A GROUP MATCHING PURSUIT ALGORITHM FOR SPARSE CHANNEL ESTIMATION FOR OFDM TRANSMISSION

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ABSTRACT

Time-domain channel estimation techniques have been proposed for OFDM systems for their ability to yield relatively accurate estimates with only a few pilots. Key information needed in such techniques is the multipath delays of the channel. Prior approaches to estimation of multipath delays require regular pilot structures and may not work in slow fading. We propose a group matching pursuit technique for channel estimation. The technique is an extension of the orthogonal matching pursuit technique. It employs the pilots in several OFDM symbols to estimate the multipath delays in a sequential manner, where the pilots can have an arbitrary structure. Simulation results show that the proposed algorithm has superior performance.

1. INTRODUCTION

Coherent demodulation of orthogonal frequency-division multiplexing (OFDM) signals critically depends on proper channel estimation. Most channel estimation methods are pilot-aided. A common approach is to estimate the channel frequency response at pilot locations first, and then "extend" the estimate to other subcarrier locations. One frequently considered way of "extension" is low-order polynomial interpolation, which can take the form of one-dimensional interpolation in the frequency domain (in the span of one OFDM symbol) or two-dimensional interpolation over frequency and time (across several OFDM symbols) [1], [2]. The performance of these methods is limited by the pilot density and the channel characteristics. For example, if the channel has small coherence bandwidth (i.e., long delay spread) and low coherence time (e.g., due to fast motion) and the pilots are widely spaced in frequency, then they would have difficulty obtaining accurate channel estimates.

Another way of "extension" is based on exploiting the time-domain characteristics of the channel [3]. Since, in many cases, only a few multipaths contribute significantly to the channel response (i.e., the channel is "sparse"), the unknowns in time-domain estimation (which consist of the path coefficients of the significant multipaths if their delays are known) are usually much fewer than that in frequency-domain interpolation (which consist of the frequency response at all subcarriers). Hence the few pilots can be put to better use and result in more accurate channel estimates. This is especially the case when the pilots are very few and very widely spaced.

Evidently, a fundamental issue in time-domain channel estimation is to find the delays of the significant multipaths. In [4], an effective delay acquisition technique is developed, but the pilots need to be equally spaced. In [5], the MUSIC algorithm widely used for spectrum analysis is employed for channel estimation, but again assuming equally spaced pilots. The algorithm can be easily extended to deal with irregular pilot spacings (see Sec. 3.1 below), but the pilot locations in the multiple OFDM symbols used in channel estimation should be identical. To the best of our knowledge, so far there is no time-domain estimation technique for fading channels under arbitrarily organized pilots in multiple OFDM symbols (as, for example, in the case of the IEEE 802.16-2004 OFDMA [6]).

In this work, we propose an effective technique for time-domain sparse channel estimation based on the matching pursuit (MP) approach. MP algorithms have been used in audio and video signal processing to select the subspace bases [7], [8]. We extend the prior MP method for multipath delay estimation by jointly considering a group of OFDM symbols; thus we term the proposed algorithm a group MP (GMP) algorithm. And we design the algorithm such that it can deal with arbitrary pilot assignment that may vary from one OFDM symbol to the next. We note that, independently of the present authors, Tropp *et al.* [9] and Rao *et al.* [10] have also proposed algorithms of this type in other problem contexts.

The rest of this paper is organized as follows. Section 2 gives a mathematical formulation of the problem, which includes the OFDM transmission system model and the time-domain channel estimation technique. Section 3 introduces the proposed approach, after describing some previous approaches both for comparison and for illustration. Section 4 presents some simulation results. And finally, Section 5 gives the conclusion.

2. PROBLEM FORMULATION

2.1. OFDM Transmission System

Assume that the coherence time of the fading wireless channel is much larger than the OFDM symbol duration. Let the size of the discrete Fourier transform (DFT) used in OFDM transmission be N and let the OFDM symbol duration be T. The transmission mechanism associated with each OFDM symbol can be described in terms of matrix-vector notations as

$$\begin{aligned} \underline{y} &= XW\underline{h} + \underline{n} \\ &= X\underline{g} + \underline{n}, \end{aligned} \tag{1}$$

where $X = \mathrm{diag}(x(0), x(1), \ldots, x(N-1))$ is the diagonal matrix composed of the transmitted data, W is the Fourier transform matrix, $\underline{h}$ is the channel impulse response vector and $\underline{g}$ is the N-vector of the corresponding frequency response vector, $\underline{n}$ is the N-vector of additive noise samples (assumed white Gaussian), and $\underline{y}$ is the N-vector of received signal in the frequency domain (i.e., after DFT). The structures of $\underline{h}$ and W are as follows.

