LOW-POWER ADAPTIVE FIR EQUALIZER VIA SOFT ERROR CANCELLATION

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ABSTRACT

In this paper, we present an adaptive FIR equalizer which reduces power dissipation by employing a new algorithmic error correction technique. Building on the *voltage overscaling* (VOS) technique, we formulate the statistical estimation of timing errors that may be caused by VOS, called *soft errors* to detect and cancel them at a system level. We derive a minimum variance unbiased estimator, and develop an adaptive and power-optimized algorithm for an adaptive equalizer. Up to 30% power savings are demonstrated with negligible performance loss for an example, 16-tap minimum mean square error (MMSE) FIR equalizer.

1. INTRODUCTION

Due to growing power demands of portable and wireless applications, low-power techniques in digital signal processing (DSP) area gained importance. A wide variety of techniques have been proposed to reduce power in DSP systems [1]-[5]. Among them, supply voltage scaling is often used to achieve the significant power savings due to the quadratic dependency of power on supply voltage [1, 2].

In practice, due to increased execution delay at reduced voltage, the extent of supply voltage reduction is limited by timing constraints. Therefore, current voltage scaling methods have performed supply voltage reduction up to the point that the critical path delay at the given supply voltage is equal to the sampling period, to avoid timing errors.

However, an approach referred to as voltage overscaling (VOS) [3, 4] has shown that the supply voltage might be scaled further for additional power savings, i.e.

$$V_{dd} = k_{\text{vos}} V_{dd-\text{crit}} \ , \ 0 < k_{\text{vos}} < 1 \tag{1}$$

where k_{vos} is voltage overscaling factor and $V_{dd-\text{crit}}$ is the supply voltage at which the critical path delay equals to the sampling period. The VOS technique is often used in conjunction with algorithmic noise tolerance (ANT) which mitigates transient errors caused by timing violations, called *soft errors* [3]. Mitigation of soft errors is crucial for successful power savings via VOS.

Previous techniques to mitigate soft errors includes *prediction based error control* (PEC) [3], which removes soft errors at the digital filter output by using a forward linear predictor. A *reduced precision redundancy* (RPR) [4] approach replaces the potentially corrupted output with the output of a reduced precision replica of the main system, when an error is detected.

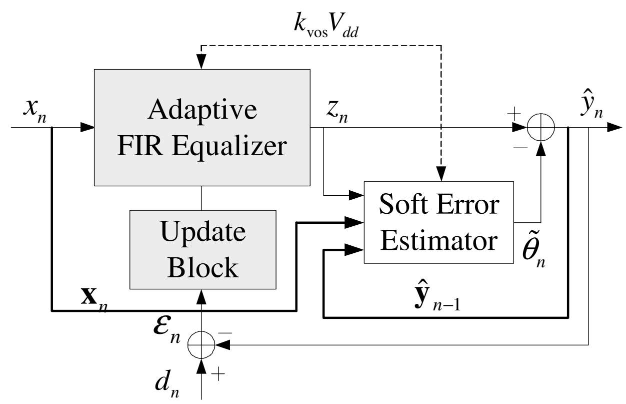

Fig. 1. Proposed soft error canceller.

In this paper, we introduce a low-power adaptive FIR equalizer which employs an iterative and power-optimized soft error canceller. In this technique, soft errors are estimated based on a subset of observed signals, and once detected, they are removed from the output of equalizer, as shown in Fig 1.

The remainder of this paper is organized as follows. In Section 2, we derive the soft error estimate based on the minimum variance criterion and provide its performance analysis. In Section 3, simulations and results are presented and in Section 4, some conclusions are given.

