Fuzzy Adaptive Blind Equalizer Using Extended Kalman Filter Based Adaptation Algorithm For Powerline Channel

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Abstract
Fuzzy adaptive equalizer (FAE) is a knowledge based equalizer operating on linguistic variables. The advantages of using fuzzy logic adaptation scheme with respect to more traditional adaptation schemes in powerline communication system are the simplicity of the approach and the use of knowledge (fuzzy IF-THEN rules and input output pairs information) about the communication medium. This paper presents a new adaptive blind equalization method based on fuzzy logic for powerline channel. We introduce a new type of fuzzy adaptive blind equalizer (FABE) using extended Kalman filter (EKF) based adaptation algorithm for powerline channel equalization. The proposed blind equalizer for powerline channel has the following merits: It is new and simple in design, and it does not requires training sequence. In a changeable distorted powerline channel, data transmission is continuous and do not stop for training the equalizer. The performance of EKF-based FABE is compared with two other types of FABEs based on the recursive least squares (RLS) and the least mean squares (LMS) adaptation algorithm. The simulation results show that EKF-based FABE has faster convergent and lower steady state probability of error compared to the other two FABEs. The bit error rate (BER) of the EKF-based FABE is close to that of the optimal equalizer.

I. INTRODUCTION

The term equalizer or filter in powerline communication system is commonly referred to a device that is designed to perform signal processing operations by extracting information about a prescribed quantity of interest from noisy and intersymbol interfered data [1]. In conventional equalization process, initial acquisition of the equalizer's parameters is usually accomplished using learning sequences transmitted periodically in time. However, for the sake of simplicity in system design and convenience in system implementation, it is sometimes desirable to let the receivers start up without the aid of the transmitter. For certain powerline communication system, it is impractical to utilize a training sequence of long duration and the system cost involved to train the equalizer with repeated transmission of a known sequence is typically high. Hence a brilliant way to overcome such problems is by applying blind equalization technique.

Blind channel equalization, or more formally, unsupervised equalization, is a type of equalization technique that depends only on the received samples and assumptions on the input data to identify the channel [1]. The idea of blind equalization systems dates back to the pioneering works of Sato [2], [3] and Godard [4], [5]. The Sato algorithm for blind equalization was introduced originally to deal with one-dimensional multilevel pulse amplitude modulation (M-ary PAM) signals, which is more robust than a decision-directed algorithm [1]. Godard algorithm is considered to be the most successful among the Bussgang family of blind equalization algorithms, as demonstrated by the comparative studies done in [6] and [7]. The Godard algorithm is more robust than other Bussgang algorithm due to the fact that cost function used for its derivation is based solely on received signals' amplitudes [8]. Under steady state conditions, blind equalizer using Godard algorithm attains a mean square error that is the lowest among all the Bussgang algorithm existed in the world nowadays [8].

In this paper, we modify on a class of Bussgang-type equalizers that employ the Godard algorithm. In particular, we have replaced the transversal filter in the conventional Bussgang-type equalizer with a fuzzy adaptive filter. The modified Bussgang type equalizers (i.e., fuzzy adaptive blind equalizer (FABE)) operate on the received signals sampled at the baud rate. Unlike most of the blind equalizers recently proposed, FABE is a knowledge based system operating on linguistic variables. Linguistic information (fuzzy IF-THEN rules) and numerical information (input-output pairs) can be combined into the blind equalizer. We further derive a FABE that uses an extended Kalman filter (EKF) adaptation algorithm [9] for equalization of powerline channel. Simulation results show that this type of FABE has faster convergent speed compared to both RLS-based FABE and LMS-based FABE.

The paper is organized as follows: Section II briefly comments on powerline data transmission system using FABE. Section III describes the structure of the proposed FABE and the adaptation algorithm. Section IV contains some computer simulation results and Section V summarizes the work.

II. POWERLINE DATA TRANSMISSION SYSTEM USING FABE

The baseband powerline data transmission system using FABE is shown in Fig. 1.

