing MSE during the tap selection process, where g_0 is a constraining value. Given column $\mathbf{y}_k$ to be chosen at the n-th iteration and the signal estimation as $\hat{\mathbf{x}}^{(n)} = \mathbf{y}_k f_n$ where f_n is the n-th selected tap coefficient for $\mathbf{f}$, with QC, the estimation problem becomes,

$$\min_{f_n} \|\mathbf{x}_{res}^{(n-1)} - \hat{\mathbf{x}}^{(n)}\|^2 \;\; subject\ to \;\; \|\mathbf{f}\|^2 = g_0 \tag{9}$$

As in MP algorithm, filter coefficients $\{f_k,\ k=1,\cdots,n_S\}$ are estimated one by one separately in each iteration. At the n-th iteration, QC can be rewritten as $f_n^2 = g_0 - \sum_{k=1}^{n-1} f_k^2 = g_n$, where g_n is the constraining value for f_n^2. Using method of Lagrange multipliers [8], the new cost function for minimization is given by

$$\mathcal{E}_n^{(QC)} = (\mathbf{x}_{res}^{(n-1)} - \mathbf{y}_k f_n)^T(\mathbf{x}_{res}^{(n-1)} - \mathbf{y}_k f_n) + \lambda\,({f_n}^2 - g_n) \tag{10}$$

where λ is a Lagrange multiplier. This can be solved for f_n as follows,

$$\hat{f}_n^{(QC)} = \frac{\mathbf{y}_k^T\mathbf{x}_{res}^{(n-1)}}{\|\mathbf{y}_k\|^2 + \lambda} \tag{11}$$

Let $\lambda = n_{tr}\sigma_L^2$, where σ_L^2 is defined as the loading level. Incorporating QC can be regarded as introducing Diagonal Loading (DL) [10], which has been a popular tool for adding robustness to adaptive array beamforming, into MP based tap selection algorithm. Defining Loading-to-Noise Ratio (LNR) as $LNR = \frac{\sigma_L^2}{\sigma_n^2}$, the DL parameter LNR can be optimally decided by L-Curve analysis as given in [11]. For simplicity, we found that LNR can be determined empirically through simulation, that is, an appropriate $LNR(\approx 10 \sim 30)$ can be chosen after a few trials.

The DL technique mitigates the problem due to mismatch between training samples and test samples. This can be interpreted as to artificially introduce a higher fixed noise floor (σ_L^2) that has the effect of preventing the algorithm from selecting the insignificant column $\mathbf{y}_k$ when $\|\mathbf{y}_k\|^2 \ll n_{tr}\sigma_L^2$. Substituting (11) into (10) and discarding the last term ($\lambda\, g_n$) which is a constant, the cost function with DL becomes,

$$\mathcal{E}_n^{(DL)} = (\mathbf{x}_{res}^{(n-1)})^T\mathbf{x}_{res}^{(n-1)} - (\mathbf{x}_{res}^{(n-1)})^T\mathbf{P}_{\mathbf{y}_k}^{(DL)}\mathbf{x}_{res}^{(n-1)} \tag{12}$$

where $\mathbf{P}_{\mathbf{y}_k}^{(DL)}$ is given by

$$\mathbf{P}_{\mathbf{y}_k}^{(DL)} = \frac{\mathbf{y}_k\mathbf{y}_k^T}{\|\mathbf{y}_k\|^2 + n_{tr}\sigma_L^2} \tag{13}$$

Thus, *MP Based Tap Selection Method with DL* is obtained by replacing matrix $\mathbf{P}_{\mathbf{y}_k}$ with $\mathbf{P}_{\mathbf{y}_k}^{(DL)}$ in *MP Based Tap Selection Method*.

Due to the introduction of DL, matrix $\mathbf{P}_{\mathbf{y}_k}^{(DL)}$ no longer represents an orthogonal projection onto the subspace of vector $\mathbf{y}_k$. But it may still be considered as an orthogonal projection onto the subspace of a virtual vector $\mathbf{y}_k' = \mathbf{y}_k + \mathbf{m}_k$ assuming the introduction of an artificial noise vector $\mathbf{m}_k$. In addition, the reselection of basis [9] should be avoided in the DL modified MP algorithm due to the introduction of virtual basis. The convergence of MP algorithm is shown [7] to depend on the correlation between the residue ($\mathbf{x}_{res}$) and the basis ($\mathbf{y}_k'$). This can be insured by choosing an appropriate loading level for σ_L^2. It is easy to show that excess loading ($\sigma_L^2 \rightarrow \infty$) will result in convergence to the *Strongest Paths* based tap selection method.

