# PARTICLE FILTERS FOR BLIND FIR CHANNEL EQUALIZATION IN NON-GAUSSIAN NOISE

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## ABSTRACT

In this work, we propose new particle filter based blind equalization algorithms for FIR channels subject to additive noise of arbitrary distributions. These algorithms employ artificial evolution methods to jointly generate samples from the missing data and from the unknown channel parameters, which are assumed time-invariant. To achieve these results we introduce a new importance function, which leads to greatly improved performance compared to more obvious alternatives as verified via numerical simulations using Weibull envelope noise processes, in which the performance of the trained MLSE equalizer is approached to a narrow margin.

## 1. INTRODUCTION

Particle filters have extensively been applied to solving blind equalization [1] and allied communications problems [2]. Most of the literature on this subject, however, assumes that the additive noise distribution is gaussian (or at most a sum of gaussians) [3], both of whose observed signal likelihood functions admit sufficient statistics, so that the unknown channel parameters can be integrated out (Rao-Blackwellized) thereby leading to improved performance. Though gaussian noise models can be adjusted to reflect the impulsive nature of disturbances observed in many realistic communications systems, this work examines new particle filtering algorithms to directly solve the blind equalization problem of FIR channels under non-gaussian distributed additive noise.

Despite the long time recognition of the potential of particle filters to solve non-gaussian estimation problems, their application to blind equalization seems to be missing arguably due to practical and theoretical reasons like (i) the noise found in many practical communications systems may be well approximated by gaussian models and (ii) non-gaussian models lead to estimation problems in which (static) parameters are part of the state. As pointed out in [4], particle filters cannot be shown to uniformly converge to the exact solution of this class of estimation problems, and, in fact, have been observed to diverge on some occasions.

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The algorithms developed in this work are based mainly on artificial parameter evolution SIS methods [5, Ch. 10], which possess smaller computational complexity (per particle) and superior performance, at least under the present models. To address the problem, we introduce a new importance function that approximates the optimal one, and, via numerical simulations, quantify the performance of the resulting algorithms under Weibull envelope noise, for differential QPSK modulation.

The remainder of this article is organized as follows: in Sec. 2, we introduce the adopted signal models and estimation objectives. In Sec. 3 we give a brief overview of parameter artificial evolution particle filtering methods applied to blind equalization and, in Sec. 4, describe the proposed methods. Finally Sec. 5 contains some simulation results, followed by conclusions in Sec. 6.

## 2. SIGNAL MODEL AND PROBLEM STATEMENT

Consider a communication system that differentially encodes a sequence of equiprobable i.i.d QPSK symbols $b_{0:k} \triangleq \{b_0, \dots, b_k\}$, resulting in the (also i.i.d. and equiprobable) sequence $x_{0:k}$, which is transmitted over a FIR additive noise frequency selective channel with known order. The baud-rate received signal sample $y_k$ at instant $k$ is then assumed to obey the relation

$$y_k = \sum_{i=0}^{L-1} x_{k-i} h_{k,i} + v_k = h_k^H X_k + v_k \ . \qquad (1)$$

where $h_k = [\ h_{k,0}\ h_{k,1}\ \dots\ h_{k,L-1}\ ]^T$, $h_{k,j}$, $0 \le j < L$ are the channel impulse response terms at instant $k$, $X_k = [\ x_k\ \dots\ x_{k-L+1}\ ]^T$, $L$ is the channel order, and $v_k$ is an i.i.d noise process with known distribution $g(\cdot)$.

We further assume that the channel parameter vector is time-invariant, statistically independent from the other variables, and that it has a Gaussian prior distribution. Our aim is hence to approximate the posterior density $p(b_k|y_{0:k})$, wherefrom MAP estimates of the transmitted bits can be obtained.

