ASYMPTOTICALLY EFFICIENT PHASE RECOVERY FOR QAM COMMUNICATION SYSTEMS

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ABSTRACT

We introduce a new blind phase offset estimator for general Quadrature Amplitude Modulated (QAM) signals. The estimator is based on the computation of a suitable phase distribution that we call "Signature". The Signature is defined as the phase-dependent distribution of the received signal magnitude after the application of a nonlinear transformation.

The Signature of a QAM signal is constituted by a discrete number of pulses and it has good autocorrelation properties in the sense of maximum/side-lobe ratio. Since the effect of a phase offset is a cyclic shift of the Signature, the phase offset can be estimated by searching for the maximum of the cyclic cross-correlation between the zero-phase Signature of the expected constellation, and the Signature calculated on the received signal. The resulting estimator is characterized by a low computational complexity and does not need gain control. The comparison shows that the presented estimator is asymptotical efficient and outperforms existing estimators for medium to high values of SNR, especially for complex constellations.

1. INTRODUCTION

The problem of phase estimation in baseband QAM systems is here addressed. This problem is very important in synchronous systems using high-speed signaling. In the recent literature several approaches for blind phase estimation have been proposed. In [1] an estimator based on a set of fourth-order statistics, which does not need any gain control, is derived. In [2] the estimator described in [1] has been proved to be equivalent to the fourth-power estimator presented in [3], that in turn approximates the maximum-likelihood estimator in the limit of small signal to noise ratios (SNRs). In [4] an estimator based on eighth-order statistics gives improved performance for cross QAM systems with respect to the fourth power phase estimator [1], while less observed samples are needed. A phase estimator based on a modification of the received constellation is presented in [5], while in [7] a nonlinear filtering is performed in order to retain only constellation points more "reliable" for the phase estimation. Recently, in [6] a phase estimator based on evaluating the concentration ellipses of the fourth power of the received data is presented. The estimator outperforms in large SNR ranges the estimators [1], [4].

This paper introduces a new blind phase offset estimator for general Quadrature Amplitude Modulated (QAM) signals, based on the observation of the distribution of the power of the received signal samples, transformed by a suitable nonlinear function. The paper is organized as follows. In section II we introduce the model of the received signal and in section III we describe the phase estimator. The effect of additive noise is addressed in section IV. The analytical evaluation of performance is reported in section V and finally section VI shows results of numerical experiments and comparison with selected existing estimators.

2. DISCRETE-TIME SIGNAL MODEL

The analytical model of the signal, resulting from the sampling at symbol rate of the complex low-pass version of the receiver output, can be written as follows:

$$Y[n] = G_C\, e^{j\theta}\, X[n] + W[n] \tag{1}$$

where G_C is the overall gain seen by transmitted symbols $X[n]$ drawn from a QAM constellation indicated as $\mathcal{A}$ normalized to have unitary variance, and the unknown carrier phase offset that has to be estimated is denoted by θ.

It is further assumed that $W[n]$ is a realization of circularly distributed complex Gaussian stationary process, statistically independent of $X[n]$. The signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} \stackrel{\mathrm{def}}{=} G_C^2/\sigma_W^2$, being $\sigma_W^2 \stackrel{\mathrm{def}}{=} \mathrm{E}\left\{|W[n]|^2\right\}$ the noise variance.

Suppose we have a sample of N consecutive observations $Y[n]$, let's say for $n = 0, \cdots, N-1$, that can be used to estimate the carrier phase offset.

The phase offset estimate is affected by a $\pi/2$ ambiguity interval caused by the quadrant constellation symmetry; hence, the estimator extracts θ in the interval $-\pi/4, \pi/4$, leaving to other training-based procedure the recover of the complete phase offset.