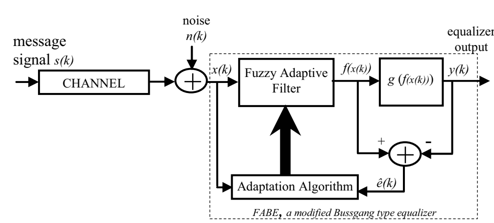

Fig 1: Powerline data transmission with fuzzy adaptive blind equalization

The channel includes the effects of the transmission filter, powerline communication (PLC) channel and reception filter. The transmitted data sequences *s(k)* are binary sequences, taking values from {-1,1}with equal probability. The inputs to the FAE [*x(k)*, *x(k-1)*,…, *x(k-n+1)*] are the channel outputs corrupted by noise. The task of the equalizer at the sampling instant *k* is to produce an estimate of the transmitted symbol $y(k-\tau)$, using the information contained in [*x(k)*, *x(k-1)*,…, *x(k-n+1)*] where the integer *n* and *τ* are the order and lag of the equalizer respectively. The modified Bussgang equalizer is similar to the ordinary Bussgang equalizer except that transversal equalizer is now replaced by FAE.

The FAE output *f(x(k))* is applied to a zero-memory nonlinear estimator *g*(*f(x(k))*), producing the estimate *y(k)* for data symbol *s(k)*. *ê(k)* is the estimation error for the iterative equalization process. Fast converging adaptation algorithm is used for initial startup and later switched to a Bussgang algorithm for tracking. A switching scheme as proposed in [10] can be used in the receiver to detect significant channel changes.

The PLC channel model suggested in [11] is given by:

$$H(f)=\sum_{i=1}^{N} g_i . A(f,d_i).e^{-j2\pi f\tau_i} \tag{1}$$

where g_i is the weighting factor that summarizes the reflection and transmission factors along the *i*-th path, $A(f,d_i)$ is the cable attenuation that increases with length and frequency, and τ_i is the propagation delay of the *i*-th multipath. The parameters g_i and $A(f,d_i)$ can be derived from measured transfer function [11].

The two-term Gaussian mixture model can be used to model the powerline noise [12]. The probability density function (pdf) of this noise model has the form:

$$f=(1-\epsilon)N(0,v^2)+\epsilon N(0,Kv^2) \tag{2}$$

with the variance $v^2>0$, and ϵ representing the probability that impulses occur, $0<\epsilon\leq 1$. Here, $N(0,v^2)$ represents the nominal background noise, and $N(0,Kv^2)$ represents an impulsive component, with *K* representing the index multiple of impulsive noise power. The variance of the additive noise is:

$$v_a^2=(1-\epsilon)v^2+\epsilon K v^2 \tag{3}$$

III. EXTENDED KALMAN FILTER BASED FUZZY ADAPTIVE BLIND EQUALIZER

A.) FAE using EKF algorithm

The fuzzy adaptive filter using extended Kalman filter (EKF) adaiptation algorithm is constructed with three steps [13]:

1.) Define *M* fuzzy sets F_i^l for each interval $[C_i^-,C_i^+]$ of the input space, *U*, with Gaussian membership functions [9]:

$$\mu_{F_i^l}=\exp\left[-\frac{1}{2}\left(\frac{x_i-\tilde{x}_i^{\,l}}{\sigma_i^{\,l}}\right)^2\right] \tag{4}$$

where $l=1,2,\ldots,M$, $i=1,2,\ldots,n$, $x_i=x(k-i+1)$ is the input to the equalizer, $\tilde{x}_i^{\,l}$ is the center of the *i*-th membership function in the *l*-th rule and $\sigma_i^{\,l}$ represents the width of the *i*-th membership function in the *l*-th rule. $\tilde{x}_i^{\,l}$ and $\sigma_i^{\,l}$ are free parameters which will be optimized using the EKF algorithm.

2.) Construct a set of changeable fuzzy IF-THEN rules either by linguistic information or numerical information from the matching input-output data pairs:

$$R^l: \text{IF } x_1 \text{ is } F_1^l \text{ and } \ldots \text{ and } x_n \text{ is } F_n^l, \text{ THEN } d \text{ is } G^l. \tag{5}$$

where $d\in R$ is the desired output, F_i^l s' are defined in step 1, G^l's are fuzzy sets defined in *R* which are determined as follows : if there are linguistic rules in the form of (5), set F_i^ls and G^l to be the labels of these linguistic rules, otherwise, choose μ_{G^l} and the parameters $\tilde{x}_i^{\,l}$ and $\sigma_i^{\,l}$ arbitrarily. These parameters will change during the adaptation process.

3.) Construct the equalizer *f(**x**)* based on the set of *M* rules by using product inference and centroid defuzzification [14]:

$$f(\mathbf{x})=\frac{\sum_{l=1}^{M}\theta^l\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)} \tag{6}$$

where $\boldsymbol{x}=[x_1,\ldots,x_n]^T$, $\mu_{F_i^l}$ s are the Gaussian membership functions defined in (2), and $\theta^l\in R$ is the value which μ_{G^l} achieves its maximum.

