THEORETICAL ANALYSIS OF TIMING ERROR IN A DIRECT TRANSMITTER SELF-CALIBRATION SYSTEM

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ABSTRACT

This paper studies the performance of a least-square-based self-calibration technique for a direct frequency conversion quadrature modulator in the presence of timing error. The technique uses the in-phase (I) and quadrature (Q) signals and their corresponding instantaneous power measurement at the transmitter output to estimate the gain/phase/DC-offset coefficients. The time synchronization between the I/Q signals and the power measurement is important for accurate coefficient estimation. In this paper we theoretically analyze the effect of the time synchronization error on the coefficient estimation. It is shown by using Taylor series expansion that the estimation errors of the gain/phase/DC-offset coefficients will be proportional to τ^2 , where τ is the timing error. Numerical simulations are used to validate the analysis.

1. INTRODUCTION

In the wireless communication industry there has been considerable effort to revolutionize transceiver architectures to reduce cost, physical size, and power consumption. Direct conversion transceiver is an example of such an efficient architecture, which performs frequency conversion between the radio frequency and the baseband in one stage, thus avoiding any use of intermediate frequencies [1],[2]. However, a direct conversion transceiver is susceptible to the gain/phase imbalances between in-phase (I) and quadrature (Q) channels, and to DC voltage offsets in the analog modulator and demodulator circuits. In the transmitter, these gain/phase imbalances produce an unwanted residual sideband (RSB), which is known to degrade the overall communication link performance [3]. In addition, the DC-offsets may cause local oscillator (LO) leakage at the output of the modulators. The DC-offset and the gain/phase imbalances distort the transmitted spectrum, making it difficult to meet the spectrum mask requirement. Therefore, it is desirable to minimize these imbalances and offsets at the transmitter.

A novel self-calibration technique described in [4] deals with the gain/phase imbalances and DC-offsets in the quadrature modulator circuit, based on baseband processing at the transmitter. The technique maximizes the LO and RSB suppression by estimating the gain/phase imbalances and DC-offsets from the power measurement at the modulator output using a least-square-based (LSbased) technique, and compensating the I and Q signals at baseband accordingly. The architecture of the direct transmitter with self-calibration is shown in Figure 1.



Figure 1. Transmitter model with self-calibration.

The performance of the LS-based self-calibration technique has been analyzed when there exist distortions in the power measurement circuit including circuit noise, quantization error, detector modeling error, and filtering [5]. Another factor affecting the calibration performance is the synchronization between the I/O signals and their associated power measurement. As we can see from Fig. 1, the power measurement signal is attributed to the I/Q signals going through various analog components, such as vector modulator, coupler, and diode detector, while the estimation block obtains the I/Q signals from the modulator block directly. Thus the power measurement data will arrive at the estimation block later than the corresponding I/Q signals. Their synchronization is of importance for the accurate estimation of gain/phase/DC-offset coefficients. In this paper, we investigate the effect of time synchronization error on the gain/phase/DC-offset coefficient estimation, and show by using Taylor series expansion that the coefficient estimation errors are proportional to τ^2 , where τ is the time synchronization error.

2. TRANSMITTER MODEL AND PARAMETER ESTIMATION

2.1. Signal Model

To represent the gain/phase imbalances and DC-offsets in the transmitter, the following model of a carrier-modulated signal is adopted

$$s(t) = [i(t) + c_i]\cos(\omega t) - \alpha[q(t) + c_q]\sin(\omega t + \phi)$$
(1)

where i(t) and q(t) are the I and Q modulating signals, each having a unity power. ω is the carrier frequency. α and ϕ represent the

gain and phase imbalances between the I and Q channels, while c_i and c_q are the DC-offsets in the I and Q channels, respectively.

By using a trigonometric identity, Eq. (1) can be expressed as

$$s(t) = u(t)\cos(\omega t) - v(t)\sin(\omega t)$$
⁽²⁾

where $u(t) = [i(t) + c_i] - \alpha[q(t) + c_q] \sin \phi$ and $v(t) = \alpha[q(t) + c_q] \cos \phi$. For $\phi \neq 0$, the I and Q channel signals become correlated, and have different power levels.

