Blind MIMO FIR Channel Identification by Exploiting Channel Order Disparity

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***Abstract*— In this paper, we consider the problem of blind multiple-input multiple-output (MIMO) finite impulse response (FIR) channel identification driven by spatially uncorrelated and temporally white input signals. A method that can entirely identify the MIMO channel based only on the second-order statistics (SOS) of the observed data is proposed. The complete identification of the convolutive mixture is accomplished by exploiting the diversity of the channel orders. The uniqueness of the proposed solution is proved. Numerical simulation results are presented to illustrate the performance of the proposed algorithm.**

I. INTRODUCTION

Blind identification of MIMO FIR channel arises in a wide variety of communication and signal processing applications, which include speech enhancement, wireless mobile communications and brain signal analysis. Thus far, there have been a lot of research works [1]–[4] on blind channel identification driven by spatially uncorrelated and temporally white input signals. In this case, most existing SOS-based methods can only identify the channel up to an unknown unitary matrix. To further resolve this instantaneous mixture, additional information such as higher order statistics (HOS) or signal constellation features needs to be exploited. Naturally, the following question arises: is it possible for us to completely identify the MIMO channel based solely on the second-order statistics of the observed data without utilizing the additional information of the transmitted signals? The answer is positive and it seems that the first solution to this problem was proposed in [5]. In [5], it was shown that the entire identification of the convolutive mixture can be achieved via the second-order statistics of the observed data by exploiting the channel order disparities, i.e. the channel orders of each pair of users are different from each other. Later in the work [6], the authors proposed a modified matrix outer-product decomposition method which can identify and equalize the user channel that has the longest channel order using the second-order statistics of the channel output. It is also noticed that some results in [7] can be reformulated into the linear MIMO setting and the reformulated results also show that the diversity of the channel orders suffices for complete channel identifiability via second-order statistics.

In this paper, we propose a SOS-based method that exploits the diversity of the channel orders for complete channel identification. This work is a further result of our previous work [8] that considers the colored source signals with *a priori* known statistics. In fact, in some sense, white signals can be deemed as a special form of the colored signals with its power spectrum being flat and its statistics known *a priori*. Thus the proposed method in [8] can be directly extended to our case and we will show that, given that the channel orders of each pair of users are different, the MIMO channel can be completely identified. By exploiting the derived property of the one-lag down and up shift square matrices, we provide a proof for the uniqueness of the system solution, which serves as a theoretical basis for our method.

We adopt the following notations throughout this paper. The notations $[\cdot]^T$, $[\cdot]^*$, $[\cdot]^H$ and $[\cdot]^\dagger$ stand for transpose, complex conjugate, Hermitian transpose and the Moore-Penrose pseudo-inverse, respectively. $E[\cdot]$ represents the mathematical expectation. $\|\mathbf{A}\|$ ($\|\mathbf{a}\|$) denotes the Frobenius norm (vector 2-norm) of matrix $\mathbf{A}$ (vector $\mathbf{a}$). The symbol $\mathbf{J}_n$ stands for the $n\times n$ one-lag down shift square matrix whose first sub-diagonal entries below the main diagonal are unity, whereas all remaining entries are zero; $\mathbf{I}_n$ denotes the $n\times n$ identity matrix. Let $\mathbf{A}[r_1 : r_2, c_1 : c_2]$ denote the sub-matrix of $\mathbf{A}$ from r_1^{th} row to r_2^{th} row and from c_1^{th} column to c_2^{th} column. $\mathbb{C}^{n\times m}$ and $\mathbb{C}^n$ denote the set of $n\times m$ matrices and the set of n-dimensional column vectors with complex entries, respectively.