B.) EKF based adaptation algorithm

The parameters of the filter, θ^l, $\tilde{x}_i^{\,l}$ and $\sigma_i^{\,l}$, are updated using the EKF algorithm [9]:

<u>Correction</u>

$$\varpi(k)=\varpi(k)+\mathbf{G}_{\varpi}(k)[d(k)-f_k(x(k))] \tag{7}$$

<u>Kalman Gain Matrix</u>

$$\mathbf{G}_{\varpi}(k)=\mathbf{E}_{\varpi}(k-1)\mathbf{\Omega}_{\varpi}^{T}(k)\left[\mathbf{\Omega}_{\varpi}(k)\mathbf{E}_{\varpi}(k-1)\mathbf{\Omega}_{\varpi}^{T}(k)+N\right]^{-1} \tag{8}$$

<u>(Posterior) error covariance matrix</u>

$$\mathbf{E}_{\varpi}(k)=\mathbf{E}_{\varpi}(k-1)-\mathbf{G}_{\varpi}(k)\mathbf{\Omega}_{\varpi}\mathbf{E}_{\varpi}(k-1) \tag{9}$$

where *k* is the discrete time index;

ϖ *(k)* is the equalizer parameter (i.e. θ^l, $\tilde{x}_i^{\,l}$ or $\sigma_i^{\,l}$);

$\mathbf{\Omega}_{\varpi}(k)$ is the jacobian of the equalizer parameter;

x(k) is the input vector at time *k*;

d(k) is the desired output at time *k*;

f(x(k)) is the equalizer output at time *k*;

N is the measurement noise covariance.

This also involves the computation of the Jacobian Ω*(k)*, which is obtained as the linearization about the current value of the nonlinear parameters (θ^l, $\tilde{x}_i^{\,l}$ and $\sigma_i^{\,l}$)[13]:

$$\Omega_{\theta}=\frac{a^l(k-1)}{b(k-1)} \tag{10}$$

$$\Omega_{\sigma_i}=\frac{\left[\theta^l-f_k(x(k))\right]a^l(k-1)c^l(k-1)}{b(k-1)} \tag{11}$$

$$\Omega_{x_i}=\frac{\left[\theta^l-f_k(x(k))\right]a^l(k-1)e^l(k-1)}{b(k-1)} \tag{12}$$

where $$a^l(k-1)=\prod_{i=1}^{n}\exp\left[-\frac{1}{2}\left(\frac{x_i(k)-\tilde{x}_i^{\,l}(k-1)}{\sigma_i^{\,l}(k-1)}\right)^2\right] \tag{13}$$

$$b(k-1) = \sum_{l=1}^{M} a^l(k-1) \quad (14)$$

$$c^l(k-1) = \frac{\left(x_i - \tilde{x}_i^{\,l}\right)^2}{\left(\sigma_i^{\,l}\right)^3} \quad (15)$$

$$e^l(k-1) = \frac{\left(x_i - \tilde{x}_i^{\,l}\right)}{\left(\sigma_i^{\,l}\right)^2} \quad (16)$$

C.) Godard Algorithm

The Bussgang algorithm used in this paper is Godard algorithm [6]. For Godard algorithm, the zero memory nonlinear function is given by [6]:

$$g\big(f(x(k))\big) = \frac{f(x(k))}{\left|f(x(k))\right|}\left(\left|f(x(k))\right| + R_p\left|f(x(k))\right|^{p-1} - \left|f(x(k))\right|^{2p-1}\right) \quad (17)$$

where $R_p = \dfrac{E\left[\left|s(k)\right|^{2p}\right]}{E\left[\left|s(k)\right|^{p}\right]}$. Since *s(k)* are equally probable data sequences taking values from {-1, 1}, $\left|s(k)\right| = 1$. Hence $E\left[\left|s(k)\right|^{p}\right] = 1$ and $E\left[\left|s(k)\right|^{2p}\right] = 1$. Therefore $R_p = 1$. In [6], two cases of Godard algorithm are suggested with specific interest in decoupling the ISI:

Case I: *p*=1, is a modification of the Sato algorithm. Equation (17) becomes: $g\big(f(x(k))\big) = \dfrac{f(x(k))}{\left|f(x(k))\right|}$ (18)

Case II: *p*=2, referred to in the literature as the Constant-Modulus algorithm (CMA). Equation (17) becomes:

$$g\big(f(x(k))\big) = f(x(k))\left[2 - \left|f(x(k))\right|^2\right] \quad (19)$$

Finally, the output of the equalizer is passed through a threshold decision device *sgn*(*y(k)*).