2.2. Parameter Estimation

The instantaneous output power of the modulator can be denoted using $p(t) = g(u^2(t) + v^2(t))$, where g is the gain of the measurement circuit. The power measurement can be expressed as [4]

$$p(t) = \mathbf{a}^{\mathrm{T}} \mathbf{x}(t) + w(t) \tag{3}$$

where the superscript "T" denotes the matrix transpose, w(t) is zero-mean measurement noise independent of i(t) and q(t), $\mathbf{a} = [a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6]^{\mathrm{T}}$ and $\mathbf{x}(t) = [i^2(t) \ i(t) \ i(t)q(t) \ q(t) \ q^2(t) \ 1]^{\mathrm{T}}$. $\{a_i\}_{i=1,2,\cdots,6}$ are functions of the parameters g, α, ϕ, c_i , and c_q , and their expressions are given in [4].

The LS-based technique is used to estimate the parameter vector **a** by minimizing the mean square error $\int [p(t) - \mathbf{a}^T \mathbf{x}(t)]^2 dt$. Its solution can be written as

$$\hat{\mathbf{a}} = \mathbf{R}_{xx}^{-1} \mathbf{r}_{xp} \tag{4}$$

where $\hat{\mathbf{a}} = [\hat{a}_1 \ \hat{a}_2, \cdots, \hat{a}_6]$ is the estimation of $\mathbf{a}, \mathbf{R}_{xx} = \mathbf{E}[\mathbf{x}(t)\mathbf{x}^{\mathrm{T}}(t)], \mathbf{r}_{xp}(\tau) = \mathbf{E}[\mathbf{x}(t)p(t)],$ and $\mathbf{E}(\cdot)$ is a mathematical expectation operator.

Once \hat{a}_i 's are obtained, the gain/phase imbalance and DC-offset estimates can be derived as follows [4]

$$\hat{\alpha} = \sqrt{\hat{a}_5/\hat{a}_1} \qquad \qquad \hat{\phi} = \arcsin -\hat{a}_3/(2\sqrt{\hat{a}_1\hat{a}_5})$$

$$\hat{c}_i = \frac{\hat{\alpha}\hat{a}_2 + \sin\hat{\phi}\hat{a}_4}{2\hat{a}_1\hat{\alpha}\cos^2\hat{\phi}} \qquad \qquad \hat{c}_q = \frac{\hat{\alpha}\sin\hat{\phi}\hat{a}_2 + \hat{a}_4}{2\hat{a}_1\hat{\alpha}^2\cos^2\hat{\phi}} \qquad (5)$$

3. THEORETICAL ANALYSIS

The LS-based self-calibration technique assumes perfect time synchronization between the I/Q signals and the corresponding power measurement. However, the power measurement is collected after the I/Q signals pass through several analog components, and usually arrives at the parameter estimation block later than the corresponding I/Q signals. Even if some alignment methods in digital circuits can be applied, there always exists residual timing error due to the continuous delay in analog components.

Assume that the timing error between the power measurement and the I/Q signals is τ . The power measurement $p(t + \tau)$ can be expressed as

$$p(t+\tau) = \mathbf{a}^{\mathrm{T}} \mathbf{x}(t+\tau) \tag{6}$$

The LS solution of the parameter vector **a** in the presence of time delay τ can be expressed as

$$\hat{\mathbf{a}}(\tau) = \mathbf{R}_{xx}^{-1} \mathbf{r}_{xp}(\tau) \tag{7}$$

where $\hat{\mathbf{a}}(\tau) = [\hat{a}_1(\tau) \ \hat{a}_2(\tau), \ \cdots, \ \hat{a}_6(\tau)]$ and $\mathbf{r}_{xp}(\tau) = \mathbf{E}[\mathbf{x}(t) \ p(t+\tau)]$.

In a practical communication system, the pulse shaping filters are usually inserted in the I/Q channel to improve the spectral efficiency. i(t) and q(t) can be expressed as a convolution of modulated signals and pulse shaping filter, h(t). That is,

$$i(t) = h(t) \otimes s_i(t)$$
 and $q(t) = h(t) \otimes s_q(t)$, (8)

where \otimes is a convolution operator, and $s_i(t)$ and $s_q(t)$ are waveforms of I and Q symbols, respectively.