II. SYSTEM MODEL AND BASIC ASSUMPTIONS

Consider a noisy linear MIMO channel with p inputs, $s_i(n), i \in \{1, 2, \cdots, p\}$, and q outputs $\mathbf{x}(n) \triangleq [x_1(n) \ \cdots \ x_q(n)]$

$$\mathbf{x}(n) = \sum_{i=1}^{p}\sum_{l=0}^{L_i}\mathbf{h}_i(l)s_i(n-l) + \mathbf{w}(n) \tag{1}$$

where $\{\mathbf{h}_i(l)\}$ denotes the multichannel filter corresponding to the i^{th} user, L_i represents the channel order corresponding to the i^{th} user. By stacking the channel output vector $\mathbf{x}(n)$ and defining $\vec{\mathbf{x}}(n) \triangleq [\mathbf{x}^T(n) \ \mathbf{x}^T(n-1) \ \dots \ \mathbf{x}^T(n-N)]^T$, $\vec{\mathbf{s}}_i(n) \triangleq [s_i(n) \ s_i(n-1) \ \cdots \ s_i(n-N-L_i)]^T$ and $\vec{\mathbf{w}}(n) \triangleq [\mathbf{w}^T(n) \ \mathbf{w}^T(n-1) \ \dots \ \mathbf{w}^T(n-N)]^T$, we can rewrite Eqn.(1) as

$$\vec{\mathbf{x}}(n) = \sum_{i=1}^{p}\mathcal{H}_i\vec{\mathbf{s}}_i(n) + \vec{\mathbf{w}}(n) = \mathcal{H}\vec{\mathbf{s}}(n) + \vec{\mathbf{w}}(n) \tag{2}$$

where $\mathcal{H}_i \in \mathbb{C}^{(N+1)q\times d_i}$ is a block Toeplitz matrix written as follows with $d_i \triangleq N + L_i + 1$

$$\mathcal{H}_i \triangleq \begin{bmatrix} \mathbf{h}_i(0) & \dots & \mathbf{h}_i(L_i) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_i(0) & \dots & \mathbf{h}_i(L_i) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{h}_i(0) & \dots & \mathbf{h}_i(L_i) \end{bmatrix}$$

$$\mathcal{H} \triangleq \begin{bmatrix} \mathcal{H}_1 & \mathcal{H}_2 & \cdots & \mathcal{H}_p \end{bmatrix}$$

$$\vec{\mathbf{s}}(n) \triangleq \begin{bmatrix} \vec{\mathbf{s}}_1^T(n) & \vec{\mathbf{s}}_2^T(n) & \cdots & \vec{\mathbf{s}}_p^T(n) \end{bmatrix}^T$$

Some basic assumptions are adopted as follows. A1) The number of sources is known *a priori*, and there are more outputs than inputs,

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i.e. $q > p$. A2) Channel is irreducible and column-reduced. A3) The channel order of each source is assumed to be known *a priori*. A4) The sources are zero-mean spatially uncorrelated and temporally white signals. A5) Additive noises are spatially uncorrelated and temporally white noises, and they are statistically independent of the sources. As a consequence of A2, the MIMO channel matrix $\mathcal{H}$ is full column rank if the stack number N is chosen to satisfy $N+1 \geq \sum_{i=1}^{p} L_i$ [9]. In the sequel, we assume that $\mathcal{H}$ is full column rank.

III. Proposed Channel Identification Method

We begin by defining the source autocorrelation matrices as follows

$$\mathbf{R}_{s_i}[k] \triangleq E[\vec{\mathbf{s}}_i(n)\vec{\mathbf{s}}_i^H(n-k)] \tag{3}$$

$$\mathbf{R}_{s}[k] \triangleq E[\vec{\mathbf{s}}(n)\vec{\mathbf{s}}^H(n-k)] \tag{4}$$

Also, in order to simplify the presentation of the proposed channel identification method, we assume the noiseless case. Thus the autocorrelation matrices of the received data $\vec{\mathbf{x}}(n)$ can be written as

$$\mathbf{R}_{x}[k] \triangleq E[\vec{\mathbf{x}}(n)\vec{\mathbf{x}}^H(n-k)] = \mathcal{H}\mathbf{R}_s[k]\mathcal{H}^H \tag{5}$$

In the following, we will show that, given that the channel orders of each pair of users are different from each other, the channel convolution matrix $\mathcal{H}$ can be identified up to a block diagonal matrix $\mathcal{D} \triangleq \mathrm{diag}(\lambda_1 \mathbf{I}_{d_1}, \cdots, \lambda_p \mathbf{I}_{d_p})$ instead of an unknown unitary matrix by utilizing the estimated channel output autocorrelation matrices $\mathbf{R}_x[k], k = \{0, \pm 1\}$, where $d_i \triangleq N + L_i + 1$. We commence by introducing the following lemma.