2. SOFT ERROR CANCELLATION

In this section, we first describe a soft error model. Then, we introduce a *minimum variance unbiased soft error estimator*, called MVU-SE, and its adaptive and energy minimum

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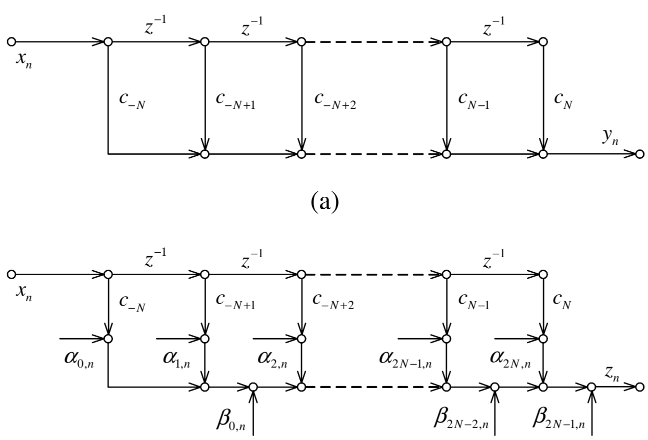
algorithm.

2.1. Soft Error Model

In many traditional designs, soft errors will tend to appear near the most significant bit (MSB) in the binary representation of the output signal, due to the use of least significant bit (LSB)-first computation for most arithmetic units. Specifically, some MSBs whose increased path delays are larger than the sampling period may not retain proper values, possibly causing soft errors. Hence, the output bits of the arithmetic unit become divided into two sets : error prone bits (EB) where the timing condition may be violated and safe bits (SB) where the timing relation is guaranteed. Soft errors can be expressed as a combination of bits from the EB region, and their magnitudes become a multiple of $2^M/2^B$ where B and M are the number of output bits and SBs, respectively. This property, which we refer to as a spacing property, plays a key role in estimating soft errors.



(a)

(b)

Fig. 2. Flow graph of a direct form I FIR filter : (a) ideal model (b) soft error model.

To illustrate how soft errors appear at the output of FIR filter, we consider a $2N+1$ coefficients FIR filter depicted in Fig 2 (a). An equivalent additive linear model of the FIR filter in the presence of soft errors can be represented in Fig 2 (b). In this model, soft error signal denoted α_i for ith multiplier and β_i for jth adder are injected to each processing unit. Hence, the soft error signal θ_n at the output can be expressed as

$$\theta_n = \sum_{i=0}^{2N} \alpha_{i,n} + \sum_{i=0}^{2N-1} \beta_{i,n}. \tag{2}$$

Due to the spacing property, θ_n takes on a value in the discrete set, $\Omega = \left\{k2^M/2^B | k \in \mathcal{Z}, k \in \left[-2^B/2^M, 2^B/2^M\right)\right\}$ where B is the number of bits of precision of the output and M is the smallest number of SBs of all processing units. In the presence of soft errors, the output z_n of a $2N+1$-tap FIR filter under VOS is written

$$z_n = y_n + \theta_n = \sum_{k=-N}^{N} c_k x_{n-k} + \theta_n, \tag{3}$$

where x_n and y_n are the nth samples of the input and error-free output respectively, and $\{c_k\}$ is the impulse response of equalizer.

2.2. Minimum Variance Unbiased Soft Error Estimator (MVU-SE)

Assume that the input and error-free output signals, x_n and y_n of the equalizer are zero mean Gaussian random process. The soft error estimator makes its decisions based on observations of a subset of $\{X = \{x_{n+N}, \cdots, x_{n-N}\}$, $\hat{Y} = \{\hat{y}_{n-1}, \cdots, \hat{y}_{n-L}\}, z_n\}$ where $\hat{y}_n$ is the nth sample of corrected output. The collection of observations is expressed in matrix form as

$$\begin{bmatrix} z_n \\ \mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1} \end{bmatrix} = \begin{bmatrix} y_n \\ \mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1} \end{bmatrix} + \theta_n \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

where $\mathbf{x}_n$ and $\hat{\mathbf{y}}_{n-1}$ are column vectors whose entries are selected from X and $\hat{Y}$, respectively. The linear unbiased estimate of θ_n is expressed as linear combination of the observations, i.e.