When the additive noise $v_k$ is Gaussian, particle filtering methods can directly approximate the density $p(b_{0:k}|y_{0:k})$ [1], giving rise to the desired density via marginalization. However, if the usual Gaussian assumption is relaxed and the noise distribution no longer admits sufficient statistics [6], Rao-Blackwellized particle filters (i.e., algorithms which integrate "nuisance" parameters) can no longer be developed. As a consequence, one must resort to methods that estimate jointly parameters and state as described in Sec. 3.

# 3. PARTICLE FILTERS FOR JOINT PARAMETER AND STATE ESTIMATION

In this section we provide a brief introduction to the use of particle filters in estimation problems with unknown fixed parameters. Please refer to [5, Ch. 2] and [4] for further information and discussion of open theoretical problems regarding the convergence of these algorithms.

Suppose that one desires to sequentially approximate the density $p(X_{0:k}, h_{0:k}|y_{0:k})$ by means of a set of weighted samples (particles). Upon observing $y_k$, one must come up with samples of the variables $X_k$ and $h_k$ such that the joint posterior distribution of $X_{0:k}$ and $h_{0:k}$ is approximated. According to the classical Gordon method, this is achieved easily by sampling from an arbitrary importance density $\pi(h_{0:k}, X_{0:k}| y_{0:k})$ defined as the product of its marginals, i.e.,

$$\pi(h_{0:k}, X_{0:k}|y_{0:k}) = \pi(h_{0:k-1}, X_{0:k-1}|y_{0:k-1}) \\ \pi(X_k, h_k|h_{0:k-1}, X_{0:k-1}, y_{0:k}),$$

and by attributing the weight

$$w_k^{(i)} = p(X_{0:k}^{(i)}, h_{0:k}^{(i)}|y_{0:k})/\pi(X_{0:k}^{(i)}, h_{0:k}^{(i)}|y_{0:k}),$$

to each sample $\{h_k^{(i)}, X_k^{(i)}\}$. These weights can be sequentially evaluated as

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(X_k^{(i)}, h_k^{(i)}, y_k|h_{0:k-1}^{(i)}, X_{0:k-1}^{(i)}, y_{0:k-1})}{\pi(h_k^{(i)}, X_k^{(i)}|h_{0:k-1}^{(i)}, X_{0:k-1}^{(i)}, y_{0:k})} . \quad (2)$$

Therefore, to approximate $p(X_{0:k}, h_{0:k}|y_{0:k})$, one must (i) evaluate and sample from the importance density and (ii) evaluate the weighting term on the numerator of the r.h.s of (2).

## 3.1. Artificial Evolution

To compute the numerator of (2), note that the prior independence of the bits and of the channel parameters leads to

$$p(X_k, h_k, y_k|h_{0:k-1}, X_{0:k-1}, y_{0:k-1}) = \\ p(y_k|h_k, X_k)p(X_k|X_{0:k-1})p(h_k|h_{0:k-1}) . \quad (3)$$

Under the assumption that the parameter vector is time-invariant, one gets that $p(h_k|h_{0:k-1}) = \delta(h_k - h_0)$ (where $\delta$ denotes the Dirac impulse function). This causes the weight of any particle for which $h_k^{(i)} \neq h_0^{(i)}$ to be zeroed, which prevents the adaption of the parameters no matter the importance function chosen. Artificial parameter evolution techniques [5, Ch. 10] provide a way to circumvent this problem by regarding the parameter vector is slowly time-varying. In general, this is done via an AR(1) Gaussian model, such that

$$p(h_k|h_{0:k-1}) = \mathcal{N}(h_k|h_{k-1}; I\rho^2), \quad (4)$$

with $\rho^2 \ll \|h_0\|^2$.

