AN EM-MAP-BLOCK ALGORITHM FOR SEMI-BLIND CHANNEL ESTIMATION

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ABSTRACT

In this paper, we propose a semi-blind MAP (maximum "a posteriori") channel estimation method based on an Expectation-Maximization Block algorithm (EM-MAP-Block). In the OFDM communication context, our algorithm still have a linear arithmetical complexity. However, it yields to an SNR improvement going up to 3.5 dB compared to classical training sequences based channel estimation method in the context of 5 GHz OFDM WLANs. The new algorithm performs better than already existing recursive algorithms and block algorithms.

1. INTRODUCTION

In OFDM transmission context, the problem of inter-symbol interference is avoided by the insertion of cyclic prefix between consecutive transmitted blocks. At the receiver, this results in the frequency domain, after demodulation by the FFT, to scalar multiplication channel effect. Each subcarrier is attenuated by the corresponding narrowband subchannel coefficient to estimate.

Classical method estimates these coefficients relying on known training sequences, assuming that the channel does not vary between two training sequences. In order to cope with Doppler effect due to the mobility of wireless systems, the reference sequences must be repeated more often resulting in a significant loss in the useful bit-rate. Alternatively, performance and mobility can be further enhanced by refining the channel coefficients by maximum likelihood estimation (EM-ML) or by maximum "a posteriori" estimation (EM-MAP). The EM-MAP algorithm [1] provide iteratively accurate estimates of the MAP using the training sequences as initialization of the estimation. EM-ML based semi-blind channel estimation methods have already been proposed in the OFDM context [2], [3]. In the present paper, we propose an EM-MAP-Block algorithm that outperforms the already existing ones.

The paper is organized as follows. In Section 2, we introduce the basic notations and recall the OFDM systems. Section 3 presents the proposed EM-MAP-Block algorithm. Finally in Section 4, we provide some simulations illustrating the performance of our algorithm.

2. DEFINITIONS AND NOTATIONS

We consider a conventional OFDM transceiver scheme depicted in figure 1, in which a base band discrete time model of the system is provided. In this model, some side entries of the size P IFFT are zeros and only N of the P subcarriers available are effectively used. The main idea of OFDM transmissions is to turn the channel convolution effect into a multiplicative one [4]. In this goal, the block of data $\underline{x} = [x_1, ..., x_N]^T$ is modulated in time domain by IFFT processing and some redundancy into the transmitted signal is introduced by cyclic prefix extension so that the overlapping introduced by the channel memory $h = [h_1, ..., h_L]^T$ corresponds to that of a circular convolution of $\underline{x}$ and h. Consequently, the channel is viewed in the frequency domain after demodulation by the FFT as parallel flat fading channels. Hence, the block $\underline{x}$ can easily be retrieved from the corresponding received block $\underline{y} = [y_1, ..., y_N]^T$ by FFT and IFFT. With $\underline{h} = [h_1, ..., h_N]^T$, the OFDM system we consider can be modeled by the following equation:

$$\underline{y} = Diag(\underline{H})\underline{x} + \underline{n} \tag{1}$$

Where $\underline{H} = SF\underline{h}$, F is the $P \times P$ Fourier matrix, and S is the $N \times P$ matrix selecting the N information sub-carriers $S = [\mathbf{0}_{\mathbf{N},\frac{\mathbf{P-N}}{\mathbf{2}}} \ \ \mathbf{I_N} \ \ \mathbf{0}_{\mathbf{N},\frac{\mathbf{P-N}}{\mathbf{2}}}]$.

Note that only the L first components of $\underline{h}$ are not null. L corresponds to the cyclic prefix length. OFDM systems are

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designed such that $L < P$ (in IEEE802.11a $L = P/4$). Each taps are assumed independent and Rayleigh distributed.