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Let the multipath channel have L paths with delays given by τ_l, $l = 0, \ldots, L-1$, where $0 \le \tau_l \le \tau_{\max}$ for some maximum possible path delay $\tau_{\max}$ and each τ_l may be nonsample-spaced (that is, it need not be an integer multiple of the OFDM sample spacing). Vector $\underline{h}$ has L elements, which are the complex gains of the multipaths. Matrix W has dimension $N \times L$, with its lth column given by $1/\sqrt{N}[1, e^{-j2\pi\tau_l/T}, \ldots, e^{-j2\pi(N-1)\tau_l/T}]'$ where $'$ denotes matrix transpose. Note that the lth column of W is parametrized by the path delay τ_l. The range space of W, or that of any matrix structured similarly to W, has been called a *delay subspace* [11].

Assume there are D pilot subcarriers in each OFDM symbol and assume $D \ge L$. Let S be the $D \times N$ *selection matrix* that selects the pilot locations of an N-vector. For example, $\bar{\underline{y}} \triangleq S\underline{y}$ is the vector of received pilots and $\bar{\underline{g}} \triangleq S\underline{g}$ is the vector of channel frequency response at the pilot locations. Then for the pilot locations, we have

$$\begin{aligned} \bar{\underline{y}} &= \bar{X}\bar{W}\underline{h} + \bar{\underline{n}} \\ &= \bar{X}\bar{\underline{g}} + \bar{\underline{n}} \end{aligned} \tag{2}$$

where

$$\bar{X} \triangleq SXS', \quad \bar{W} \triangleq SW. \tag{3}$$

2.2. Time-Domain Approach to Channel Estimation

Given the pilot data $\bar{X}$ and the received pilot vector $\underline{y}$, one way of time-domain channel estimation is to first derive the least-square (LS) estimates of $\bar{\underline{g}}$ and $\underline{h}$, which are given by [4]

$$\hat{\bar{\underline{g}}} = \bar{X}^{-1}\bar{\underline{y}} \tag{4}$$

and

$$\hat{\underline{h}} = (\bar{W}^H\bar{W})^{-1}\bar{W}^H\hat{\bar{\underline{g}}} \triangleq \bar{W}^{\dagger}\hat{\bar{\underline{g}}}, \tag{5}$$

respectively, where superscript H denotes Hermitian transpose. Then the estimated channel frequency response is given by

$$\begin{aligned} \hat{\underline{g}} &= W\hat{\underline{h}} \\ &= W\bar{W}^{\dagger}\bar{X}^{-1}\bar{\underline{y}}. \end{aligned} \tag{6}$$

With known pilot locations and pilot values, to complete the computation described in the right-hand side of (6), the only information that needs to be estimated is the delay subspace, or equivalently, the set of path delays $\{\tau_l\}$. To this subject we now turn in the next section.

3. ESTIMATION OF MULTIPATH DELAYS

Consider a group of L_g successive OFDM symbols and let them be indexed $j = 0, \ldots, L_g - 1$. Assume that, within the time span of these L_g symbols, i.e., L_gT, the complex multipath gains may vary due to fading, but the path delays remain the same. This assumption is appropriate because the path delays usually change much more slowly than the path gains [11]. In our earlier notations, W stays constant over this period but $\underline{h}$ may change. For convenience, we attach an index to $\underline{h}$ and let $\underline{h}^j$ denote the channel response in the jth OFDM symbol period in the group. Likewise, we also use superscript j to index other quantities that may change with symbols, such as $\hat{\bar{\underline{g}}}^j$ and S^j.

Let there be Q candidate delay values between 0 and $\tau_{\max}$ from which we will identify L for the delay subspace. One reasonable choice of these Q values is $\tau_{\max}k/Q$, $k = 0, \ldots, Q-1$. We can define an $N \times Q$ *dictionary matrix* as $V = [\underline{v}_0, \ldots, \underline{v}_{Q-1}]$ where its kth column is given by

$$\underline{v}_k = [1, e^{-j2\pi\tau_{\max}k/Q/T}, \ldots, e^{-j2\pi(N-1)\tau_{\max}k/Q/T}]'. \tag{7}$$

Define $\bar{V}^j \triangleq S^jV$.