## 3.2. Basic Blind Equalization Algorithm

For the signal model of Sec. 2, the remaining terms of (3) can be easily determined. While the likelihood term is given by

$$p(y_k|h_k, X_k) = g(y_k - h_k^H X_k), \quad (5)$$

one also obtains that

$$p(X_k|X_{0:k-1}) = p(x_k|x_{k-1}) \\ \prod_{j=k-L+1}^{k} \prod_{i=1}^{L+k-j-1} \mathcal{I}\left\{[X_j]_i = [X_{j-1}]_{i+1}\right\}, \quad (6)$$

where $\mathcal{I}\{\cdot\}$ denotes the event indicator and $[A]_i$ indicates the $i-$th element of a vector. Notice that the term that multiplies $p(x_k|x_{k-1})$ on the r.h.s. of (6) always equals 1 if the sequence $X_{0:k}$ is consistent with its definition given in Sec. 2. Therefore

$$p(X_k|X_{0:k-1}) = p(x_k) , \quad (7)$$

as the coded symbols $x_k$ are assumed i.i.d..

Observe also that (6) implies that $p(X_{0:k}) = p(x_{0:k})$, and, similarly, exploiting the fact that $b_{0:k}$ uniquely defines $x_{0:k}$, we obtain that $p(X_{0:k}, h_{0:k}|y_{0:k}) = p(b_{0:k}, h_{0:k}|y_{0:k})$. By combining (4)-(7), one can evaluate (3) and, as a result, the weight update function (2). Adopting the prior importance function,

$$\pi(h_k^{(i)}, X_k^{(i)}|h_{0:k-1}^{(i)}, X_{0:k-1}^{(i)}, y_{0:k}) = p(x_k)\mathcal{N}(h_k|h_{k-1}^{(i)}; I\rho^2) , \quad (8)$$

further simplifies (2), allowing one to estimate $b_{0:k}$ according to the algorithm of Table 1. Unfortunately, this algorithm performs rather poorly (Sec. 5) regardless of the number of particles $P$ employed, a fact that must have discouraged further research on this topic.

A few comments about the algorithm of Table 1: note that (i) the computational complexity (per particle) of this method is much smaller than that of Rao-Blackwellized [1] algorithms, for which a Kalman filter step must be evaluated for each particle at every iteration, and is dominated by the complexity of the resampling step, (ii) its memory requirements are also very small, since one must only store the variables $h_k^{(i)}$ and $X_k^{(i)}$, a total of $2LP$ numbers and finally (iii) the phase ambiguity inherent to the blind equalization problem not only makes the use of a differential coding scheme mandatory, but also prevents the direct estimation of the channel parameters, since the samples $h_k^{(i)}$ are affected by random phase rotations.

(Initialization)
-Draw $h_0^{(i)} \sim \mathcal{N}_C(h_0|\bar{h}_0;\Sigma_0)$.
-Draw $X_0^{(i)} \sim p(X_0)$.
•For $k > 0$
• For $i = 0, ..., P-1$,
-Draw $h_k^{(i)} \sim \mathcal{N}(h_k|h_{k-1}^{(i)}; I\rho^2)$.
-Draw $X_k^{(i)} \sim p(X_k^{(i)}|X_{k-1}^{(i)})$.
-Obtain $b_k^{(i)}$ by decoding $x_{0:k}^{(i)}$
-Calculate and normalize the weights
$w_k^{(i)} \propto g(y_k - h_k^{(i)H} X_k^{(i)})$ .
• End
-Resample [5] the particle set with probabilities given by the weights $w_k^{(i)}$.
-Estimate $p(b_k)$ as
$p(b_k = B|y_{0:k}) \approx \frac{1}{P}\sum_{i=0}^{P-1} \mathcal{I}\{b_k^{(i)} = B\}$ .
•End