Furthermore, an intuitive way for selecting taps can be done by simply choosing a subset with the *Strongest Projections* based on the projection power $\{\|\mathbf{P}_{\mathbf{y}_k}\mathbf{x}_{tr}\|^2 = \mathbf{x}_{tr}^T\mathbf{P}_{\mathbf{y}_k}\mathbf{x}_{tr} = \frac{|\mathbf{y}_k^T\mathbf{x}_{tr}|^2}{\|\mathbf{y}_k\|^2},\ k = 1,\cdots,n_L\}$. This intuitive method is similar to the *Strongest Paths* based tap selection method assuming simple correlation output is used as CIR estimation, that is $\{|\mathbf{y}_k^T\mathbf{x}_{tr}|^2,\ k = 1,\cdots,n_L\}$, and choosing a subset with strongest paths. The computational cost for this *Strongest Projections* or *Strongest Paths* based selection method is on the order of $(\mathcal{O}(n_{tr}n_L))$. The additional cost for selecting n_S strongest taps out of n_L is no more than $(\mathcal{O}(n_S n_L))$. Moreover, there is extra cost of $\mathcal{O}(n_L)$ division operation for the *Strongest Projections* based method. The numerical analysis shows that only a small portion of taps selected by either of these two simple methods is different from the taps selected by *MP Based Tap Selection Method with DL*. A fast heuristic tap selection method can be developed by firstly choosing a large portion of taps using simple *Strongest Projections* or *Strongest Paths* based method and then choosing the rest of taps, i.e., assuming n_{depth} taps, using MP based method. This heuristic MP based tap selection method incorporated with DL is detailed as follows.

Heuristic MP Based Tap Selection Method

- Initialize by setting $n = 0$ and $\mathbf{x}_{res}^{(0)} = \mathbf{x}_{tr}$
- Pre-selection step:
  Select an initial subset of $(n_S - n_{depth})$ taps based on the *Strongest Projections* of $\mathbf{x}_{tr}^T\mathbf{P}_{\mathbf{y}_s}^{(DL)}\mathbf{x}_{tr}$ and update the residue by $\mathbf{x}_{res}^{(n)} = \mathbf{x}_{res}^{(n-1)} - \mathbf{P}_{\mathbf{y}_s}^{(DL)}\mathbf{x}_{res}^{(n-1)}$ for all these pre-selected columns $\{\mathbf{y}_s\}$.
  Set $n = n_S - n_{depth}$
- MP selection step:
  Set $n = n + 1$.
  Select a new column $\mathbf{y}_s$ by
  $s = \arg\max_k (\mathbf{x}_{res}^{(n-1)})^T \mathbf{P}_{\mathbf{y}_k}^{(DL)}\mathbf{x}_{res}^{(n-1)}$ and
  update the residue by $\mathbf{x}_{res}^{(n)} = \mathbf{x}_{res}^{(n-1)} - \mathbf{P}_{\mathbf{y}_s}^{(DL)}\mathbf{x}_{res}^{(n-1)}$.
  Continue the MP iteration until $n = n_S$

The computational complexity for *Heuristic MP Based Tap Selection Method* is about $\mathcal{O}((1+n_{depth})n_{tr}n_L)$. The numerical analysis shows that for larger n_S, *Heuristic MP Based Tap Selection Method* is capable of making a fast selection by choosing $n_{depth} \ll n_S$ with limited performance tradeoff in comparison with *MP Based Tap Selection Method with DL*.

4. SIMULATION RESULTS

The numerical results are obtained based on 100 UWB channel realizations for CM2 [12]. The sampling duration is $\Delta\tau = 0.167ns$ and symbol rate is set as $R_s = 1/T_f = 93.56MHz$ (making $n_\tau = T_f/\Delta\tau$ an integer). Then we have $n_{ISI} = 8$. Suppose $(n_{MAI} = 3)$ MAI transmitters are presented with equal transmission power as the desired transmitter and $(n_S = 32)$ taps are selected out of total $(n_L = 512)$ samples for effective interference mitigation. In addition, for reasonable comparison, the equalizer coefficients for all the simulations with different tap selection based methods are estimated by LS estimation which has the computational complexity of $(\mathcal{O}(n_S^2(n_{tr} + n_S))$.