3. ROUGH PHASE ESTIMATION

The phase estimator description starts with the analysis of the Signature shape for QAM signals. The Signature is defined as the phase-dependent distribution of the received signal magnitude after the application of a suitable nonlinearity transformation The selected nonlinearities have the following form:

$$Z[n] = |Y[n]|^P \cdot e^{j4 \cdot \arg\{Y[n]\}} \tag{2}$$

and they have been considered also in [10]. The order P of the magnitude influences the performances with respect to the additive noise whereas the 4-fold of the phase simply reflect the estimation ambiguity interval of $\pi/2$.

Dividing the interval 2π in L bins, the received signal Signature $\hat{F}_P[k]$ can be expressed as follows:

$$\hat{F}_P[k] = \frac{1}{N} \cdot \sum_{n:\arg\{Z[n]\} \in [\frac{2\pi \cdot k}{L}, \frac{2\pi \cdot (k+1)}{L}]} |Z[n]| \tag{3}$$

where k takes values in the set $0 \ldots L-1$.

Considering the constellation $\mathcal{A} = \{S_0, .., S_{M-1}\}$, in absence of noise and with perfect points distribution, the analytical expression of the Signature for a QAM signal is given by:

$$F_P^{\mathcal{A}}[k] = \frac{1}{M} \sum_{m=0}^{M-1} |S_m|^P \mathrm{rect}_{2\pi/L} \left((\phi/4 - \arg\{S_m\} \right) \tag{4}$$

Fig.1 shows some common constellations Signatures in absence of additive noise.

The effect of a phase offset on the Signature is a cyclic shift on the finite support $0, \ldots, L-1$. Since the autocorrelation of the Signature has a clear identifiable maximum, a rough estimation of the phase offset can be obtained finding the maximum of the cyclic cross-correlation between the expected Signature and the Signature calculated on the received signal. We define this estimate as rough because, being the Signature evaluated on a discrete number of samples (L), the estimate will be affected by the corresponding quantization error.

So, the rough phase estimate $\hat{\theta}_0$ for a QAM signal using constellation $\mathcal{A}$ can be obtained as follows:

$$C[k] \stackrel{\text{def}}{=} \hat{F}_P[k] * F_P^{\mathcal{A}}[k] \tag{5}$$

$$\hat{\theta}_0 = \frac{2\pi}{L} \cdot \arg\max_k \{C[k]\} \tag{6}$$

The cyclic cross-correlation can be carried out by a single Fast Fourier Transform (FFT); moreover, the expected Signature, or its FFT, can be calculated and stored a priori.

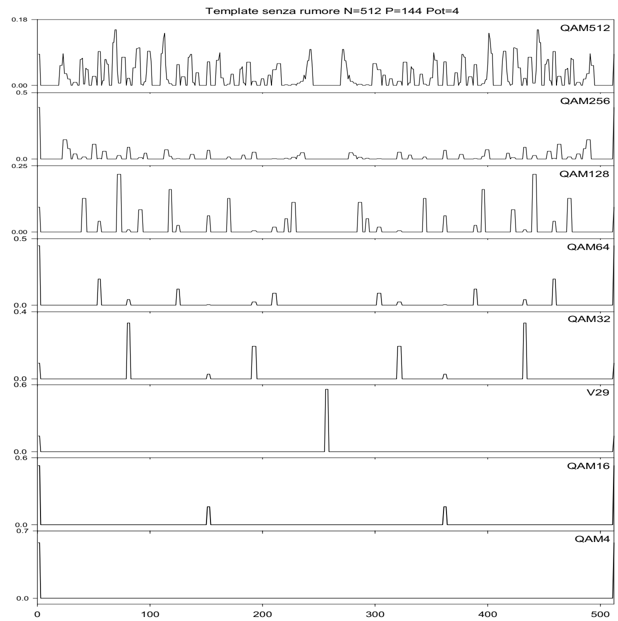

Fig. 1. *Noise free Signatures for common constellations ($L = 512, P = 4$).*

4. ESTIMATION IN PRESENCE OF ADDITIVE NOISE

In the received signal domain, the constellation points are observed in additive noise. The noise has a convolutive effect on the Signature, since it spreads the constellation point location and smooths the Signatures. Fig.2 shows the same Signature of Fig.1, now with additive noise and SNR $= 23$dB. Since the Signature are degraded by the noise, the Signature autocorrelation will result more smooth and, consequently, the phase estimate more dispersed.