Let us make a few assumptions about the characteristics of the I/Q signals, which can be easily justified for most quadrature modulation communication systems:

- 1. $s_i(t)$ and $s_q(t)$ are white processes.
- 2. The I/Q signals, i(t) and q(t), are stationary ergodic random processes with identical and symmetric probability distribution. That is, f(-i, q) = f(i, q) = f(i, -q) = f(-i, -q), where f(i, q) is the joint probability density function of the I and Q signals. It can be translated to $\mu_k = E[i^k(t)] = E[q^k(t)]$ and $\mu_{k_1,k_2} = E[i^{k_1}(t) \ q^{k_2}(t)] = 0$ for k_1 and/or k_2 odd.
- 3. The frequency response of the pulse shaping filter is approximated by the following rectangular function

$$|H(\omega)| = \begin{array}{cc} 1 & -B \le \omega \le B \\ 0 & \text{otherwise} \end{array}$$
(9)

where B is the bandwidth of the pulse shaping filter.

Using higher-order statistics methods and the above assumptions, the LS-solution to Eq. (6) can be obtained. Due to the space limitation, its final expression is given without the detailed derivation. The solution can be written as

$$\hat{\mathbf{a}}(\tau) = \begin{bmatrix} [a_1\eta_1(\tau) + a_5\eta_2(\tau)]/\Lambda \\ a_2m_2(\tau)/\mu_2 \\ a_3m_{iqiq}(\tau)/\mu_{2,2} \\ a_4m_2(\tau)/\mu_2 \\ [a_1\eta_2(\tau) + a_5\eta_1(\tau)]/\Lambda \\ (a_1 + a_5)\eta_3(\tau)/\Lambda + a_6 \end{bmatrix}$$
(10)

where $\Lambda = \mu_4^2 - 2\mu_4\mu_2^2 + 2\mu_{2,2}\mu_2^2 - \mu_{2,2}^2$, and

$$\begin{aligned} \eta_1(\tau) &= (\mu_4 - \mu_2^2) m_4(\tau) + (\mu_2^2 - \mu_{2,2}) m_{iiqq}(\tau) + \mu_2^2 [\mu_{2,2} - \mu_4] \\ \eta_2(\tau) &= (\mu_2^2 - \mu_{2,2}) m_4(\tau) + (\mu_4 - \mu_2^2) m_{iiqq}(\tau) + \mu_2^2 (\mu_{2,2} - \mu_4) \\ \eta_3(\tau) &= \mu_2(\mu_{2,2} - \mu_4) [m_4(\tau) + m_{iiqq}(\tau) - \mu_4 - \mu_{2,2}] \end{aligned}$$

 $m_2(\tau), m_4(\tau), m_{iiqq}(\tau), m_{qqii}(\tau),$ and $m_{iqiq}(\tau)$ are defined as

$$m_{2}(\tau) = E[i(t)i(t+\tau)] = E[q(t)q(t+\tau)] = \mu_{2}f_{1}(\tau)$$

$$m_{4}(\tau) = E[i^{2}(t)i^{2}(t+\tau)] = E[q^{2}(t)q^{2}(t+\tau)]$$

$$= 3(\mu_{4} - 3\mu_{2}^{2})f_{3}(\tau)/4 + 2\mu_{2}^{2}f_{2}(\tau) + \mu_{2}^{2}$$

$$m_{iiqq}(\tau) = E[i^{2}(t)q^{2}(t+\tau)]$$

$$= 3(\mu_{2,2} - \mu_{2}^{2})f_{3}(\tau)/4 + \mu_{2}^{2}$$

$$m_{qqii}(\tau) = E[q^{2}(t)i^{2}(t+\tau)] = m_{iiqq}(\tau)$$

$$m_{iqiq}(\tau) = E[i(t)q(t)i(t+\tau)q(t+\tau)]$$

$$= 3(\mu_{2,2} - \mu_{2}^{2})f_{3}(\tau)/4 + \mu_{2}^{2}f_{2}(\tau)$$

where $f_1(\tau) = \sin(B\tau)/(B\tau)$, $f_2(\tau) = f_1(\tau)$, and $f_3(\tau) = [2B\tau - \sin(B\tau)]/(B\tau)^3$.