Figure 1 and 2 illustrate the BER performance for RAKE receiver (where simply using correlation to estimate the CIR from training sequence for implementing RAKE), conventional uniformly spaced MMSE equalization (where, the number of taps is set as $n_{tap} = \frac{n_{tr}}{2}$ and n_{tap} earliest paths are utilized in consideration of exponentially decaying power law for UWB channel, assuming perfect synchronization at the receiver) and nonuniformly spaced MMSE equalization using tap selection methods (*Strongest Paths / Strongest Projections* based method, *Heuristic MP* based method and *MP with DL* based method). The computational cost for tap selection and LS estimation of equalizer coefficients is tabulated as follows.

Equalization Methods	Computational Cost $n_L = 512$, $n_S = 32$, $n_{tr} = 64 \sim 512$
Full RAKE	$\mathcal{O}(n_{tr}n_L)$
MMSE (LS est.)	$\mathcal{O}(n_{tap}^2(n_{tap} + n_{tr}))$, $n_{tap} = \frac{n_{tr}}{2}$
Strongest Paths / Strongest Project. + MMSE	$\mathcal{O}(n_{tr}n_L)$ + $\mathcal{O}(n_S^2(n_S + n_{tr}))$
Heuristic MP + MMSE	$\mathcal{O}((1 + n_{depth})n_{tr}n_L)$ + $\mathcal{O}(n_S^2(n_S + n_{tr}))$, $n_{depth} = 6$
MP + MMSE	$\mathcal{O}(n_S n_{tr} n_L)$ + $\mathcal{O}(n_S^2(n_S + n_{tr}))$

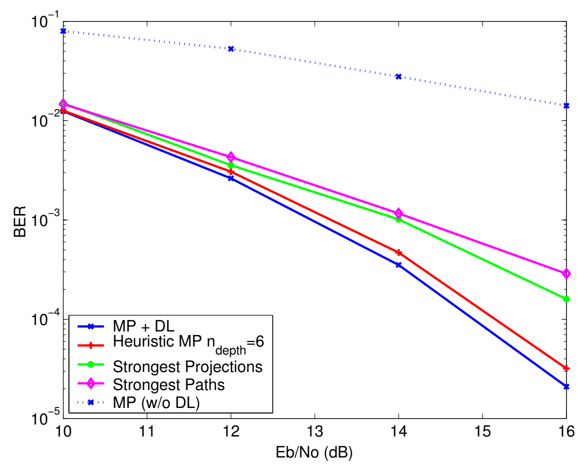

Fig. 1. Performance comparison for different tap selection methods (CM2: $R_s = 93.56Mbps$, $n_{mai} = 3$, $n_S = 32$, $n_{tr} = 64$)

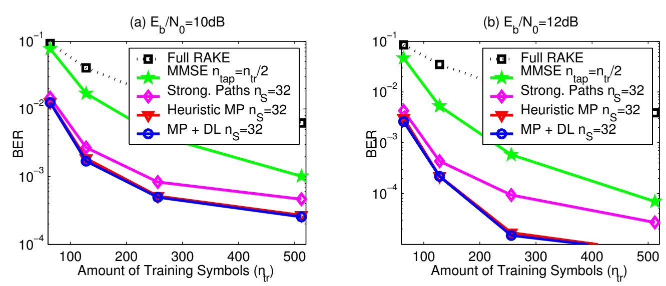

Fig. 2. Performance comparison for different tap selection methods by various amount of training symbols (CM2: $R_s = 93.56Mbps$, $n_{mai} = 3$)

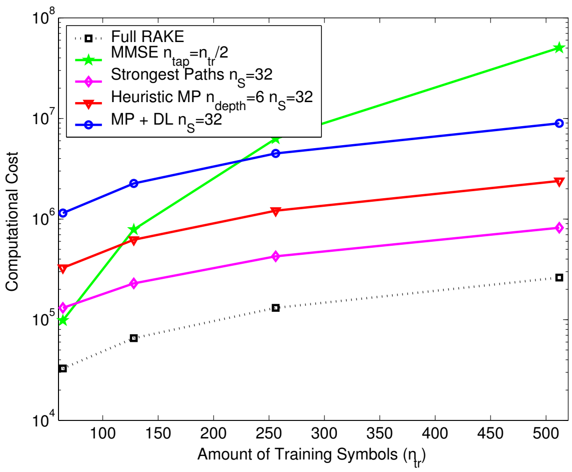

Fig. 3. Computational complexity for different equalization methods (CM2: $R_s = 93.56Mbps$, $n_{mai} = 3$)