**Table 1**. Basic Blind Equalization Algorithm (AE I)

### 3.3. Smoothing

A method described in [7] allows one to approximate the smoothing density $p(b_k|y_{0:k+d})$, $d > 0$, by a trivial modification of the algorithm of Table 1. Suppose that at instant $k$ the resampling step produces a set of indexes $j_k(i)$, $0 \leq i < P$. Now, introducing the notation $b_{k,k}^{(i)} \triangleq b_k^{(i)}$ and $b_{l,k+1}^{(i)} \triangleq b_{l,k}^{j_k(i)}$, $l \leq k$, one can show that for $d \geq 0$,

$$p(b_{k-d} = B|y_{0:k}) \approx \frac{1}{P}\sum_{i=0}^{P-1} \mathcal{I}\{b_{k-d,k}^{(i)} = B\} \ , \qquad (9)$$

i.e., smoothed estimates can be obtained by resampling $b_k^{(i)}$, $0 \leq i < P$ according to the weights calculated in successive iterations. As verified in (Sec. 5), this leads to great performance improvements at the cost of a small complexity and memory increase.

## 4. IMPROVED EQUALIZATION ALGORITHM

The bad performance of the algorithm of Table 1 induced us to look for alternative particle filtering structures to solve the target estimation problem. As the optimal importance function cannot be evaluated for the adopted signal model, we propose an alternative heuristic function that mimics the optimal, approximating the analytically unsolvable integral by a summation (Table 2).

The proposed importance function consists of two steps: in the first, $M$ samples of $h_k^{(i)}$ are drawn from a reduced variance artificial prior, forming the set $h_k^{(i,r)}$. Then, the likelihood of each of the possible pair $\{h_k^{(i,r)}, X_k^{(i)}\}$ is evaluated and normalized, and a sample is drawn from the resulting discrete joint density. The weights are then evaluated according to (2).

(Initialization)
-Draw $h_0^{(i)} \sim \mathcal{N}_C(h_0|\bar{h}_0;\Sigma_0)$.
-Draw $X_0^{(i)} \sim p(X_0)$.
•For $k > 0$
• For $i = 0, ..., P-1$,
-For $r = 0, ..., M-1$, draw
$h_k^{(i,r)} \sim \mathcal{N}_C(h_k|h_{k-1}^{(i)}; I(\rho^2/2))$.
-Draw $X_k$ and $h_k$ jointly from the **discrete density**
$\pi(X_k, h_k) \propto g(y_k - h_k^H X_k)p(X_k|X_{k-1}^{(i)})\mathcal{I}\{h_k = h_k^{(i,r)}\}$,
-Calculate and normalize the weights $w_k^{(i)} \propto$
$\sum_{r,s} g(y_k - h_k^{(r)H} X_k^{(s)}) p(X_k^{(s)}|X_{k-1}^{(i)}) \mathcal{N}(h_k^{(r)}|h_{k-1}^{(i)}; I\frac{\rho^2}{2})$ .
•End
-Obtain $b_k^{(i)}$ by decoding $x_{0:k}^{(i)}$, $i = 0, ..., P-1$.
-Resample the particle set with probabilities given by the weights $w_k^{(i)}$.
-Estimate $p(b_{k-L})$ as
$p(b_{k-L} = B) \approx \frac{1}{P}\sum_{i=0}^{P-1} \mathcal{I}\{b_{k-d,k}^{(i)} = B\}$ .
•End

**Table 2**. Modified Algorithm (AE II)

## 5. SIMULATION RESULTS

To quantify the performance of the proposed blind equalization methods, we carried out numerical simulations, evaluating the mean BER (bit error rates) along 250 independent realizations, each containing a block of 400 independently transmitted QPSK symbols. BER evaluation was made after discarding the first 100 symbols to allow for algorithm convergence. The proposed algorithms were initialized as described in Tables 1 and 2, with $\bar{h}_0 = 0$ and $\bar{\Sigma}_0 = I$.

We chose a white Weibull envelope noise model (see the Appendix), with parameter $a = 1.1$ (strongly non-gaussian), and employed order $L = 3$ normalized complex random channels, drawn from the complex Gaussian prior $\mathcal{N}(h|0; I)$. To establish a comparison basis, we also evaluated the performance of the optimal MLSE equalizer (Viterbi) with exact channel knowledge and using the Weibull envelope noise density to compute path metrics.