Lemma 1: Given $\mathbf{R}_x[k] = \mathcal{H}\mathbf{R}_s[k]\mathcal{H}^H$, $\mathcal{H}$ is full column rank and $\mathbf{R}_s[0]$ is invertible, then we have

$$\mathbf{R}_x[k]\mathbf{R}_x^{\dagger}[0] = \mathcal{H}\mathbf{R}_s[k]\mathbf{R}_s^{-1}[0]\mathcal{H}^{\dagger} \tag{6}$$

$$\mathbf{R}_x[k]\mathbf{R}_x^{\dagger}[0]\mathcal{H} = \mathcal{H}\mathbf{R}_s[k]\mathbf{R}_s^{-1}[0] \tag{7}$$

Proof: This lemma can be easily proved since we have $\mathbf{R}_x^{\dagger}[0] = (\mathcal{H}^H)^{\dagger}\mathbf{R}_s^{-1}[0]\mathcal{H}^{\dagger}$ which satisfies the four *Moore-Penrose conditions* [10]. ■

For convenience, let

$$\Upsilon_{2k-1} \triangleq \mathbf{R}_x[k]\mathbf{R}_x^{\dagger}[0] \qquad \Upsilon_{2k} \triangleq \mathbf{R}_x[-k]\mathbf{R}_x^{\dagger}[0]$$
$$\Theta_{2k-1} \triangleq \mathbf{R}_s[k]\mathbf{R}_s^{-1}[0] \qquad \Theta_{2k} \triangleq \mathbf{R}_s[-k]\mathbf{R}_s^{-1}[0]$$

We can therefore re-express Eqn.(7) (choose $K \geq k \geq 1$) as

$$\Upsilon_{\bar{k}}\mathcal{H} = \mathcal{H}\Theta_{\bar{k}} \qquad \forall\ \bar{k} \in \{1, \ldots, 2K\} \tag{8}$$

and further, for every $\bar{k} \in \{1, \ldots, 2K\}$, we have the following by exploiting the block diagonal structure of $\Theta_{\bar{k}} \triangleq \mathrm{diag}(\Theta_{\bar{k},1}, \Theta_{\bar{k},2}, \cdots, \Theta_{\bar{k},p})$ ($\Theta_{\bar{k}}$ is a block diagonal matrix because the sources are uncorrelated with each other)

$$\Upsilon_{\bar{k}}\mathcal{H}_i = \mathcal{H}_i\Theta_{\bar{k},i} \qquad \forall\ i \in \{1, \ldots, p\} \tag{9}$$

where $\Theta_{\bar{k},i} \triangleq \mathbf{R}_{s_i}[k]\mathbf{R}_{s_i}^{-1}[0]$, $k = (\bar{k}+1)/2$ if $\bar{k}$ is odd and $k = -\bar{k}/2$ if $\bar{k}$ is even. For each $i \in \{1, \ldots, p\}$, the above equation can be used to identify the channel convolution matrix of user i, i.e. $\mathcal{H}_i$, since the knowledge of $\Theta_{\bar{k},i}$ is known *a priori* and the information of $\Upsilon_{\bar{k}}$ can be obtained from the second-order statistics of the observed data. By exploiting the block Toeplitz structure of $\mathcal{H}_i$, we can rewrite Eqn.(9) as

$$\mathcal{T}_1[\Upsilon_{\bar{k}}]\mathbf{h}_i = \mathcal{T}_2[\Theta_{\bar{k},i}]\mathbf{h}_i \tag{10}$$

where $\mathbf{h}_i \triangleq \left[\mathbf{h}_i^T(0) \ \ldots \ \mathbf{h}_i^T(L_i)\right]^T$, $\mathcal{T}_1[\cdot]$ and $\mathcal{T}_2[\cdot]$ respectively represent a certain transformation on the bracketed matrix. Therefore we may estimate $\mathbf{h}_i$ by the following criterion

$$\hat{\mathbf{h}}_i = \arg\min_{\|\mathbf{h}_i\|=1} \sum_{\bar{k}=1}^{2K} \left\| \left[\ \mathcal{T}_1[\Upsilon_{\bar{k}}] - \mathcal{T}_2[\Theta_{\bar{k},i}]\ \right] \mathbf{h}_i \right\|^2 \tag{11}$$