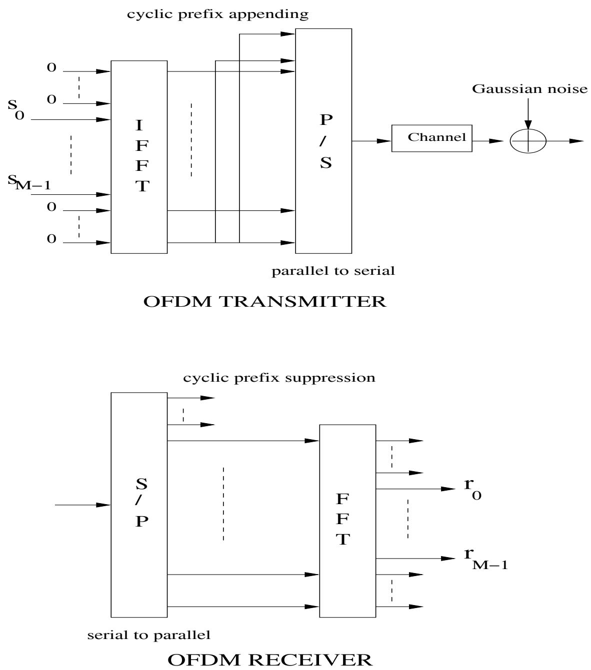

Fig. 1. Conventional OFDM Transceiver

3. THE EM-MAP-BLOCK CHANNEL ESTIMATOR FOR OFDM SYSTEM

In this section we detail our OFDM semi-blind channel estimation method. First, we describe the EM-MAP procedure.

3.1. The EM-MAP algorithm principle

The EM-MAP algorithm is a two-step iterative procedure maximizing the probability "a posteriori" [1], rather than the maximum likelihood in the traditional EM-ML case. It aims, like the EM-ML algorithm, to derive the parameters θ of a system from the observations of its output y (observable) without knowing its input x (hidden variables). The procedure is the following:

- Starting from an estimate $\hat{\theta}^{(0)}$ of the parameter θ
- At iteration i, we perform subsequently the:
  - Expectation step (E-step): compute $Q(\hat{\theta},\ \hat{\theta}^{(i)}) = \mathbb{E}_x[log(P((x,y);\hat{\theta}^{(i)}))]$
  - Maximization step (M-step): $\hat{\theta}^{(i+1)} = argmax_\theta\ Q(\theta,\ \hat{\theta}^{(i)})$

There are many practical modified and extended versions of the EM-ML traditional algorithm [1], which have been conceived to deal with many encountered problems such that local maxima, slow convergence, or to maximize others functions of interest like, in our case, a probability "a posteriori" to better use of prior information. Note that the EM-MAP algorithm use the joint probability $P((x,y);...)$, while the conditional probability $P((x|y);...)$ is used in the EM-ML case.

3.2. The EM-MAP Block algorithm

The EM-MAP algorithm is applied to solve our channel estimation problem by considering a block of observations y_m:

$$y_m = H_m x_m + n_m,\ m = 1, ..., N$$

Note that the EM-MAP estimator requires, to be consistent, many observations y_m for each parameter H_m. With this in mind, and relying on the fact that these channel coefficients H_m are in practice, like in the HIPERLAN/2 [5] wireless LAN context, strongly correlated, we can assume that

$$\underline{y} = X\underline{\tilde{H}} + \underline{n}, \tag{2}$$

where $\underline{\tilde{H}} = [H_0, ..., H_0, ..., H_{q-1}, ..., H_{q-1}]^T$ denotes the total channel vector considered as constant on each sub-block of size T, such that $N = qT$. A simple order 1 AR modelling of the channel variations between blocks is supposed: if we set $\underline{H} = [H_0, ..., H_{q-1}]^T$, one has:

$$A_\theta \underline{H} = \underline{\epsilon},\ A_\theta = \begin{pmatrix} 1 & 0 & . & . & 0 \\ -\theta & 1 & . & . & . \\ 0 & -\theta & . & . & . \\ . & . & . & . & 0 \\ 0 & . & 0 & -\theta & 1 \end{pmatrix},$$

where $\underline{\epsilon}$ is a white Gaussian noise with variance σ^2 and θ is the correlation coefficient. Then,

$$\underline{H} \propto \frac{1}{(\sigma^2)^q} \exp(-\frac{1}{\sigma^2}\,|A_\theta \underline{H}|^2) \tag{3}$$

The noise vector $\underline{n} = [n_0, ..., n_{N-1}]^T$ is supposed to be white Gaussian with variance $1/K$. The hidden variables x_i, are supposed to be uniformly distributed and take values on a M size constellation $\{s_1, ..., s_M\}$. For $\underline{m} = [m_1, ..., m_N]^T \in \{1, ..., M\}^N$, we set $S_{\underline{m}} = Diag(s_{m_1}, ..., s_{m_N})$.
Note that $\underline{\tilde{H}} = B\underline{H}$, where B is the $N \times q$ tensorial product $I_q \otimes [1, ..., 1]^T$, which satisfies for all variables $z_0, ..., z_{N-1}$:

$$B^* Diag(z_0, ..., z_{N-1})B = Diag(\sum_{l=0}^{T-1} z_l, ..., \sum_{l=(q-1)T}^{N-1} z_l) \tag{4}$$

$$B^*[z_0, ..., z_{N-1}]^T = [\sum_{l=0}^{T-1} z_l, ..., \sum_{l=(q-1)T}^{N-1} z_l]^T \tag{5}$$