3.1. Estimation Based on the MUSIC Algorithm

We mentioned that the MUSIC (MUltiple SIgnal Classification) algorithm [12] has been proposed for use in multipath delay estimation for OFDM transmission, with the assumption that the pilot locations be fixed and equal-spaced [5]. Below we outline the algorithm without giving all the details. It is written in a form applicable to the case with fixed but not necessarily equal-spaced pilots.

The fundamental idea of the MUSIC technique is to first find the null (noise) subspace based on the received signal and then project all candidate basis vectors of the delay subspace (i.e., columns of $\bar{V}^j$ or V) into the null subspace. Since the actual basis vectors of the delay subspace (which correspond to signal) do not lie in the null subspace, the reciprocals of the projections should show peak at these basis vectors. From this we can identify the delay subspace. Procedure-wise, the steps are as follows:

1. For each OFDM symbol group, collect the L_g estimated channel frequency response vectors $\hat{\bar{\underline{g}}}^j$ for the pilot locations. Solve for the projection matrix P_N for the noise subspace with rank $D - L$.
2. Project all the columns in $\bar{V}$ with P_N. (Note that we have omitted the index j for $\bar{V}$ because $\bar{V}^j$ is identical for all j. Indeed, having fixed pilot locations is a requirement of the MUSIC technique.) Find the L columns with the smallest projection magnitudes. These L columns define the desired delays.
3. Follow the procedure in Sec. 2.2 to complete the channel estimation.

Besides the limitation of fixed pilot locations, a property of the MUSIC technique is that, if some path coefficients do not change significantly over the L_g OFDM symbols, then there may be a rank-deficiency problem. The result is that these paths may not be identified and resolved properly. This property is quite undesirable, because it appears to imply the unpalatable conclusion that, in order to achieve good multipath delay estimation, we should make the OFDM symbol period a significant fraction of the channel coherence time. In the area of direction-of-arrivals estimation, this effect has been known as the problem of correlated signal sources, and it may heavily degrade the estimation performance even in high SNR [13]. A technique called spatial smoothing [5], [13] can solve the problem, but the remedy itself also requires equal-spaced pilots. Moreover, it would divide the pilots into several groups, which is an unaffordable solution when the pilots are very few.

3.2. Estimation Based on Orthogonal Matching Pursuit Employing One Single OFDM Symbol

In preparation for the description of the proposed GMP technique, we describe how conventional orthogonal matching pursuit (OMP) can be applied to multipath delay estimation with a single OFDM symbol [14].

Ideally, to choose L delays out of Q candidate values, we should try all $\binom{Q}{L}$ possible combinations. For each combination, $\hat{\bar{\underline{g}}}$ may be projected into the corresponding delay subspace. We then choose the combination with the largest projection magnitude as the estimation result. But the above exhaustive search approach is obviously impractical even with a moderate number of candidate delays Q. One suboptimal but much more efficient approach is the OMP technique [8], which employs a kind of greedy search method to determine the chosen candidates in a sequential fashion.

In applying OMP to multipath delay estimation for OFDM, we determine one path delay at a time. At each iteration, say iteration p, let U_p be the matrix containing the columns from $\bar{V}$ that define the (partial) delay subspace found so far. We project $\hat{\underline{g}}$ to the subspace and find the residual. Then from all columns of $\bar{V}$ that have not entered U_p, we choose the one that has the maximum inner product with the residual and add it to U_p. At this, we go to the next iteration until the required number of paths is found.

Mathematically, let d_p be the index of the column from $\bar{V}$ that is chosen in iteration p. Let $\underline{k}_p$ denote this vector, that is, $\underline{k}_p = \underline{\bar{v}}_{d_p}$. Let P_{U_p} denote the matrix that, when premultiplied to a vector, projects the vector onto the range space of U_p. And let $\underline{r}_p$ be the residual after the pth iteration. Then the OMP algorithm, in iteration p, works as follows:

$$d_p = \arg\max_i |\underline{r}_{p-1}^H \underline{\bar{v}}_i|, \quad 0 \le i \le Q-1, \tag{8}$$

$$\underline{k}_p = \underline{\bar{v}}_{d_p}, \quad U_p = [U_{p-1}, \underline{k}_p], \tag{9}$$

$$\underline{r}_p = (I - P_{U_p})\hat{\underline{g}}, \tag{10}$$

where $\underline{r}_0 \triangleq \hat{\underline{g}}$, $U_0 = \varnothing$, and U_{L-1} gives the desired estimaet of $\bar{W}$ (see Sec. 2.2).