The above optimization has a closed-form solution which can be obtained as the right singular vector associated with the smallest singular value. However, this criterion fails to provide the true channel estimation if the solution to Eqn.(10) is not unique, i.e. there exist other non-zero vectors, $\mathbf{g}_i$, that are linearly independent of $\mathbf{h}_i$ and also satisfy $\mathcal{T}_1[\Upsilon_{\bar{k}}]\mathbf{g}_i = \mathcal{T}_2[\Theta_{\bar{k},i}]\mathbf{g}_i$ for any $\bar{k} \in \{1, \ldots, 2K\}$. Hence we are faced with the following problem, that is, under what conditions the solution of Eqn.(10) will be unique. This problem is studied in the following and we will show that, under the condition that *the channel order of user i is different from the channel orders of other users*, the uniqueness of the solution to Eqn.(10) can be established by utilizing only $\mathbf{R}_x[0]$ and $\mathbf{R}_x[\pm 1]$, i.e. the uniqueness of the solution can be guaranteed by choosing $\bar{k} = 1, 2$ in Eqn.(10). Under the assumption A4 that the sources are spatially uncorrelated and temporally white, we have

$$\Theta_1 = \mathbf{R}_{s_i}[1]\mathbf{R}_{s_i}^{-1}[0] = \mathrm{diag}\left(\mathbf{J}_{d_1}, \mathbf{J}_{d_2}, \cdots, \mathbf{J}_{d_p}\right) \tag{12}$$

and

$$\Theta_2 = \mathbf{R}_{s_i}[-1]\mathbf{R}_{s_i}^{-1}[0] = \mathrm{diag}\left(\mathbf{J}_{d_1}^T, \mathbf{J}_{d_2}^T, \cdots, \mathbf{J}_{d_p}^T\right) \tag{13}$$

In order to prove the uniqueness of the solution to Eqn.(10), we, firstly, introduce the following lemma that exploits the properties of the one-lag down and up shift square matrices.

Lemma 2: Given that $\mathbf{Y} \in \mathbb{C}^{m \times n}$ satisfies the following two equations

$$(a)\ \ \mathbf{J}_m\mathbf{Y} = \mathbf{Y}\mathbf{J}_n \quad (b)\ \ \mathbf{J}_m^T\mathbf{Y} = \mathbf{Y}\mathbf{J}_n^T \tag{14}$$

then we have

- If $m = n$, then $\mathbf{Y} = \lambda\mathbf{I}$, where λ could be any complex scalar including zero.
- If $m \neq n$, then $\mathbf{Y} = \mathbf{0}$.

Proof: See Appendix A. ■

We now prove the uniqueness of the system solution to Eqn.(10) by utilizing the above lemma. We, firstly, prove that the solution to Eqn.(9) is unique (up to a scalar factor). The problem is formulated as the following theorem.

Theorem 1: Given that (note that the following two equations are directly from Eqn.(6))

$$(a)\ \ \Upsilon_1 = \mathcal{H}\Theta_1\mathcal{H}^{\dagger} \quad (b)\ \ \Upsilon_2 = \mathcal{H}\Theta_2\mathcal{H}^{\dagger} \tag{15}$$

If $\mathcal{H}$ is full column rank and the channel order of user i is different from those of other users, i.e. $L_i \neq L_j$ for $j \in \{1, \ldots, i-1, i+1, \ldots, p\}$, then any non-zero matrix $\mathcal{G}_i$, that has the same block Toeplitz structure as $\mathcal{H}_i$ and also satisfies Eqn.(9) for $\bar{k} = 1, 2$, i.e. $\Upsilon_1\mathcal{G}_i = \mathcal{G}_i\Theta_{1,i}$ and $\Upsilon_2\mathcal{G}_i = \mathcal{G}_i\Theta_{2,i}$, can be written as $\mathcal{G}_i = \lambda_i\mathcal{H}_i$, where λ_i is a non-zero complex scalar.