These important relations will be used to show that the EM-MAP-Block algorithm still have low (linear) arithmetical complexity as recursive algorithms [2].
Using (2) and (3), the auxiliary function for the MAP reads:

$$Q_{MAP}(\underline{H},\underline{H}^{(i)},\sigma,\sigma^{(i)},\theta,\theta^{(i)})$$
$$= \sum_{\underline{m}} \log(P(\underline{y},S_{\underline{m}},\underline{H};\sigma,\theta))P(S_{\underline{m}}|\underline{y},\underline{H}^{(i)},\sigma^{(i)},\theta^{(i)})$$
$$= -K\sum_{\underline{m}} |\underline{y}-S_{\underline{m}}B\underline{H}|^2 P(S_{\underline{m}}|\underline{y},\underline{H}^{(i)},\sigma^{(i)},\theta^{(i)})$$
$$-\frac{1}{\sigma^2}\mid A_\theta \underline{H}\mid^2 - q\log(\sigma^2)+cte$$

In the following asterix denotes trans-conjugated value. By setting to zero the derivative of $Q_{MAP}(\underline{H},\underline{H}^{(i)},\sigma,\sigma^{(i)},\theta,\theta^{(i)})$ with respect to θ, σ^2 and $\underline{H}$, one obtains:

$$\theta = \frac{\sum_{j=1}^{q-1} H_{j-1}^* H_j}{\sum_{j=1}^{q-1}\mid H_{j-1}\mid^2},\quad \sigma^2=\frac{1}{q}\left|A_\theta \underline{H}\right|^2, \tag{6}$$
$$E(\theta,\sigma^2)\underline{H}=V, \tag{7}$$

where $E(\theta,\sigma^2)=\frac{1}{K\sigma^2}A_\theta^* A_\theta + D$,

$$D=\sum_{\underline{m}\in\{1,\dots,M\}^N} B^* S_{\underline{m}}^* S_{\underline{m}} B P(S_{\underline{m}}|\underline{y},\underline{H}^{(i)},\sigma^{(i)},\theta^{(i)}),$$

and $$\underline{V}=\sum_{\underline{m}\in\{1,\dots,M\}^N} B^* S_{\underline{m}}^* \underline{y} P(S_{\underline{m}}|\underline{y},\underline{H}^{(i)},\sigma^{(i)},\theta^{(i)}).$$

Our goal, now, is to solve simply (with linear arithmetical complexity) the equations (6) and (7). Note that, without simplifications, the sums over the multi-index $\underline{m}$ in the expressions of D and $\underline{V}$ lead to exponential arithmetical complexity. However, using equation (4), the matrix $B^*S_{\underline{m}}^*S_{\underline{m}}B$ is diagonal and its j-th component is given by:

$$D_j=\sum_{l=jT}^{(j+1)T-1}\sum_{\underline{m}\in\{1,\dots,M\}^N}\mid s_{m_l}\mid^2 P(S_{\underline{m}}|\underline{y},\underline{H}^{(i)},\sigma^{(i)},\theta^{(i)})$$

As $P(S_{\underline{m}}|\underline{y},\underline{H}^{(i)},\sigma^{(i)})=\prod_{k=1}^N P(s_{m_k}|y_k,\underline{H}^{(i)},\sigma^{(i)},\theta^{(i)})$,

$$D_j=\sum_{l=jT}^{(j+1)T-1}\sum_{m_l=1}^{M}\mid s_{m_l}\mid^2 P(s_{m_l}|y_l,\underline{H}^{(i)},\sigma^{(i)},\theta^{(i)})$$
$$=\sum_{l=jT}^{(j+1)T-1}\frac{\sum_{m=1}^M \mid s_m\mid^2 P(s_m)\exp(-K\mid y_l-H_j^{(i)}s_m\mid^2)}{\sum_{m=1}^M P(s_m)\exp(-K\mid y_l-H_j^{(i)}s_m\mid^2)}$$