$$\hat{\theta}_n = a_0 z_n + \begin{bmatrix} \overline{a}^T & \overline{b}^T \end{bmatrix} \begin{bmatrix} \mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1} \end{bmatrix} \tag{5}$$

where $\overline{a} = [a_{-N_1}, \cdots, a_{N_1}]^T$, and $\overline{b} = [b_1, \cdots, b_{N_2}]^T$. If we assume that the system is operating in a low-error-rate regime, such that the vector $\hat{\mathbf{y}}_{n-1}$ does not contain errors, then we may assume that $\hat{\mathbf{y}}_{n-1}$ has the same statistic as $\mathbf{y}_{n-1}$. Therefore, to satisfy the unbiased constraint, it is clear that $a_0 = 1$. The vectors, $\overline{a}$ and $\overline{b}$ are determined to minimize the variance of the estimator $\hat{\theta}_n$, i.e.,

$$\text{VAR}(\hat{\theta}_n) = E\left[\left(y_n + \begin{bmatrix} \overline{a}^T & \overline{b}^T \end{bmatrix} \begin{bmatrix} \mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1} \end{bmatrix}\right)^2\right]. \tag{6}$$

The estimator coefficients $\overline{a}$ and $\overline{b}$ are obtained as minimum mean square error (MMSE) estimate of y_n based on $\mathbf{x}_n$ and $\hat{\mathbf{y}}_{n-1}$. The optimal coefficients, $\overline{a}_0$ and $\overline{b}_0$ are given by

$$\begin{bmatrix} \overline{a}_0 \\ \overline{b}_0 \end{bmatrix} = -\text{Cov}\left(\begin{bmatrix} \mathbf{x}_n \\ \mathbf{y}_{n-1} \end{bmatrix}\right)^{-1} \text{Cov}\left(y_n \begin{bmatrix} \mathbf{x}_n \\ \mathbf{y}_{n-1} \end{bmatrix}\right) \tag{7}$$

where $\mathbf{y}_{n-1}$ consists of the error-free output signal corresponding to $\hat{\mathbf{y}}_{n-1}$. The MVU-SE exploits the correlation structure between the input and output signals in the equalizer to correct errors. However, since θ_n takes on a value in the set Ω, the estimate of θ_n should be in Ω. Therefore, the final estimate of θ_n, or $\widetilde{\theta}_n$ is obtained by mapping of $\hat{\theta}_n$ into the closest value in Ω, i.e.,

$$\widetilde{\theta}_n = \frac{2^M}{2^B} i$$
$$\text{where} \quad \left|\hat{\theta}_n - \frac{2^M}{2^B} i\right| \leq \left|\hat{\theta}_n - \frac{2^M}{2^B} j\right| \quad \text{for } \forall j. \quad (8)$$

The corrected output of the equalizer is then obtained by subtracting $\widetilde{\theta}$ from the noisy output, z_n, i.e.,

$$\hat{y}_n = z_n - \widetilde{\theta}_n = y_n + (\theta_n - \widetilde{\theta}_n). \quad (9)$$

The term $\theta_n - \widetilde{\theta}_n$ is the residual error which will corrupt the desired output y_n. It should be noted that MVU-SE employs the previous decisions to estimate the current soft error signal, and hence operates in a *decision feedback* mode.

2.3. Residual Mean Square Error (RMSE) Analysis

In this subsection, we derive an estimate of the mean squared residual error, $\theta_n - \widetilde{\theta}_n$, (RMSE). Since the amplitude of residual error is a multiple of $2^M/2^B$, an upper bound on the RMSE is given by

$$E\left[(\theta_n - \widetilde{\theta}_n)^2\right] \leq \sum_{k=-\infty}^{\infty} (\eta k)^2 P\left(\left(\theta_n - \widetilde{\theta}_n\right) = \eta k\right) \quad (10)$$

where $\eta = 2^M/2^B$. If we assume that the estimator coefficients $\overline{a}$ and $\overline{b}$ converge to $\overline{a}_0$ and $\overline{b}_0$ in (7), then the variance of $\hat{\theta}_n$ is given by

$$\text{VAR}(\hat{\theta}_n) = E\left[y_n^2\right] - \begin{bmatrix}\overline{a}_0 \\ \overline{b}_0\end{bmatrix}^T \text{Cov}\left(\begin{bmatrix}\mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1}\end{bmatrix}\right) \begin{bmatrix}\overline{a}_0 \\ \overline{b}_0\end{bmatrix} \quad (11)$$