3.3. The Group Matching Pursuit Algorithm for Multipath Delay Estimation

Now we turn to the proposed GMP algorithm for estimation of multipath delays based on observation of one OFDM symbol group. While the multipath delays and the delay subspace characterized by W are (assumed to be) fixed within one OFDM symbol group, the changing pilot locations result in different $\bar{W}^j$ and different $\bar{V}^j$. One approach, based on OMP, to address this condition is to perform L_g OMP operations, one for each OFDM symbol, and combine the results. But how the results can be combined poses a problem, because the estimated delays may be different for different OFDM symbols.

The idea of GMP is to make use of the whole set of $\hat{\underline{g}}^j$, $j = 0, \ldots, L_g - 1$, and obtain a jointly optimal delay estimation in some sense. This results in the following steps for iteration p of the algorithm:

$$d_p = \arg\max_i \sum_{j=0}^{L_g-1} |(\underline{r}_{p-1}^j)^H \underline{\bar{v}}_i^j|, \quad 0 \le i \le Q-1, \tag{11}$$

$$\underline{k}_p^j = \underline{\bar{v}}_{d_p}^j, \quad U_p^j = [U_{p-1}^j, \underline{k}_p^j], \quad 0 \le j \le L_g - 1, \tag{12}$$

$$\underline{r}_p^j = (I - P_{U_p^j})\hat{\underline{g}}, \quad 0 \le j \le L_g - 1. \tag{13}$$

(See the beginning paragraph of Sec. 3 for the meaning of the superscript j.) As in OMP, U_p^j, $j = 0, \ldots, L_g - 1$, give the the desired estimates of $\bar{W}^j$, $j = 0, \ldots, L_g - 1$, that define the delay subspace and can be used as described in Sec. 2.2 to obtain a channel estimate for each OFDM symbol in the group. Note that the channel estimates may be different for different symbols, because the channel is subject to fading, but the delay subspace remains the same.

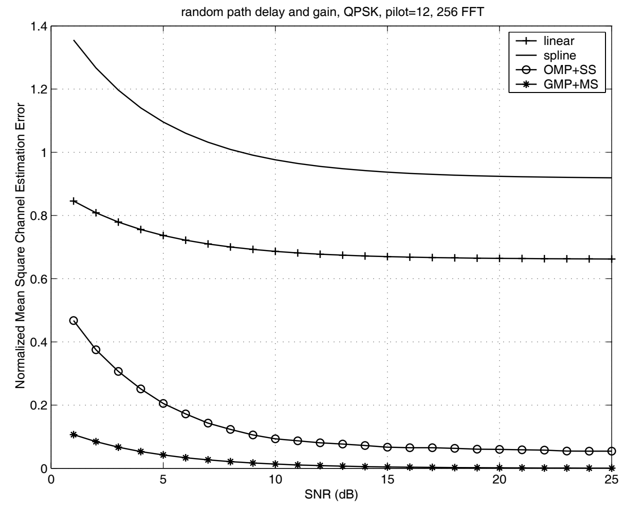

Fig. 1. Normalized mean-square channel estimation errors of different channel estimation methods, i.e., $(\hat{\underline{g}} - \underline{g})^H(\hat{\underline{g}} - \underline{g})/\underline{g}^H\underline{g}$.

3.4. Number of Path Delays to Estimate

Throughout the paper, we have assumed that the number of path delays to be estimated is known. This information can be obtained through other means of channel analysis [4] or empirical data. Even if the number of estimated delays is different from the actual number, in many cases it should not be critical. For example, if we have estimated less path delays than the actual but have captured the most significant paths, then the loss may be acceptable. Conversely, if we have estimated several more path delays than the actual, the resulting enhancement in noise may have little implication as long as its correlation with the actual delay subspace is small [14]. In any case, the number of multipath delays that can be estimated with the proposed technique is upper bounded by D, for otherwise we would have an underdetermined set of equations for $\hat{\underline{h}}$ (see, e.g., (5)).

4. SIMULATION RESULTS

Let the DFT size in OFDM be 256, with 12 subcarriers assigned for pilots. The pilot locations are randomly determined. Let QPSK be employed for each data subcarrier. Consider transmission over a 4-path channel. The path coefficients vary randomly from one OFDM symbol to another, each following a complex Gaussian distribution. Besides the first path delay $\tau_0 = 0$, other path delays are uniformly distributed in the range $[0, \tau_{\max})$, but stay constant during the OFDM symbol group used in GMP channel estimation. We let $\tau_{\max} = 25$ and $L_g = 10$.