Proof: Suppose a non-zero matrix $\mathcal{G}_i \in \mathbb{C}^{(N+1)q \times d_i}$ with the same block Toeplitz structure as $\mathcal{H}_i$ also satisfies Eqn.(9) for $\bar{k} = 1, 2$, then we have

$$\Upsilon_1\mathcal{G}_i = \mathcal{G}_i\Theta_{1,i} \Rightarrow \mathcal{H}\Theta_1\mathcal{H}^{\dagger}\mathcal{G}_i = \mathcal{G}_i\Theta_{1,i} \Rightarrow \Theta_1\mathcal{H}^{\dagger}\mathcal{G}_i = \mathcal{H}^{\dagger}\mathcal{G}_i\Theta_{1,i} \tag{16}$$

$$\Upsilon_2\mathcal{G}_i = \mathcal{G}_i\Theta_{2,i} \Rightarrow \mathcal{H}\Theta_2\mathcal{H}^{\dagger}\mathcal{G}_i = \mathcal{G}_i\Theta_{2,i} \Rightarrow \Theta_2\mathcal{H}^{\dagger}\mathcal{G}_i = \mathcal{H}^{\dagger}\mathcal{G}_i\Theta_{2,i} \tag{17}$$

Let $\mathbf{X} \triangleq \mathcal{H}^\dagger \mathcal{G}_i \triangleq \begin{bmatrix} \mathbf{X}_1^T & \cdots & \mathbf{X}_p^T \end{bmatrix}^T$, where $\mathbf{X}_j \in \mathbb{C}^{d_j \times d_i}$, then we have

$$\Theta_{1,j}\mathbf{X}_j = \mathbf{X}_j\Theta_{1,i} \;\; \forall \;\; j \in \{1, \dots, p\} \tag{18}$$

$$\Theta_{2,j}\mathbf{X}_j = \mathbf{X}_j\Theta_{2,i} \;\; \forall \;\; j \in \{1, \dots, p\} \tag{19}$$

Since $\Theta_{1,i} = \mathbf{J}_{d_i}$, $\Theta_{2,i} = \mathbf{J}_{d_i}^T$ and $d_i \neq d_j$ for $j \in \{1, \dots, i-1, i+1, \dots, p\}$ (note that $L_i \neq L_j$ is equivalent to $d_i \neq d_j$ since $d_i = N + L_i + 1$), by utilizing Lemma 2, we have $\mathbf{X}_j = \mathbf{0}$ for any $j \neq i$ and $\mathbf{X}_j = \lambda_i \mathbf{I}_{d_i}$ for $j = i$, i.e.

$$\mathcal{H}^\dagger \mathcal{G}_i = \begin{bmatrix} \mathbf{0} & \cdots & \lambda_i \mathbf{I}_{d_i} & \cdots & \mathbf{0} \end{bmatrix}^T \triangleq \lambda_i \mathbf{E}_i \tag{20}$$

Therefore we only need to prove that the solution of $\mathcal{G}_i$ that satisfies Eqn.(20) is unique and $\mathcal{G}_i = \lambda_i \mathcal{H}_i$. Notice that $\mathcal{G}_i$ has the same block Toeplitz structure as $\mathcal{H}_i$. If we write $\mathcal{H}^\dagger \triangleq [\mathbf{V}_0 \; \cdots \; \mathbf{V}_N]$, we can transform $\mathcal{H}^\dagger \mathcal{G}_i = \lambda_i \mathbf{E}_i$ as

$$\mathcal{V} \begin{bmatrix} \mathbf{g}_i(0) \\ \vdots \\ \mathbf{g}_i(L_i) \end{bmatrix} = \mathrm{vec}(\lambda_i \mathbf{E}_i) \tag{21}$$

where $\mathbf{g}_i(0), \cdots, \mathbf{g}_i(L_i)$ are the corresponding column vectors used to construct the block Toeplize matrix $\mathcal{G}_i$ in the way as we define $\mathcal{H}_i$ using $\mathbf{h}_i(0), \cdots, \mathbf{h}_i(L_i)$, $\mathcal{V} \in \mathbb{C}^{d_i(d_1 + \cdots + d_p) \times (L_i+1)q}$ is a block Toeplitz matrix written as

$$\mathcal{V} = \begin{bmatrix} \mathbf{V}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \mathbf{V}_0 & \ddots & \vdots \\ \mathbf{V}_N & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_N & \ddots & \mathbf{V}_0 \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{V}_N \end{bmatrix}$$