The effect of the additive noise can be in part compensated modifying the Signature and using, as expected Signature, the one that corresponds to the known level of SNR.

The shape of the noisy Signature can be analytically evaluated. Details of this derivation are not reported here, but the general expression of the noisy Signature can be found to be a continous function of the phase ψ ($L \to \infty$), namely:

$$\begin{aligned} f_P^{\mathcal{A}}(\psi) = \frac{1}{M} \sum_{m=0}^{M-1} & e^{-\frac{\rho_m^2}{\sigma_W^2}} \cdot G_P^0(\psi, \rho_m, \theta_m, \sigma_W) + \\ & e^{-\frac{\rho_m^2 \sin^2((\phi - 4\theta_m)/4)}{\sigma_W^2}} G_P^S(\psi, \rho_m, \theta_m, \sigma_W) + \\ & e^{-\frac{\rho_m^2 \cos^2((\phi - 4\theta_m)/4)}{\sigma_W^2}} G_P^C(\psi, \rho_m, \theta_m, \sigma_W) \end{aligned}$$

where the functions $G_P^0(\cdots)$, $G_P^S(\cdots)$, and $G_P^C(\cdots)$ depend on the order P of the magnitude nonlinearity.

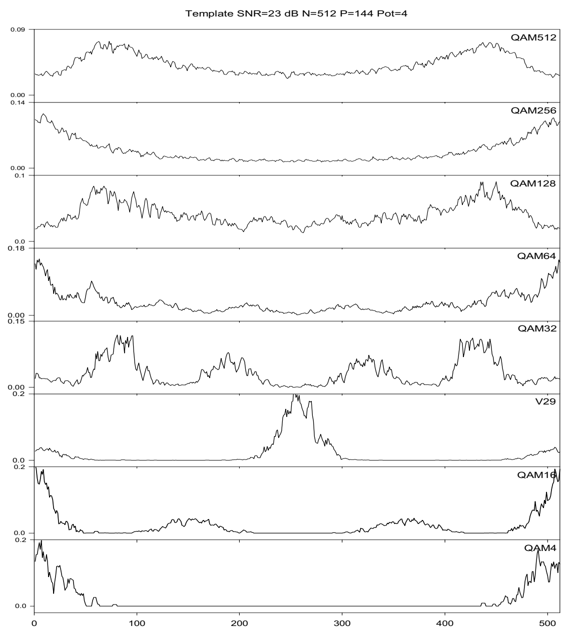

Fig. 2. *Noisy templates for common constellations (L=512,P=4,SNR $= 23$dB).*

The evaluation of $f_P^{\mathcal{A}}(\psi)$ need the knowledge of the SNR, that can be estimated through channel measure in absence of signals.

The Signature in case of L finite but large enough can be approximated by sampling of $f_P^{\mathcal{A}}(\psi)$ in the points $\psi = 2\pi k/L$ with $k = 0 \ldots L-1$.

The quantization error of the rough estimate can be reduced through an interpolation of the values assumed by the cross-correlation around the maximum. For example a parabolic interpolation of the cross-correlation function brings to the following formula for the fine estimate $\hat{\theta}_1$:

$$\hat{\theta}_1 = \hat{\theta}_0 + \frac{2\pi}{L} \frac{C[\hat{k}+1] - C[\hat{k}-1]}{C[\hat{k}+1] + C[\hat{k}-1] - 2C[\hat{k}]} \tag{7}$$

where $\hat{k}$ is the lag of the cross-correlation maximum value.

We have also conducted an analytical evaluation of performance that is not reported here due to the space limitation.