IV. NUMERICAL RESULTS

In this section, we investigate the performance of the proposed blind equalizer in terms of convergence speed and steady state probability of error, and compare the results to other related adaptation algorithms. We consider two arbitrary powerline channels' transfer functions: $h_1(z) = 1 + 0.5z^{-1}$ and $h_2(z) = 0.5 + z^{-1}$. The order and lag of the equalizer are *n*=2 and *τ*=0 respectively. We have compared the performance of EKF-based FABE with two types of fuzzy adaptive blind equalizers, namely RLS-based FABE and LMS-based FABE. The noise in powerline channel is based on the Gaussian mixture model as stated in equation (2) with ε=0.01, *K*=5. The EKF-based FABE and LMS-based FABE have *M*=25 rules while the RLS-based FABE has $m_1 \times m_2 = 5 \times 5$ rules.

Example 1: We first consider a single static channel, with $h_1(z)$ transfer function. We randomly set $\theta^l(0)$ in [-0.5 0.5], $\tilde{x}_i^{\,l}(0)$ in [-2.0 2.0] and $\sigma_i^{\,l}(0)$ in [0.1 0.3]. For EKF-based FABE, we set *N*=0.999 and *E*(0)=*I*, where *I* is an identity matrix of size $M \times M$. For LMS-based FABE, we set the step size for adaptation, α =0.05. For RLS-based FAE, we set the forgetting factor, *λ*=0.999 and **σ**=0.01. Case I and Case II Godard algorithms are used for comparison which we set the zero memory nonlinear function according to equation (18) and (19) respectively. The FAEs' parameters θ^l, $\tilde{x}_i^{\,l}$ and $\sigma_i^{\,l}$ are adapted at baud rate until a fixed numbers of iterations. The parameters are then fixed for symbol detection. It turned out that around 1500 iterations are enough to make the FABE converges. Using more iterations for FABE's equalization does not lead to lower probability of error but may lead to divergence which causes higher probability of error afterwards. When signal to background noise ratio equals to 10dB, we plot the convergence curves for the three blind equalizers in Fig. 2a and 2b. It can be seen that FABEs using Case II Godard algorithm converges much faster than that using Case I Godard algorithm. Among the adaptation algorithms, EKF achieves the fastest convergent. EKF-based FABE also outperforms the RLS-based FABE and LMS-based FABE in term of steady state probability of error.

Example 2: Next, we consider that after 4000 iterations, the channel has been switched from $h_1(z)$ to $h_2(z)$. The FABEs switch back to adaptation stage at that moment. After 1500 iterations, adaptation switch off and the fixed parameters (θ^l, $\tilde{x}_i^{\,l}$ and $\sigma_i^{\,l}$) are used for the rest of the equalization. We plot the convergence curves for the three FABEs in Fig. 3a and 3b. By using FABE in powerline communication, an advantage observed here is that the data transmission along a time varying powerline channel need not stop for a training sequence of long duration. Data transmission is continuous but pays a cost in high probability of error before the equalizer converged. Therefore, fast convergent adaptation algorithms have to be considered to minimize the bit error rate. In Fig. 3a and 3b, we observed that EKF-based FABE achieves the fastest convergence and lowest steady state probability of error.

Example 3: Finally, the bit error rate of EKF-based FABE is compared to the maximum-a-posteriori probability (MAP) equalizer that has the minimum probability of error in a single static channel ($h_1(z)$) after 1500 iterations. Fig. 4 shows the bit error rates of the optimum MAP equalizer and the FABEs for different signal to noises (background and impulsive) ratios. We see that the bit error rate of the EKF-based FABE is very close to the optimal one.

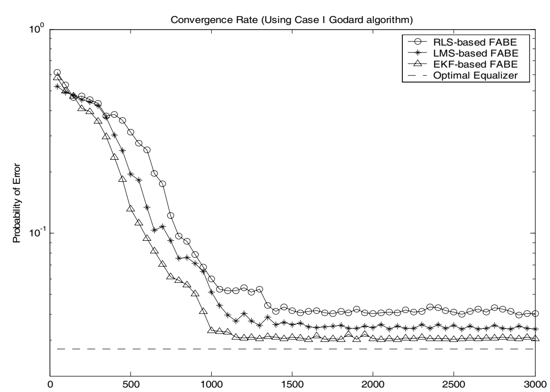

Fig.2a : Convergence rate for 3 types of FABEs using Case I Godard algorithms in a single static channel.