Obviously, from Eqn.(21) we know that $\mathcal{G}_i$ can be uniquely determined if $\mathcal{V}$ has full column rank. Recalling Theorem 1 in [11], $\mathcal{V}$ has full column rank if the following condition holds, i.e. there exists a nonzero z_0 (including ∞) such that the polynomial matrix $\mathbf{V}(z_0)$ has full column rank, where $\mathbf{V}(z) \triangleq \mathbf{V}_0 + \mathbf{V}_1 z^{-1} + \cdots + \mathbf{V}_N z^{-N}$. This mild condition can be satisfied with probability one since generally, when $L \geq 1$, the entries of matrix $\mathcal{H}^\dagger$ can be considered as randomly generated. Note that the polynomial matrix $\mathbf{V}(z)$ is of dimension $(d_1 + \cdots + d_p) \times q$ and we can guarantee this matrix to be a tall matrix be choosing a proper N. Thus we can conclude that the solution of $\mathcal{G}_i$ is unique and $\mathcal{G}_i = \lambda_i \mathcal{H}_i$. Note that λ_i can not be zero here because $\mathcal{G}_i$ would be zero under the condition $\lambda_i = 0$, which contradicts our previously made assumption $\mathcal{G}_i \neq \mathbf{0}$. The proof is completed here. ■

Since Eqn.(9) and Eqn.(10) can be derived from each other, it implies that the solution to Eqn.(10) is unique up to a scaling constant of the "true" channel $\mathbf{h}_i$. Therefore $\mathbf{h}_i$ can be estimated by the criterion in Eqn.(11) with $K = 1$, i.e.

$$\hat{\mathbf{h}}_i = \arg \min_{\|\mathbf{h}_i\|=1} \left\| \begin{bmatrix} \mathcal{T}_1[\Upsilon_1] - \mathcal{T}_2[\mathbf{J}_{d_i}] \\ \mathcal{T}_1[\Upsilon_2] - \mathcal{T}_2[\mathbf{J}_{d_i}^T] \end{bmatrix} \mathbf{h}_i \right\|^2 \tag{22}$$

As mentioned before, the above optimization has a closed-form solution which can be obtained as the right singular vector associated with the smallest singular value. Also, as we can see, the channel of the desired user i, $\mathcal{H}_i$, can be estimated up to a nonzero complex scalar from the second-order statistics of the received data if the channel order of user i is different from those of other users. Clearly, given that the channel orders of each pair of users are different, then the channel convolution matrix $\mathcal{H}$ can be identified up to a block diagonal matrix $\mathcal{D} = \mathrm{diag}(\lambda_1 \mathbf{I}_{d_1}, \cdots, \lambda_p \mathbf{I}_{d_p})$, where λ_i for each $i \in \{1, \dots, p\}$ is an unknown nonzero complex scalar.

In the following, we would like to discuss the case where the identifiability condition imposed on the channel orders is not satisfied in Theorem 1, that is, the channel order of the desired user i is the same as those of some other users. For simplicity, we only assume that the channel order of the desired user i is the same as that of another user l, i.e. $L_i = L_l$ and $L_i \neq L_j$ for $j \in \{1, \dots, p\}, j \neq i, l$. In this case, by following the similar steps as that in proof of Theorem 1, we can conclude that $\mathcal{G}_i = \lambda_i \mathcal{H}_i + \lambda_l \mathcal{H}_l$, where λ_i and λ_l can be any complex scalar including zero but λ_i and λ_l can not be zero at the same time. Obviously, $\mathcal{H}_i$, $\mathcal{H}_l$ and the linear combinations of $\mathcal{H}_i$ and $\mathcal{H}_l$ can all satisfy Eqn.(9). Hence the solution to Eqn.(10) is not unique because $\mathbf{h}_i$, $\mathbf{h}_l$ and their linear combinations $\lambda_i \mathbf{h}_i + \lambda_l \mathbf{h}_l$ all satisfy Eqn.(10). Consequently, the criterion of Eqn.(11) admits two independent solutions, say, $\mathbf{g}_i$ and $\mathbf{g}_l$, which are all expressed by the linear combinations of $\mathbf{h}_i$ and $\mathbf{h}_l$, that is, $\mathbf{g}_i = a_{11}\mathbf{h}_i + a_{12}\mathbf{h}_l$ and $\mathbf{g}_l = a_{21}\mathbf{h}_i + a_{22}\mathbf{h}_l$. As we can see, in this case, the unknown instantaneous mixture ambiguity arises again.