and $\hat{\theta}_n$ approximately follows $\mathcal{N}(\theta_n, \text{VAR}(\hat{\theta}_n))$ in the low error-rate regime. Based on the rule, (8), the bound of RMSE can be given by

$$E\left[(\theta_n - \widetilde{\theta}_n)^2\right] \leq 2\eta^2 \sum_{k=0}^{\infty} k^2 Q_k\left(\frac{\eta}{\text{VAR}(\hat{\theta}_n)}\right), \quad (12)$$

where $Q_k(x) = Q\left(x(k - \frac{1}{2})\right) - Q\left(x(k + \frac{1}{2})\right)$ and $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt$. Note that the RMSE bound decreases monotonically as $\text{VAR}(\hat{\theta}_n)$ decreases.

2.4. Energy Minimum Soft Error Canceller

In this subsection, we develop an adaptive procedure to approximates the optimal estimate for an equalizer whose coefficients vary with time. This can be formulated as an algorithm for adapting $\overline{a}$ and $\overline{b}$. The simple least mean square (LMS) algorithm can be used to approximately find the MMSE estimate in (7) in an iterative manner :

$$\begin{bmatrix}\overline{a} \\ \overline{b}\end{bmatrix}_{n+1} = \begin{bmatrix}\overline{a} \\ \overline{b}\end{bmatrix}_n + \mu e_n \begin{bmatrix}\mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1}\end{bmatrix} \quad (13)$$

where the LMS error signal, e_n is given by

$$e_n = \left(\hat{y}_n - \begin{bmatrix}\overline{a} \\ \overline{b}\end{bmatrix}_n^T \begin{bmatrix}\mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1}\end{bmatrix}\right), \quad (14)$$

and μ is the step size parameter.

A conflicting goal is to optimize the power dissipation of the error canceller. The soft error estimator (SE) can be optimized by minimizing the power dissipation of the SE block subject to a performance constraint i.e.

$$\text{Minimize} : P(N_1, N_2) \quad (15)$$
$$\text{Subject to} : E\left[(\theta_n - \widetilde{\theta}_n)^2\right] \leq \tau \quad (16)$$

where $P(N_1, N_2)$ is the power dissipation of SE block and τ is a desired power of residual error. We assume that the power dissipation, $P(N_1, N_2)$ is proportional to the number of coefficients, $N_1 + N_2$ and replace $N_1 + N_2$ in (15). Based on the following relationship,

$$\theta_n - \widetilde{\theta}_n = \hat{y}_n - y_n \quad (17)$$
$$\simeq \hat{y}_n - E\left[y_n \middle| \begin{bmatrix}\mathbf{x}_n \\ \hat{\mathbf{y}}_{n-1}\end{bmatrix}\right] = e_n, \quad (18)$$

the power of residual error can be estimated by time-averaging the squared LMS error signal, i.e.,

$$P_{n+1} = \delta P_n + (1 - \delta)e_n^2 \quad (19)$$

where P_n is the residual error power estimate at the time n and δ is a constant between 0 and 1. To control power dissipation of SE block, we define a control vector $\overline{c} = [c_1, \cdots, c_{N_1}]^T$. The ith coefficient a_i and its update sub-block are powered on if $c_i = 1$ and off if $c_i = 0$. The estimate, P_n is monitored in real time and compared with the preset threshold, τ. Starting from $\overline{c} = [1, \cdots, 1]$, we set $c_i = 0$ if P_n is smaller than $\tau - \lambda$ where i corresponds to smallest $|a_i|$ and λ is some positive constant. On the contrary, we set $c_i = 1$, if P_n becomes larger than τ. Note that whenever $\overline{c}$ changes, the adaptive algorithm runs until the $\overline{a}$ and $\overline{b}$ vectors converge. This procedure is summarized in Table 1.

A block diagram of the MVU-SE is depicted in Fig 3. The mapping, (8) is realized by a *minimum distance detector* (MDD) which rounds the input signal into the nearest integer multiple of $2^M/2^B$ [5].

