A NOVEL SYNCHRONIZATION SCHEME FOR OFDM OVER FADING CHANNELS

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ABSTRACT

This paper presents a novel preamble-based synchronization scheme for orthogonal frequency division multiplexing systems over time-variant multipath fading channels. A new timing metric is derived with the use of a local synchronizing sequence (LSS). We propose one specifically designed training sequence which consists of two segments of equal length where each segment is constructed from a different pseudo-noise (PN) sequence. The advantages of using the LSS are twofold: multipath effects are reduced in the new timing metric and the metric trajectory is impulse-like. The new algorithm is shown to provide excellent performance in timing estimation even in severe time-variant fading channels. As for frequency synchronization, a two-step approach handles both fractional and integer frequency offset providing a large frequency acquisition range without loss of estimation accuracy.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM), because of its ability to combat inter-symbol interference (ISI), immunity to multipath fading and increased spectral efficiency, has been widely used in broadband communication systems (such as DAB/DVB, IEEE 802.11a/g, and ETSI HiperLAN/2) and is increasingly considered as a promising technology for the next generation communication systems.

A major drawback of OFDM is its sensitivity to timing and frequency synchronization errors [1]-[2]. Incorrect timing of OFDM symbol will introduce ISI. Frequency mismatch between the transmitter and receiver will destroy the orthogonality of subcarriers thus introducing inter-carrier interference (ICI). Therefore, the synchronization is of vital importance to OFDM systems. Most existing synchronization schemes use special training symbols (preambles) with repetitive or symmetric segments [1]-[3]. For timing synchronization, [1] uses a training symbol with two identical segments. The timing metric in [1] has a plateau which may result in large timing estimation variance. In [2], Minn proposed a preamble structure using more identical segments. The timing metric has much sharper trajectory. Park further proposes a Hermitian symmetric preamble structure [3], the corresponding timing metric has an impulse-like trajectory. However, Minn's method and Park's method suffer from the multi-path effects and the channel variation in fast-fading environments. For frequency estimation, the method in [1] has a high estimation accuracy but relatively small acquisition range. An improved method is proposed in [2] where the training symbol consists of L identical segments and the acquisition range is L times higher.

The work described in this paper was fully supported by a grant from City University of Hong Kong (Project No. 7001851).

In this paper, we present a novel synchronization scheme providing both timing and frequency estimations. We propose a new training symbol structure where the preamble consists of two segments of equal length. Each segment is constructed from a different pseudo-noise (PN) sequence. A new timing metric, using the so-called local synchronizing sequence (LSS) correlator, is derived to have an impulse-like trajectory. It is shown that the proposed method is robust even in the dispersive and fast fading channels. We apply a two-step scheme to estimate both the fractional and integer frequency offset. The acquisition range can reach to $\pm N/4$ subcarrier spacing (where N is the total number of subcarriers) with no loss of accuracy.

The rest of the paper is organized as follows. The system model is described in Section 2. In Section 3, we present our new synchronization scheme. Numerical results are shown and discussed in Section 4. We conclude in Section 5.

2. SYSTEM MODEL

The discrete samples of the transmitted OFDM symbols can be written as

$$s(k) = \frac{1}{\sqrt{N}} \sum_{p=-\infty}^{+\infty} \sum_{q=0}^{N-1} c_{p,q} e^{j(2\pi/N)q(k-pM)} g(k-pM) \quad (1)$$

where $c_{p,q}$ is the frequency domain data symbol which modulates the subcarriers, N is the number of subcarriers and M is the number of samples in an OFDM symbol (usually M is larger than N so that the $M-N$ samples at the head and/or tail of the symbol are transmitted as the guard interval), and $g(k)$ is the pulse shaping function.