IV. Simulation Results

In this section, we present simulation results to illustrate the performance of our proposed algorithm. We compare our method to the subspace (SS) method developed in [5]. In our simulations, the additive noise $\mathbf{w}(n)$ is taken as spatial-temporal white complex Gaussian noise with zero mean and variance σ_w^2. The signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR} = 10 \cdot \log \frac{E[\|\mathcal{H}\vec{\mathbf{s}}(n)\|^2]}{E[\|\vec{\mathbf{w}}(n)\|^2]}$$

We consider $p = 2$ sources arriving at $q = 3$ sensors via a multipath channel. The source signals are independent and identically distributed (i.i.d.) information sequences drawn from a 4-QPSK constellation $\mathcal{S} = \{1, -1, i, -i\}$. The channel is randomly generated as

$$\{\mathbf{h}_1(l)\} = \begin{bmatrix} 0.2885 & 0.4926 & 0.2480 & 0 \\ 0.1714 & -0.2387 & 0.1945 & 0 \\ 0.0455 & -0.0463 & -0.0256 & 0 \end{bmatrix}$$

$$\{\mathbf{h}_2(l)\} = \begin{bmatrix} 0.0572 & 0.2074 & -0.0466 & 0.1085 \\ 0.2475 & -0.1004 & 0.0213 & -0.2331 \\ 0.0968 & -0.2527 & -0.3888 & 0.2701 \end{bmatrix}$$

It can be seen that the channel orders corresponding to these two users are different and this suffices for the complete channel identification. Once the channel has been estimated by our algorithm, we can compute the zero-forcing equalizers and the minimum mean-squared error (MMSE) equalizers respectively as

$$\mathcal{E}_{\mathrm{ZF}} = \hat{\mathcal{H}}^\dagger$$
$$\mathcal{E}_{\mathrm{MMSE}} = \mathcal{E}_{\mathrm{ZF}}(\mathbf{I} - \sigma_w^2 \hat{\mathbf{R}}_x^{-1}[0])$$

where $\hat{\mathbf{R}}_x[0]$ is the undenoised estimated autocorrelation matrix. The inherent phase ambiguity of equalizers per user is removed before we perform the equalization. For the subspace method [5], the source signals can be recovered directly by using a two-step estimation procedure, without the need to identify the channel in advance. In the simulations, we choose stack number $N = 5$. The channel order of each user is assumed known *a priori*. Results are averaged over

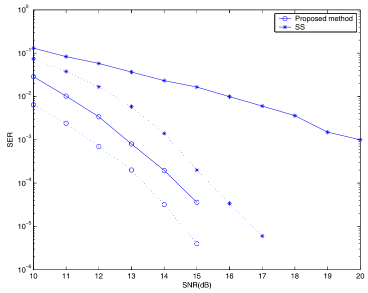

Fig. 1: SER versus SNR, $T_s = 1000$. Solid lines for user 1; dotted lines for user 2.

500 Monte Carlo runs. Figure 1 shows the symbol error rates (SER) as a function of SNR with $T_s = 1000$ data samples being used for statistics estimation. The MMSE equalizer with delay 5 is used for our proposed algorithm. We can see that our proposed method presents a clear performance advantage over the subspace method. The results also validate our previous claim that the channel can be completely identified by exploiting the channel order disparity.

V. Conclusion

In this paper, we propose a SOS-based method that admits a closed-form solution for blind identification of MIMO FIR channel driven by white inputs. By exploiting the disparity of the users' channel orders, the proposed method identifies the channel of each user up to a complex scalar instead of identifying the channel up to an unknown nonsingular matrix. Simulation results show that the proposed method compares favorably with the subspace method [5].

Appendix A
Proof of Lemma 2

We present our proof in the following three steps.