Similarly, using (5), the j-th component of $\underline{V}$ reads:

$$V_j=\sum_{l=jT}^{(j+1)T-1} y_l\frac{\sum_{m=1}^M s_m^* P(s_m)\exp(-K\mid y_l-H_j^{(i)}s_m\mid^2)}{\sum_{m=1}^M P(s_m)\exp(-K\mid y_l-H_j^{(i)}s_m\mid^2)}$$

Now, by setting $e_j=\frac{1+\theta^2}{K\sigma^2}+D_j$ for $j=0,\dots,q-2$, and $e_{q-1}=\frac{1}{K\sigma^2}+D_{q-1}$, the matrix $E(\theta,\sigma^2)$ reads:

$$E(\theta,\sigma^2)=\begin{pmatrix} e_0 & -\theta & 0 & . & 0\\ -\theta & e_1 & -\theta & . & .\\ 0 & -\theta & . & . & 0\\ & . & . & . & -\theta\\ 0 & . & 0 & -\theta & e_{q-1}\end{pmatrix}$$

In order to solve (6) and (7), a fixed-point research method is applied: for given values of σ^2 and θ, we set $H_j=a_j+b_jH_0$. Then, from (7) we get:

$$a_0=0,\ a_1=-\frac{v_1}{\theta},\ b_0=1,\ b_1=\frac{e_1}{\theta},$$

and for $j=1,\dots,q-2$,

$$a_{j+1}=-\frac{v_j}{\theta}-a_{j-1}+\frac{e_j}{\theta}a_j,\ b_{j+1}=-b_{j-1}+\frac{e_j}{\theta}b_j.$$

The last coordinate in (7) reads $-\theta H_{q-1}+e_qH_q=v_q$, which gives $H_0=\frac{v_{q-1}+\theta a_{q-2}-e_{q-1}a_{q-1}}{e_{q-1}b_{q-1}-\theta b_{q-2}}=\frac{a_q}{b_q}$. So, the sequences a_j and b_j are calculated recursively and the channel coefficients are obtained. We, then, update the variance and correlation values using (6). A new iteration (inner iteration for the fixed-point research) is, then, restarted for computing the channel coefficients, and so on. Let $a_j^{(i)}$ and $b_j^{(i)}$ be the coefficients obtained after some number (three in our case) of these inner iterations, then the update formula, at iteration $i+1$ of the EM-MAP-Block algorithm, are the following:

$$H_0^{(i+1)}=\frac{v_{q-1}+\theta a_{q-2}^{(i)}-e_{q-1}a_{q-1}^{(i)}}{e_{q-1}b_{q-1}^{(i)}-\theta b_{q-2}^{(i)}}=\frac{a_q^{(i)}}{b_q^{(i)}},$$
$$H_j^{(i+1)}=a_j^{(i)}+b_j^{(i)}H_0^{(i+1)},\ j=1,\dots,q-1 \tag{8}$$
$$\theta^{(i+1)}=\frac{\sum_{j=0}^{q-2}(H_j^{(i+1)})^* H_{j+1}^{(i+1)}}{\sum_{j=0}^{q-2}\mid H_j^{(i+1)}\mid^2} \tag{9}$$
$$(\sigma^2)^{(i+1)}=\frac{1}{q}\left|A_{\theta^{(i+1)}}\underline{H}^{(i+1)}\right|^2 \tag{10}$$

This results in a linear arithmetical complexity proportional to $qTM=NM$ for the computation of the $H_j^{(i+1)}$'s, $(\sigma^2)^{(i+1)}$ and $\theta^{(i+1)}$. Thus, the complexity of the EM-MAP-Block still linear .

4. SIMULATIONS

The simulations have been performed in the HIPERLAN/2 [5] wireless LAN context: a $N=64$ carrier 20 MHz bandwidth broadband wireless system operating in the 5 GHz band using a 16 sample CP. A rate $R=1/2$, constraint length $l=7$ Convolutional Code (CC) (171/133) is used before bit interleaving followed by 16-QAM mapping.