Substituting the LS-solution of Eq. (10) into Eq. (5), we obtain the gain/phase/DC-offset estimates in the presence of the time delay τ , i.e., $\hat{\alpha}(\tau)$, $\hat{\phi}(\tau)$, $\hat{c}_i(\tau)$, and $\hat{c}_q(\tau)$.

It can be shown from Eq. (10) that $\hat{\mathbf{a}}(0) = \mathbf{a}$, and thus the estimation of the gain/phase/DC-offset coefficients are unbiased. More precisely, $\hat{\alpha}(0) = \alpha$, $\hat{\phi}(0) = \phi$, $\hat{c}_i(0) = c_i$, and $\hat{c}_q(0) = c_q$.

The Taylor series expansion method is used to investigate the timing sensitivity of the LS-based estimation technique. As shown in the Appendix that the first order derivatives of the $\hat{a}(\tau)$ w.r.t. $\tau = 0$ are zeros, it can be easily proven that

$$\hat{\alpha}'(0) = \hat{\phi}'(0) = \hat{c}'_i(0) = \hat{c}'_q(0) = 0 \tag{11}$$

where "'" means the first order derivative w.r.t. τ .

The second order derivatives of the gain/phase/DC-offset coefficients w.r.t. τ at $\tau = 0$ are given by

$$\hat{\alpha}^{''}(0) = [\hat{a}_{5}^{''}(0) - \alpha^{2} \hat{a}_{1}^{''}(0)]/(2g\alpha)$$

$$\hat{\phi}^{''}(0) = -[\alpha^{2} \sin(\phi) \hat{a}_{1}^{''}(0) + \sin(\phi) \hat{a}_{5}^{''}(0) + \alpha \hat{a}_{3}^{''}(0)]$$
(12)

$$/(2g\alpha^2\cos(\phi)) \tag{13}$$

$$\hat{c}_{i}^{''}(0) = \left[-2c_{i}\alpha \hat{a}_{1}^{''}(0) + \alpha \hat{a}_{2}^{''}(0) - (c_{i}\sin\phi + \alpha c_{q})\hat{a}_{3}^{''}(0) + \sin\phi \hat{a}_{4}^{''}(0) - 2c_{q}\sin\phi \hat{a}_{5}^{''}(0)\right] / (2q\alpha\cos^{2}\phi)$$
(14)

$$\hat{c}_{q}^{''}(0) = \left[-2\alpha c_{i}\sin\phi\hat{a}_{1}^{''}(0) + \alpha\sin\phi\hat{a}_{2}^{''}(0) - 2c_{q}\hat{a}_{5}^{''}(0) + \right]$$

$$\hat{a}_{4}^{''}(0) - (c_i + \alpha c_q \sin \phi) \hat{a}_{3}^{''}(0)] / (2g\alpha^2 \cos^2 \phi)$$
(15)

where "" means the second order derivative w.r.t. τ , and the expressions of $\hat{a}''_i(0)$, $i = 1, 2, \dots, 5$ are given in the Appendix.

Then the gain/phase/DC-offset coefficients can be approximated using Taylor series expansions as

$$\hat{\alpha}(\tau) = \hat{\alpha}(0) + \hat{\alpha}'(0)\tau + \frac{1}{2}\hat{\alpha}''(0)\tau^2 + O(\tau^3)$$
$$= \alpha + \hat{\alpha}''(0)\tau^2/2 + O(\tau^3)$$
(16)

$$\hat{\phi}(\tau) = \hat{\phi}(0) + \hat{\phi}'(0)\tau + \frac{1}{2}\hat{\phi}''(0)\tau^2 + O(\tau^3)$$
$$= \phi + \hat{\phi}''(0)\tau^2/2 + O(\tau^3)$$
(17)

$$\hat{c}_i(\tau) = \hat{c}_i(0) + \hat{c}'_i(0)\tau + \frac{1}{2}\hat{c}''_i(0)\tau^2 + O(\tau^3)$$

$$\hat{c}_i(\tau) + \hat{c}''_i(0)/2\tau^2 + O(\tau^3)$$
(18)

$$\hat{c}_q(\tau) = \hat{c}_q(0) + \hat{c}'_q(0)\tau + \frac{1}{2}\hat{c}''_q(0)\tau^2 + O(\tau^3)$$
(15)

$$= c_q + \hat{c}_q''(0)\tau^2/2 + O(\tau^3)$$
(19)

From Eqs. (16)-(19), we observe that in the presence of time synchronization error, the estimation errors of the gain/phase/DC-offset coefficients are independent of g because all of the $\hat{a}''_i(0)$'s have a factor of g to eliminate the factor of 1/g in Eqs.(12)-(15). The estimation errors depend on the higher-order statistics characteristics of i(t) and q(t), i.e., μ_2 , μ_4 and $\mu_{2,2}$, and also depend on the gain/phase/DC-offset coefficients. Meanwhile, they also show that the estimation errors are proportional to τ^2 .