The impulse response of the time-variant fading channel considered in this paper can be expressed as

$$h(t,\tau) = \sum_{l} h_l(t)\delta(\tau-\tau_l) \quad (2)$$

where l is the channel path index, $\delta(\tau)$ is Delta Function, τ_l is the lth path delay, and $h_l(t)$ is the time-variant channel gain on the lth path. We define $R_{h_l h_m}(t,\tau) = h_l^*(t)h_m(t+\tau)$. For wide sense stationary uncorrelated scattering (WSSUS) channel, the correlation function of $h_l(t)$ can be expressed as

$$E\{R_{h_l h_m}(t,\tau)\} = \begin{cases} \sigma_l^2 R_t(\tau), & for \quad l=m \\ 0, & for \quad l \neq m \end{cases} \quad (3)$$

where σ_l^2 is the average channel power gain of the lth path, and $R_t(\tau) = J_0(2\pi f_m \tau)$ where $J_0(\cdot)$ is the 0th-order Bessel function of the first kind, and f_m is the maximum doppler shift [6].

The received signal samples can be expressed as

$$r(n) = e^{j(2\pi n\theta_e/N+\phi)}w(n-n_e) + \rho(n) \tag{4}$$

where N is the total number of subcarriers, n_e is the timing error, θ_e is the frequency offset (normalized by the subcarrier spacing), $\rho(n)$ is the additive noise, ϕ is a random phase which is uniformly distributed within $[-\pi, \pi]$, and

$$w(n) = \sum_{l=0}^{J-1} h_l(n)s(n-\tau_l) \tag{5}$$

where J is the number of channel paths. Consider the following correlation of the received signal

$$\begin{aligned}\gamma(n,K) &= r^*(n)r(n+K) \\ &= e^{j2\pi\theta_e K/N}\sum_{l=0}^{J-1} R_{h_l h_l}(n,K) \\ &\quad \times P_K(n-n_e-\tau_l) + \xi(n)\end{aligned} \tag{6}$$

where $\xi(n)$, containing the cross terms and noise terms, is expressed as

$$\begin{aligned}\xi(n) &= e^{j2\pi\theta_e K/N}\sum_{l,m=0,l\neq m}^{J-1} R_{h_l h_m}(n,K) \\ &\quad \times P_{K+\tau_l-\tau_m}(n-n_e-\tau_l) \\ &\quad + e^{j[2\pi(n+K)\theta_e/N+\phi]}\rho^*(n)w(n+K-n_e) \\ &\quad + e^{-j(2\pi n\theta_e/N+\phi)}\rho(n+K)w^*(n-n_e) \\ &\quad + \rho^*(n)\rho(n+K)\end{aligned} \tag{7}$$

where $P_K(n)$ is the correlation of the transmitted samples and is defined as

$$P_K(n) = s^*(n)s(n+K) \tag{8}$$

According to (3) and (6), the autocorrelation function of the received signal can be expressed as

$$\begin{aligned}E\{\gamma(n,K)\} &= e^{j2\pi\theta_e K/N}R_t(KT_s) \\ &\quad \times \sum_{l=0}^{J-1}\sigma_l^2 P_K(n-n_e-\tau_l) + R_\rho(KT_s)\end{aligned} \tag{9}$$

where T_s is the sample rate, $R_t(\tau)$ has been described in (3), and $R_\rho(\tau)$ is the autocorrelation function of $\rho(n)$, for white Gaussian noise $R_\rho(\tau) = \begin{cases}\sigma_\rho^2, & \tau = 0 \\ 0, & otherwise\end{cases}$, where σ_ρ^2 is the noise power. In the rest of the paper, we only consider the white Gaussian noise.

3. PROPOSED SYNCHRONIZATION SCHEME

3.1. Timing Synchronization

The preamble consists of two segments of length $K = N/2$. Let $\mathbf{s}_1 = [s(0), \cdots, s(K-1)]$ and $\mathbf{s}_2 = [s(K), \cdots, s(N-1)]$ denote these two segments respectively. Therefore $\mathbf{s} = [\mathbf{s}_1, \mathbf{s}_2]$ denotes the transmitted training symbol. Other $s(i)$ for $i \in (-\infty, 0) \cup [N, +\infty)$ are arbitrary OFDM samples which assume to be independent zero-mean random variables, thus $P_K(j)$ for $j \in (-\infty, 0) \cup [K, +\infty)$ as defined in (8) are also zero-mean random variables. We define $\mathbf{P} = [P_K(0), P_K(1), \cdots, P_K(K-1)]$ as the local synchronizing sequence (LSS). We use a LSS correlator at the receiver, which means we consider the following function