Figure 1 shows the importance and effectiveness of introducing DL into MP based tap selection algorithm. *MP w/o DL* performs poorly due to the mismatch between training samples and test data. On the other hand, under appropriate loading σ_L^2, *MP + DL* (MP with DL) achieves much better performance with increased SNR when compared with *Strongest Paths / Strongest Projections* based method. Since the capability of tap selection method for ISI and MAI mitigation becomes more noticeable in the case of higher SNR. From Figure 1 or Figure 2, it is also observed that *RAKE* receiver exhibits performance floor and breaks down in the presence of severe ISI and MAI, although it has the lowest computational cost. In addition, *Strongest Paths / Strongest Projections + MMSE* significantly outperforms conventional *MMSE* equalizer under finite training sample support and is with lower computational cost (when $n_S < n_{tap}$). Moreover, *MP+DL + MMSE* achieves the best performance given the same amount of training symbols (n_{tr}). It outperforms *Strongest Paths / Strongest Projections + MMSE* but is with higher computational cost for tap selection. Furthermore, *Heuristic MP* exhibits only limited performance degradation but with reduced complexity. Figure 3 illustrates the estimated computational complexity for different equalization methods. As the amount of training symbols increases, the computational cost for tap selection based methods increases at a slower pace in comparison with the conventional uniformly spaced MMSE equalizer which requires more taps for achieving a reasonable performance.

5. CONCLUSIONS

In this paper, a novel training sequence based tap selection technique is proposed for UWB channel equalization in the presence of ISI and MAI. This technique effectively reduces the requirement for training symbols and lowers the complexity for the equalizer. For tap selection techniques, it is found that the simple *Strongest Paths / Strongest Projections* based method is preferred under lower SNR level due to its low complexity. On the other hand, MP based tap selection method with DL achieves improved performance with increased SNR when compared with the *Strongest Paths / Strongest Projections* based method. Furthermore, *Heuristic MP* based method is developed for fast tap selection implementation with only limited performance tradeoff. Simulation results show that the proposed tap selection based equalization method significantly outperforms the conventional MMSE equalizer under finite training sample support.

6. REFERENCES

[1] R. A. Scholtz, “Multiple access with time-hopping impulse modulation,” in *Proc. IEEE MILCOM*, 1993, vol. 2, pp. 447-450.

[2] M. Z. Win and R. A. Scholtz, “Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications”. *IEEE Trans. on Commun.*, vol. 48, no. 4, pp. 679-691, Apr. 2000.

[3] Anuj Batra, “Multi-band OFDM Physical Layer Proposal,” Nov. 2003, http://grouper.ieee.org/groups/802/15/pub/.

[4] Q. Li and L. A. Rusch, “Multiuser detection for DS-CDMA UWB in the home environment,” *IEEE J. Select. Areas Commun.*, vol. 20, issue 9, pp. 1701 - 1711, Dec. 2002.

[5] Frederick K. H. Lee and Peter J. McLane, “Design of nonuniformly spaced tapped-delay-line equalizers for sparse multipath channels,” *IEEE Trans. on Commun.*, vol. 52, no. 4, Apr. 2004, pp. 530-535.

[6] Sirikiat Ariyavisitakul, Nelson R. Sollenberger and Larry J. Greenstein, “Tap-selectable decision-feedback equalization,” *IEEE Trans. on Commun.*, vol. 45, no. 12, Dec. 1997, pp. 1497-1500.

[7] S. G. Mallat and Z. Zhang, “Matching pursuits with time-frequency dictionaries,” *IEEE Trans. Signal Processing*, vol. 41, no. 12, pp. 3397-3414, Dec. 1993.

[8] S. Haykin, *Adaptive Filter Theory*. 4th ed., New Jersey: Prentice Hall, 2002.

[9] S. F. Cotter, *Subset Selection Algorithms with Applications*, PhD Dissertation, University of California, San Diego, 2001.

[10] B. D. Carlson, “Covariance matrix estimation errors and diagonal loading in adaptive arrays,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 24, pp.397-401, Jul. 1988.

[11] P.C. Hansen, *Rank-Deficient and Discrete Ill-Posed Problems : Numberical Aspects of Linear Inversion*. SIAM, Philadelphia, 1998.

[12] J. Foerster, “Channel Modeling Sub-Committee Report Final,” Feb. 2003, http://grouper.ieee.org/groups/802/15/pub/.