4. NUMERICAL SIMULATIONS

A numerical method is used to validate the above analysis. An 8PSK modulation is used with a square-root raised cosine (SRC) filter with a rolloff factor of 0.25 as the pulse shaping function. The bandwidth B of the SRC filter is one half of a symbol rate R_s . Two cases of gain/phase/DC-offset parameters in the modulator

are considered, representing a typical scenario (C1) and a worst case scenario (C2):

C1:
$$\alpha = 0.9, \phi = 10^{\circ}, c_i = 0.1, \text{ and } c_q = -0.1$$

C2:
$$\alpha = 1.4, \phi = 20^{\circ}, c_i = 0.3, \text{ and } c_q = -0.3$$

In our simulation, the I/Q symbols are upsampled by a factor of K to obtain the power measurement at different time delays with the delay stepsize of $1/(KR_s)$. To derive the second-order approximation of the gain/phase/DC-offsets in Eqs. (16)-(19), μ_2 , $\mu_{2,2}$, and μ_4 are calculated from 20,000 samples. The numerical results of the gain/phase/DC-offset estimation error are obtained by averaging 100 solutions with 20,000 samples of Eq. (4).

Figure 2 shows the theoretical second-order approximation and numerical results of the estimation errors of the gain/phase/DC-offsets for the first set of parameters C1. The estimation error is normalized in the figure. For example, $\frac{\hat{\alpha}(\tau) - \alpha}{\alpha}$ is for the parameter α . It is the same for the parameters ϕ , c_i , and c_q . It can be seen that the second-order approximation results are consistent with the numerical simulation when the time delay is smaller than 10% of the symbol interval. The difference becomes noticeable when the time delay increases beyond this value. This degradation may come from the SRC filter approximation in Eq. (9), and the neglected higher-order terms in Eqs. (16)-(19).



Figure 2. Estimation error of gain/phase imbalances and DC-offsets for the parameter set C1.

Figure 3 shows the theoretical and numerical results of the estimation errors in the gain/phase/DC-offsets for the second set of parameters C2. Similar to Fig. 2, it can be seen that the theoretical analyses are consistent with the numerical results when the time delay is small.

Both cases show that the second-order approximation results are quite close to numerical results when the delay is smaller than 10% of the symbol interval. It is this time delay range that we are interested in. The reason is the coefficient estimation error caused by the time synchronization error becomes larger when the time delay is greater than 10% of the symbol interval. The large estimation error would largely degrade the LO and RSB suppression performance. It should be noted that the estimation error of the LSbased self-calibration technique is not sensitive to a small timing error.



Figure 3. Estimation error of gain/phase imbalances and DC-offsets for the parameter set C2.

5. CONCLUSIONS

This paper analyzed the estimation error of a least-square-based direct transmitter self-calibration technique. A second-order approximation of the gain/phase/DC-offset coefficients was obtained using Taylor series expansion. Numerical simulations validated the accuracy of the derivation. The results showed that the analytical result was accurate for a timing error up to 10% of the symbol interval. This analysis is helpful for system designers to specify the timing requirement between the power measurement and the I/Q signals.