Two MP-based approaches are simulated: OMP and GMP. As mentioned previously, to the best of our knowledge there does not exist prior techniques suitable for subspace-based OFDM channel estimation under arbitrary pilot assignments that may vary from symbol to symbol. Thus we cannot compare with eigen-decomposition based schemes such as that in [4] or [5]. However, we simulate channel estimation methods based on linear interpolation and spline interpolation, for a comparison.

Figure 1 shows the mean-square channel estimation errors of different approaches, and Fig. 2 the average symbol error rates for each simulated scheme. In the figures, the labels "GMP+MS" and "OMP+SS" mean "GMP approach for multi-symbol estimation" and

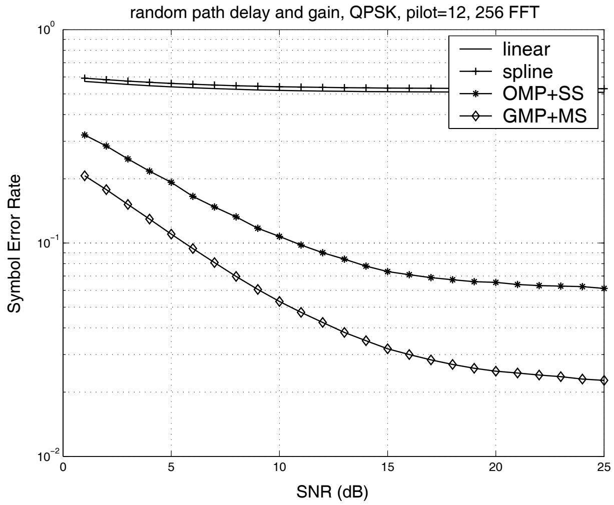

Fig. 2. Average symbol error rates (SERs) at data subcarriers with different channel estimation methods.

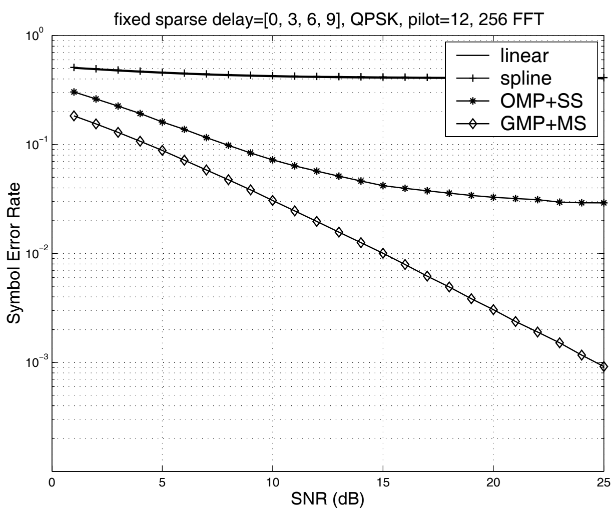

Fig. 3. Average SERs at data subcarriers with different channel estimation methods when multipath delay are spaced apart.

"OMP for single-symbol estimation," respectively. While interpolation-based methods suffer from scarcity of pilots and are not able to estimate the channel frequency responses accurately, MP-based methods can use the limited resource (pilots) efficiently and result in clearly superior estimation. The proposed GMP algorithm enjoys the greatest "diversity gain" from multi-symbol processing and thus has the better performance among all.

Figure 3 shows the average symbol error rates when the four path delays are fixed at [0, 3, 6, 9]. The simulation demonstrates even better performance for GMP than that in Fig. 2. This is because subspace-based algorithms for OFDM channel estimation has a resolution limitation depending on the pilot ratio [5]. When some paths are close together, as occasionally happened in the simulation resulting in Fig. 2, MP-based schemes may have difficulty telling them apart. But this is certainly not the case with the simulation resulting in Fig. 3, for the paths are well separated.

5. CONCLUSION

Time-domain channel estimation techniques can obtain relatively accurate channel estimates for OFDM transmission with relatively few pilot subcarriers. But it requires knowledge of the multipath delays. We proposed a group matching pursuit technique for multipath delay estimation. Unlike previous techniques, the proposed technique allows arbitrary pilot structures that may vary from one OFDM symbol to the next. Simulation results show that the proposed algorithm has superior estimation performance.

6. REFERENCES

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