Finally, let us observe that the non-equal population of the received signal constellation points gives rise to the so-called self-noise, present also in high SNR. However, since the self-noise modifies the relative height of the different Signature pulses, but does not modify the position of the pulses, it is implicitly rejected by the cross-correlation based estimator.

5. NUMERICAL EXPERIMENTS

Some numerical experiments have been conducted to compare the performance of presented estimator with selected existing estimators. Figs. 3-8 shows the asymptotical standard deviation of phase estimate versus SNR, the experiment consist in 500 Monte Carlo trials and the phase offset is random trial by trial. Figs. 3, 5 and 7, that corresponds to square constellation, have been obtained with $N = 500$, whereas Figs. 4, 6 and 8, that corresponds to cross constellation, have been obtained with $N = 2000$. Note the excellent agreement between analytical performance and numerical results.

All the figures shows a substantial performance gain for all the constellations at medium-high SNR values; the gain is effective for complex constellations. This is mainly due to the self-noise compensation, in fact the maximum of cross-correlation between Signatures does not vary with the constellation points distribution. The method [10] shows comparable results only for the QAM16 constellation.

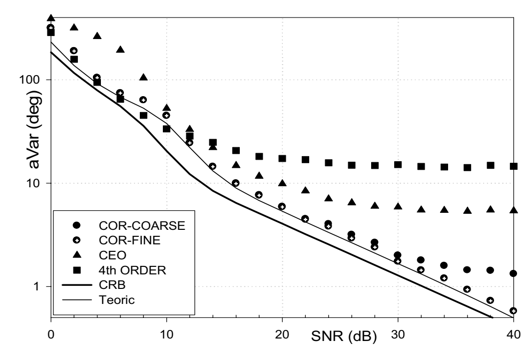

Fig. 3. *Phase estimator standard deviation vs. SNR for QAM16 constellation ($N = 500, L = 512, P = 1$).*

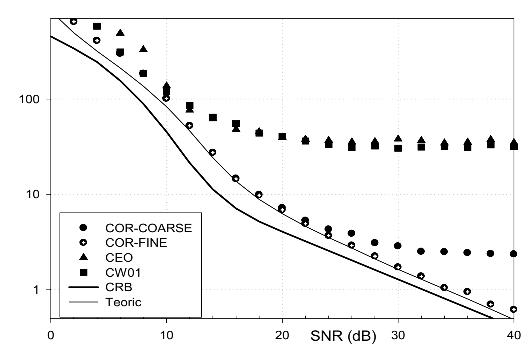

Fig. 4. *Phase estimator standard deviation vs. SNR for QAM32 constellation ($N = 2000, L = 512, P = 1$).*

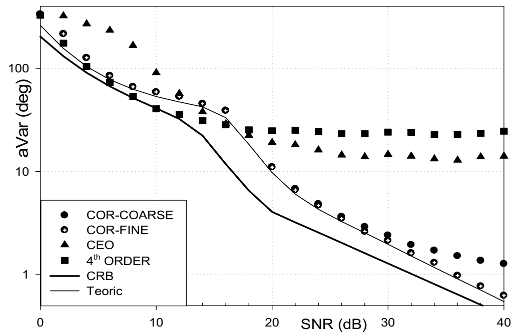

Fig. 5. *Phase estimator standard deviation vs. SNR for QAM64 constellation ($N = 500, L = 512, P = 1$).*

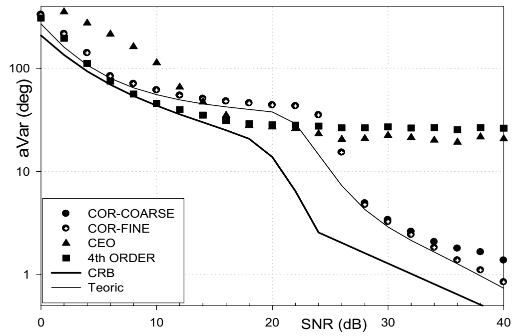

Fig. 7. *Phase estimator standard deviation vs. SNR for QAM256 constellation ($N = 500, L = 512, P = 1$).*

6. REFERENCES

[1] K. V. Cartwright, “Blind phase recovery in general QAM communication systems using alternative higher order statistics”, *IEEE Signal Processing Letters*, vol.6, Dec 1999.