APPENDIX: The 1st and 2nd derivatives of $\mathbf{a}(\tau)$ w.r.t. τ

It can be shown that the first order and second order derivatives of $f_1(\tau)$ w.r.t. τ at $\tau = 0$ can be obtained as

$$f_1'(0) = \lim_{\tau \to 0} f_1'(\tau) = \lim_{\tau \to 0} \frac{B\tau \cos(B\tau) - \sin(B\tau)}{B\tau^2} = 0 \quad (20)$$

and

$$f_{1}^{''}(0) = \lim_{\tau \to 0} f_{1}^{''}(\tau) = \lim_{\tau \to 0} \frac{1}{B\tau^{3}} [-B^{2}\tau^{2}\sin(B\tau) - 2B\tau\cos(B\tau) + 2\sin(B\tau)] = -B^{2}/3 \quad (21)$$

The first order and second order derivatives of $f_2(\tau)$ w.r.t. τ at $\tau = 0$ are given by

$$f_2'(0) = \lim_{\tau \to 0} 2f_1(\tau) f_1'(\tau) = 0$$
(22)

and

$$f_2''(0) = \lim_{\tau \to 0} 2[f_1'(\tau)]^2 + 2f_1(\tau)f_1''(\tau) = -2B^2/3 \qquad (23)$$

The first order and second order derivatives of $f_3(\tau)$ w.r.t. τ at $\tau = 0$ are shown to be

$$f_3'(0) = \lim_{\tau \to 0} \frac{-2B\tau \cos(2B\tau) + 3\sin(2B\tau) - 4B\tau}{B^3\tau^4} = 0 \quad (24)$$

and

$$f_{3}^{''}(0) = \lim_{\tau \to 0} \frac{1}{B^{3}\tau^{5}} [4B^{2}\tau^{2}\sin(2B\tau) + 12B\tau\cos(2B\tau) - 12\sin(2B\tau) + 12B\tau] = -8B^{2}/15$$
(25)

Then it can be shown that the first-order derivatives of $m_2(\tau)$, $m_4(\tau)$, $m_{iiqq}(\tau)$, $m_{qqii}(\tau)$, and $m_{iqiq}(\tau)$ w.r.t. τ at $\tau = 0$ are zero. That is, $m'_2(0) = m'_4(0) = m'_{iiqq}(0) = m'_{qqii}(0) = m'_{iqiq}(0) = 0$. Their second order derivatives are given as

$$m_2''(0) = -\mu_2 B^2 / 3 \tag{26}$$

$$n_4''(0) = -(6\mu_4 + 2\mu_2^2)B^2/15$$
 (27)

$$m_{iiqq}''(0) = m_{qqii}''(0) = -2(\mu_{2,2} - \mu_2^2)B^2/5$$
 (28)

$$m_{iqiq}^{\prime\prime}(0) = -(6\mu_{2,2} + 4\mu_2^2)B^2/15$$
⁽²⁹⁾

Furthermore we can obtain that the first order derivatives of $\hat{a}_i(\tau)$'s w.r.t. τ at $\tau = 0$ are zero. That is

$$a_1'(0) = a_2'(0) = a_3'(0) = a_4'(0) = a_5'(0) = 0$$
 (30)

Their second order derivatives w.r.t. τ at $\tau=0$ are

$$\hat{a}_{1}''(0) = g[\eta_{1}''(0) + \alpha^{2}\eta_{2}''(0)]/\Lambda$$
(31)

$$\hat{a}_{2}(0) = 2g(c_{i} - \alpha c_{q} \sin \phi)m_{2}(0)/\mu_{2}$$
(32)

$$\hat{a}_{3}(0) = -2g\alpha \sin\phi m_{iqiq}(0)/\mu_{2,2}$$
 (33)

$$\hat{a}_{4}(0) = 2g\alpha(\alpha c_{q} - c_{i}\sin\phi)m_{2}(0)/\mu_{2}$$
 (34)

$$\hat{u}_5(0) = g[\eta_3(0) + \alpha^2 \eta_1(0)] / \Lambda$$
 (35)

where

1

n

$$\begin{aligned} &\eta_{1}^{''}(0) &= (\mu_{4} - \mu_{2}^{2})m_{4}^{''}(0) + (\mu_{2}^{2} - \mu_{2,2})m_{iiqq}^{''}(0) \\ &\eta_{2}^{''}(0) &= (\mu_{2}^{2} - \mu_{2,2})m_{4}^{''}(0) + (\mu_{4} - \mu_{2}^{2})m_{iiqq}^{''}(0) \\ &\eta_{3}^{''}(0) &= \mu_{2}(\mu_{2,2} - \mu_{4})[m_{4}^{''}(0) + m_{iiqq}^{''}(0)] \end{aligned}$$

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