In all simulations, we adopted $\rho^2 = 0.05$ and employed the residual resampling algorithm [5, Ch. 1]. In Fig. 1 we show the mean BER obtained by the algorithms with a smoothing lag of $d = 10$. As one can readily verify, the algorithm of Table 1 (AE I) performed very poorly (the same was verified for $d = 0$, not shown in the graphic). While the algorithm of Table 2 (AE II) also performed poorly for $M = 1$, its performance was much improved for $M = 5$, approaching that of the trained MLSE equalizer by a margin of 2-3 dB. As one might expect, increasing the number of particles lead to improved results.

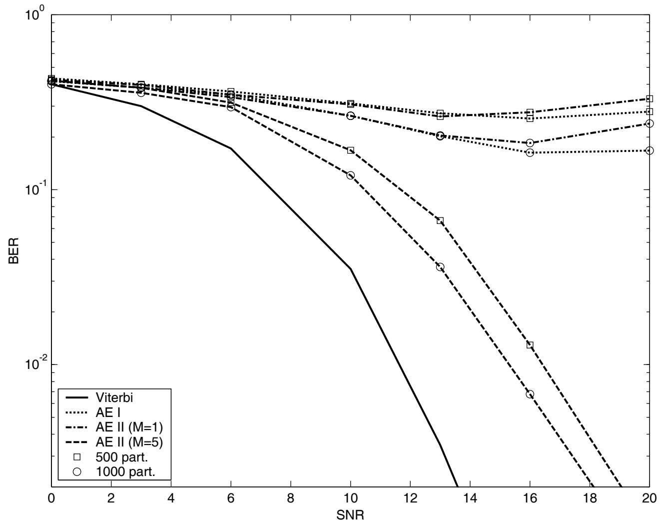


**Fig. 1**. Performance of blind equalization algorithms (smoothing lag $d = 10$) under Weibull envelope white noise as a function of the signal-to-noise ratio (SNR) and of the number of particles employed, compared to the performance of the optimal MLSE detector (Viterbi).

## 6. CONCLUSIONS

In this work we proposed and evaluated the performance of blind equalization algorithms for known-order FIR channels subject to additive non-gaussian noise. We introduced a new importance function that led to greatly improved performance under the simulation scenarios considered, in which QPSK differentially modulated symbols are transmitted under Weibull envelope noise.

Due to computational restraints, we did not evaluate the performance of the proposed methods in steady state. However, we did not notice convergence issues when the proposed algorithms are run for up to 10.000 iterations.

## 7. REFERENCES


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## Appendix: Weibull envelope noise

Consider the complex random variable $v = v_R + jv_I$ obtained [8] from two independent zero-mean real Gaussian variables $x, y$ of variance $\sigma^2$ via the transform

$$\begin{aligned} v_R &= x(x^2+y^2)^{1/a-1/2} \\ v_I &= y(x^2+y^2)^{1/a-1/2} \end{aligned}$$

for $a > 1$. The joint probability function of $(v_R, v_I)$ is given by

$$p(v_R, v_I) = \frac{a}{4\pi\sigma^2}(v_R^2+v_I^2)^{a/2-1}\exp\left(-\frac{1}{2\sigma^2}(v_R^2+v_I^2)^{a/2}\right), \tag{10}$$

and the envelope $|v|$ is Weibull distributed

$$p(|v|) = \frac{a}{2\sigma^2}|v|^{a-1}\exp\left(-\frac{1}{2\sigma^2}|v|^a\right). \tag{11}$$

The kurtosis of $v$ (as well as its variance) is a function of the parameter $a$. It can be verified that for $1 < |a| \le 2$, $v$ is supergaussian, characteristic that is emphasized when $a \to 1$. Although Weibull envelope processes have long been used to model heavy-tailed disturbances found, for instance, in radar detection problems [9], most authors in the communications literature tend to resort to gaussian sum models for the same purpose.