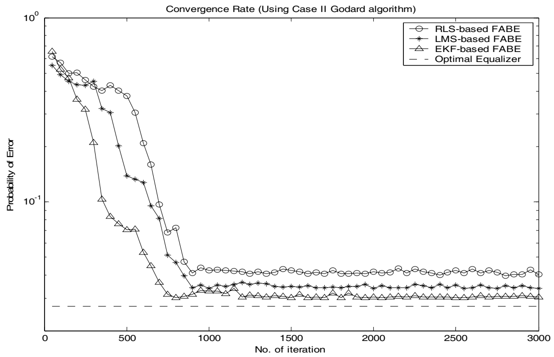

Fig.2b : Convergence rate for 3 types of FABEs using Case II Godard algorithms in a single static channel.

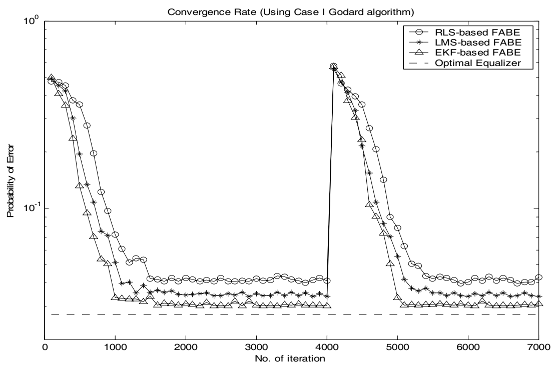

Fig.3a : Convergence rate for 3 types of FABEs using Case I Godard algorithms in changeable channels.

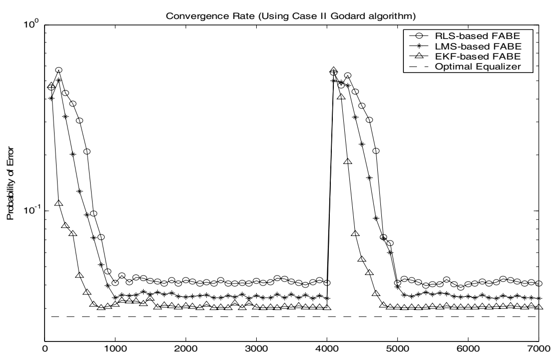

Fig.3b : Convergence rate for 3 types of FABEs using Case II Godard algorithms in changeable channels.

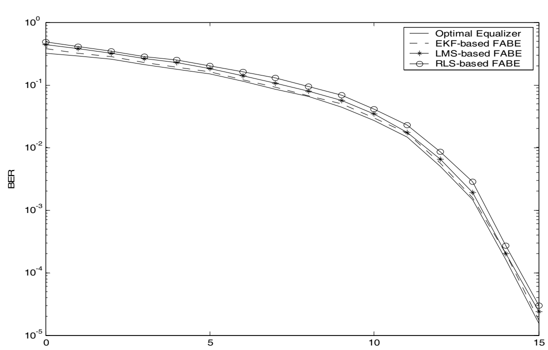

Fig. 4: The bit error rate vs. signal to noise ratio plot for the EKF-based FABE and the optimal equalizer in a single static channel.

V. CONCLUSION

In this paper, we have integrated fuzzy logic into the blind equalization techniques. We used fuzzy adaptive filter, extended Kalman filter based adaptation algorithm and Godard algorithms to design an extended Kalman filter based fuzzy adaptive blind equalizer (EKF-based FABE) for powerline channel. The EKF-based FAE has the fuzzy system's capabilities to deal with imprecise information using "common sense" fuzzy rules. It is able to cope with changeable distorted powerline channel, as shown in example 2, and it does not need a training sequence. The merits of EKF-based FABE as compared to two others FABEs (RLS-based FABE and LMS-based FABE) are faster convergent and lower steady state probability of error. Simulation results show that higher reliability is obtained using the EKF adaptation algorithm together with Case II Godard algorithm. The steady state BER of EKF-based FABE is very close to that of the optimal equalizer. Future works include considerations of more realistic powerline channels and new adaptation algorithm with even faster convergent than EKF.

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