Step 1: For notational convenience, let $\mathbf{G}_1 \triangleq \mathbf{J}_m\mathbf{Y} = \mathbf{Y}\mathbf{J}_n$, $\mathbf{G}_2 \triangleq \mathbf{J}_m^T\mathbf{Y} = \mathbf{Y}\mathbf{J}_n^T$. Considering the relationship of $\mathbf{G}_1$, we have

$$\begin{aligned}\mathbf{G}_1[2:m,n] &= \begin{bmatrix} y_{1,n} & y_{2,n} & \cdots & y_{m-1,n}\end{bmatrix}^T \\ &= \begin{bmatrix} 0 & 0 & \cdots & 0\end{bmatrix}^T \end{aligned} \tag{23}$$

$$\begin{aligned}\mathbf{G}_1[1,1:n-1] &= \begin{bmatrix} 0 & 0 & \cdots & 0\end{bmatrix} \\ &= \begin{bmatrix} y_{1,2} & y_{1,3} & \cdots & y_{1,n}\end{bmatrix} \end{aligned} \tag{24}$$

Similarly, considering the relationship of $\mathbf{G}_2$, we have

$$\begin{aligned}\mathbf{G}_2[1:m-1,1] &= \begin{bmatrix} y_{2,1} & y_{3,1} & \cdots & y_{m-1,1}\end{bmatrix}^T \\ &= \begin{bmatrix} 0 & 0 & \cdots & 0\end{bmatrix}^T \end{aligned} \tag{25}$$

$$\begin{aligned}\mathbf{G}_2[m,2:n] &= \begin{bmatrix} 0 & 0 & \cdots & 0\end{bmatrix} \\ &= \begin{bmatrix} y_{m,1} & y_{m,2} & \cdots & y_{m,n-1}\end{bmatrix} \end{aligned} \tag{26}$$

Therefore we can conclude that all entries located at the edges of the matrix $\mathbf{Y}$ are zero except the entries $y_{1,1}$ and $y_{m,n}$.

Step 2: Now we consider the sub-matrix of $\mathbf{G}_1$ from second row to m^{th} row and from first column to $(n-1)^{th}$ column, denoted by $\mathbf{G}_1[2:m,1:n-1]$. This sub-matrix can be easily computed as if we write $\mathbf{J}_m$ and $\mathbf{Y}$ as follows

$$\mathbf{J}_m = \begin{bmatrix} \mathbf{0}_{1\times(m-1)} & 0 \\ \mathbf{I}_{(m-1)\times(m-1)} & \mathbf{0}_{(m-1)\times 1}\end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}[1:m-1,1:n-1] & \mathbf{Y}[1:m-1,n] \\ \mathbf{Y}[m,1:n-1] & y_{m,n}\end{bmatrix}$$

Obviously from $\mathbf{G}_1 = \mathbf{J}_m\mathbf{Y}$ we have

$$\mathbf{G}_1[2:m,1:n-1] = \mathbf{Y}[1:m-1,1:n-1] \tag{27}$$

On the other hand, we can write $\mathbf{J}_n$ and $\mathbf{Y}$ as

$$\mathbf{Y} = \begin{bmatrix} y_{1,1} & \mathbf{Y}[1,2:n] \\ \mathbf{Y}[2:m,1] & \mathbf{Y}[2:m,2:n]\end{bmatrix}$$

$$\mathbf{J}_n = \begin{bmatrix} \mathbf{0}_{1\times(n-1)} & 0 \\ \mathbf{I}_{(n-1)\times(n-1)} & \mathbf{0}_{(n-1)\times 1}\end{bmatrix}$$

Then from $\mathbf{G}_1 = \mathbf{Y}\mathbf{J}_n$ we have

$$\mathbf{G}_1[2:m,1:n-1] = \mathbf{Y}[2:m,2:n] \tag{28}$$

By combining Eqn.(27) and Eqn.(28), we can conclude that

$$y_{i,j} = y_{i+1,j+1} \tag{29}$$

for $i \in \{1,\ldots,m-1\}, j \in \{1,\ldots,n-1\}$, which shows that $\mathbf{Y}$ has a Toeplitz form.

Step 3: If $m = n$, based on the above derived results, it is easy to know that all entries on the main diagonal are equal, and all entries off the main diagonal are zero. Therefore we conclude that $\mathbf{Y} = \lambda\mathbf{I}$, where λ could be any complex scalar including zero. If $m \neq n$, since $\mathbf{Y}$ has a Toeplitz form and all entries located at the edges of the matrix $\mathbf{Y}$ are zero (note that $y_{1,1}$ and $y_{m,n}$ can be easily proved to be zero by utilizing the Toeplitz form when $m \neq n$), hence $\mathbf{Y} = \mathbf{0}$. The proof is completed here.

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