[2] E. Serpedin, P. Ciblat, G. B. Giannakis, and P. Loubaton, “Performance analysis of blind carrier phase estimators for general QAM constellations”, *IEEE Transactions on Signal Processing*, vol.49, August 2001.

[3] M. Moeneclaey and G. de Jonghe, “ML-oriented NDA carrier synchronization for general rotationally symmetric signal constellations”, *IEEE Transactions on Communications*, vol.42, August 1994.

[4] K. V. Cartwright, “Blind phase recovery in cross QAM communication cystems with eighth-order statistics”, *IEEE Signal Processing Letters*, vol.8, December 2001.

[5] K. V. Cartwright, “Fourth power phase estimation with alternative two-dimensional odd-bit constellations”, *IEEE Communications Letters*, vol.4, June 2000.

[6] P. Campisi, G. Panci, S. Colonnese, G. Scarano, *Blind Phase Recovery for QAM Communication Systems*, IEEE Transactions on Signal Processing, May, 2005.

[7] C. N. Georghiades, “Blind carrier phase acquisition for QAM constellations”, *IEEE Transactions on Communications*, vol.45, November 1997.

[8] M. Morelli, U. Mengali, A.N. D’Andrea, “Feedforward estimation techiniques for carrier recovery in 16-QAM modulation, in *Broadband Wireless Communications*, M. Luise and S. Pupolin eds. New York: Springer-Verlag 1988.

[9] Y. Wang, E. Serpedin, “A class of blind phase recovery techniques for higher order QAM modulations: estimators and bounds”, *IEEE Signal Processing Letters*, vol.9, October 2002.

[10] Y. Wang, E. Serpedin, P. Ciblat, “Optimal Blind Nonlinear Least-Squares Carrier Phase and Frequency Offset Estimator for General QAM Modulation”, *IEEE Transactions on Wireless Communications*, vol.2, September 2003.

[11] U. Mengali, A.N. D’Andrea, *Synchronization Techniques for Digital Receivers*, New York:Plenum, 1997.

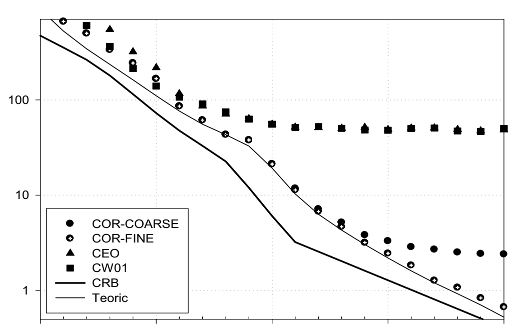

Fig. 6. *Phase estimator standard deviation vs. SNR for QAM128 constellation ($N = 2000, L = 512, P = 1$).*

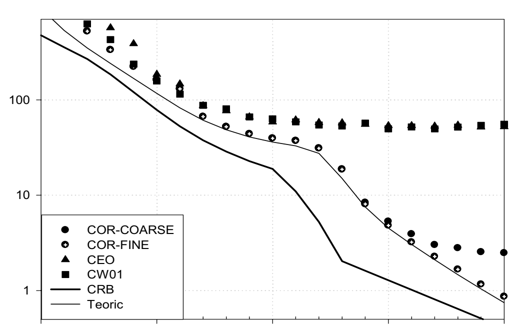

Fig. 8. *Phase estimator standard deviation vs. SNR for QAM512 constellation ($N = 2000, L = 512, P = 1$).*