Monte Carlo simulations are run and averaged over 5000 realizations of a BRANC [6] frequency selective channel in order to obtain BER curves. Each frame processed contains 2 known training symbols, followed by 100 OFDM data symbols. The channel estimation process is made using the EM-MAP-Block algorithm described in section 3, with 4 iterations in which the symbol probability $P(s_m)$ is given by the product of the coded bits probabilities composing the symbol s_m: $P(s_m)=\prod_{l=1}P(b_l^m)$. The bit probabilities $P(b_l^m)$ estimates are performed in the E-step by two iterations of the turbodemodulation process [7], which is an iterative joint demapping algorithm yielding a better estimation of $P(b_l^m)$

than classical methods like BJCR algorithm [8]. The block size is set $T = 10$. Note that our results are robust to the block size parameter choice. Indeed, similar results are obtained with blocks of size $T = 4$.

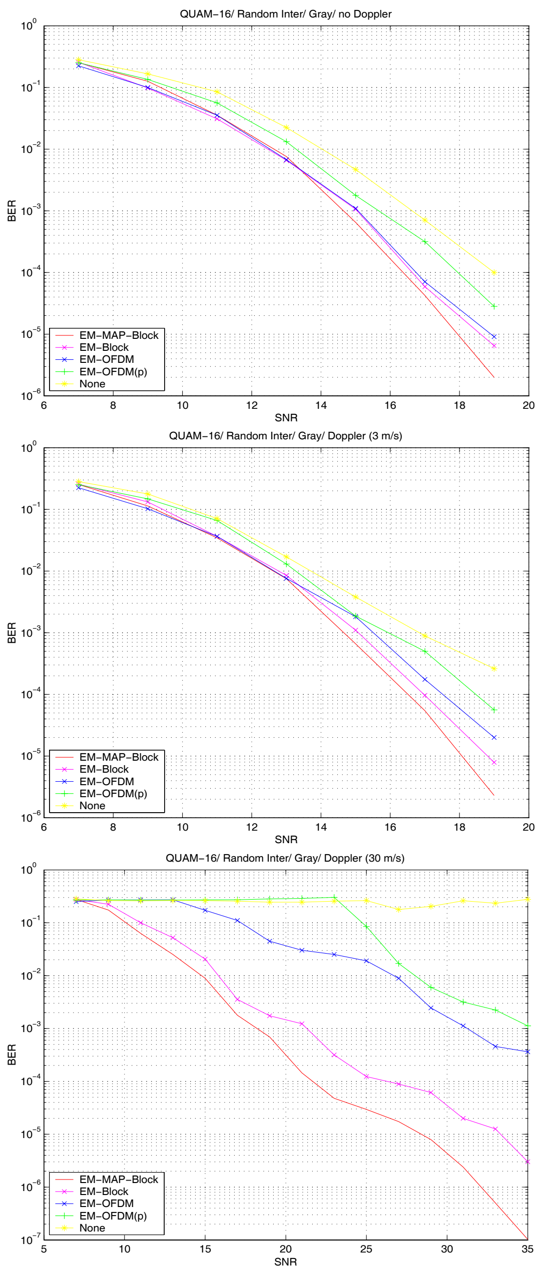

Fig. 2. Simulation results for a BRAN C channel with a mobility of: Top: 0 m/s, Middle: 3 m/s, Bottom: 30 m/s

Figure 2 represent, respectively from the top to the bottom, the results obtained for a static channel and for a channel mobility of 3 m/s and 30 m/s. The curves obtained by the EM-MAP-Block algorithm are compared to a system without tracking between training sequences called 'none' and to curves obtained by two EM-based channel estimation recursive algorithms presented in [2] called 'EM-OFDM' and 'EM-OFDM(p)'. The 'EM-OFDM' algorithm suppose perfect knowledge of the channel variances of the channel coefficients H_m, while in the 'EM-OFDM(p)' case these variances are supposed to follow a linear decreasing profile. The comparison is also made with the results of the EM-Block algorithm [3], which is a traditional EM-ML algorithm associated to (2).

We observe that our EM-MAP-Block algorithm enables a gain of 3.5 db compared to the classical ('none') method and of 2 db, going up to 2.5 db, compared to the EM-OFDM(p). Note that in the case of low mobility, the performances of the EM-MAP-Block algorithm are as good as those of 'EM-Block' and 'EM-OFDM'. The last one, however, is relying on perfect knowledge of the channel coefficients variances, while the EM-Block algorithm enhance performances in channel estimation without the need of such an unrealistic assumption. In the case of high mobility, the EM-MAP-Block algorithm is clearly better than all the others. For example, to obtain a BER of 10^{-14}, the EM-MAP-Block requires an SNR of $22\,dB$, while an SNR of $26\,dB$ is required by EM-Block.

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