Table 1. Power optimization algorithm

step 1	Start with $\overline{c} = \overline{1}$, $\overline{a} = [c_{n-N}, \cdots, cn+N]^T$, and $\overline{b} = \overline{0}$.
step 2	If the equalizer coefficients converge, then set $c_i = 0$ if $P_n - \tau \leq -\lambda$ for the smallest $\lvert a_i \rvert$.
step 3	Set $c_i = 1$ and $a_i = 0$ if $P_n - \tau > 0$
step 4	Run (13) until $\overline{a}$ and $\overline{b}$ converges. Then go to step 2.
step 5	If the equalizer coefficients begin updating, go to step 1.

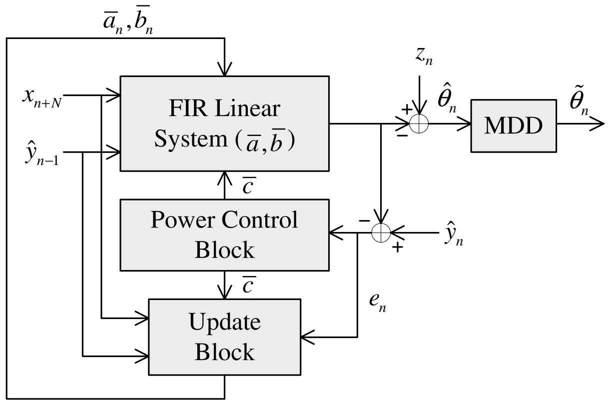


Fig. 3. Block diagram of soft error estimator.

3. SIMULATIONS

As an example, a 16-tap adaptive FIR MMSE equalizer is employed in a binary phase shift keying (BPSK) system. We assume a moderate dispersive intersymbol interference (ISI) channel whose span is within 4 sample periods with additive white Gaussian noise (AWGN). All computations in the equalizer are performed with 8 bit two's complement arithmetic. Soft errors are emulated by computing timing constraints via 0.25μm, 2.5V CMOS process logic-level simulations.

Table 2. MSE and energy savings for several k_{vos}s.

k_{vos}	MSE	Energy Savings (%)
1.0	-20 dB	0.0 %
0.9	-20 dB	17.7 %
0.8	-19.8 dB	23.3 %
0.7	-18.7 dB	37.5 %

The numerical precision of the soft error estimator is set to 4 bits. We set N_2 to 2 to limit the effect of error propagation and adjust N_1 to minimize power dissipation. Table 2 provides the mean squared error (MSE) of the MMSE equalizer after it converges and the measured power savings when employing the MVU-SE. It is shown that we can achieve up to 37 % power savings with only a 1.3 dB loss in MSE.

Fig 4 shows that the power optimization and corresponding soft error estimator applied for a the time-varying channel. Note that power optimization algorithm adapts the number of estimator coefficients to the change of the equalizer.

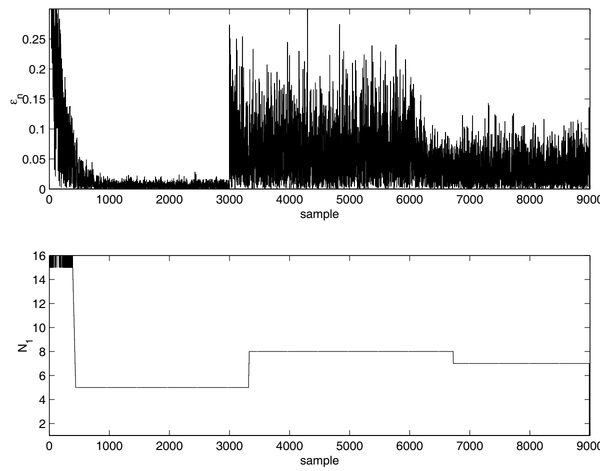


Fig. 4. (a) Instantaneous absolute value of error signal ε_n of the 16-tap equalizer, and (b) change of the number of coefficients $\overline{a}$, or N_1. The underlying channel abruptly changes at sample $n = 3000$ and sample $n = 6000$.

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