$$X(d) = \sum_{n=0}^{K-1}\gamma(n+d,K)\cdot P_K^*(n) \tag{10}$$

According to (9), we obtain

$$E\{X(d)\} = e^{j2\pi\theta_e K/N}R_t(KT_s)\sum_{l=0}^{J-1}\sigma_l^2 C_p(d-n_e-\tau_l)$$

where $C_p(d)$, defined as aperiodic autocorrelation function (AACF) [4] of the sequence **P**, can be written as

$$\begin{aligned}C_p(d) &= \sum_{i=0}^{K-d-1} P_K(i)^* P_K(i+d) \\ &= \sum_{i=0}^{K-d-1} s_1(i)s_1^*(i+d)s_2^*(i)s_2(i+d)\end{aligned} \tag{11}$$

where $s_j(n)$ denotes the nth element of $\mathbf{s}_j$, $j \in \{1,2\}$. Next, we assume that the 0th channel tap has the largest power, i.e $\sigma_l^2 \leq \sigma_0^2$, for $l = 1, 2, \cdots, J-1$. From the frame detection point of view, the actual timing offset is $\tau_o + n_e$ because we always want to receive signals on the strongest path. It is desirable that the LSS is a PN sequence so that **P** has an impulse-like AACF property which maximizes $|E\{X(\tau)\}|^2$ when $\tau = \tau_0 + n_e$ thus identifying the correct timing position. A timing metric used for sliding search of the correct timing position can be derived as

$$\Lambda(d) = \frac{|X(d)|^2}{(\Gamma(d))^2} \tag{12}$$

where

$$\begin{aligned}X(d) &= \sum_{n=0}^{K-1}\gamma(n+d,K)\cdot P_K^*(n) \\ \Gamma(d) &= \sum_{n=0}^{K-1}|\gamma(n+d,K)|^2\end{aligned} \tag{13}$$

where $\Gamma(d)$ is the normalizing factor to reduce the false detection caused by noise [2]. An estimation of the correct timing position is found to be

$$\widehat{d}_{opt} = arg\max_d \Lambda(d) \tag{14}$$

3.2. Training Symbol Construction

There are two issues need to be considered when designing the preamble. First, in OFDM systems small peak-to-mean envelope power ratio (PMEPR) of the the transmitted signal $s(n)$ is required in order to ease the amplifier specification at transmitters. Second, the LSS is preferred to have an impulse-like AACF property. To fulfill the requirements, we apply Golay sequence [4] to construct the preamble.

For OFDM-type frequency domain (FD) preamble, $\mathbf{s}_1$ and $\mathbf{s}_2$ are generated by K-point inverse Fourier transform (IFFT) of two different bipolar Golay sequences of length K respectively. As for single-carrier-type time domain (TD) preamble [2], $\mathbf{s}_1$ and $\mathbf{s}_2$ are directly chosen from two different Golay sequences.

Under the assumptions of high SNR and static channel condition, we have the necessary condition of LSS, that is

$$C_p(0) \geq \frac{(\beta^2 - \sigma_{1^{st}}^2)C_{min} - (\beta^2 - \sigma_{2^{nd}}^2)C_{max}}{(\sigma_{1^{st}}^2 - \sigma_{2^{nd}}^2)} \tag{15}$$

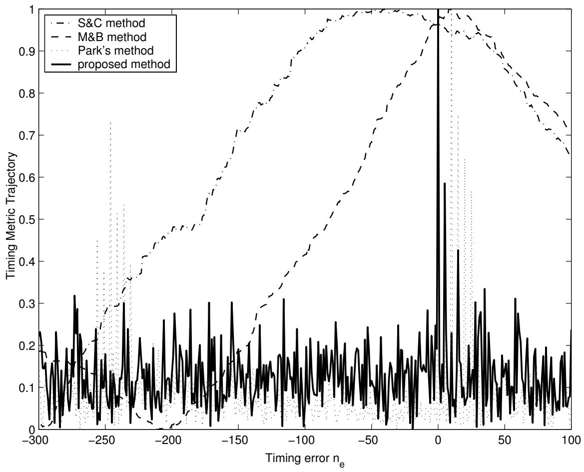

Fig. 1. Comparison of timing metric for different methods in multipath fading channel, $v = 80km/hr, SNR = 0dB$.

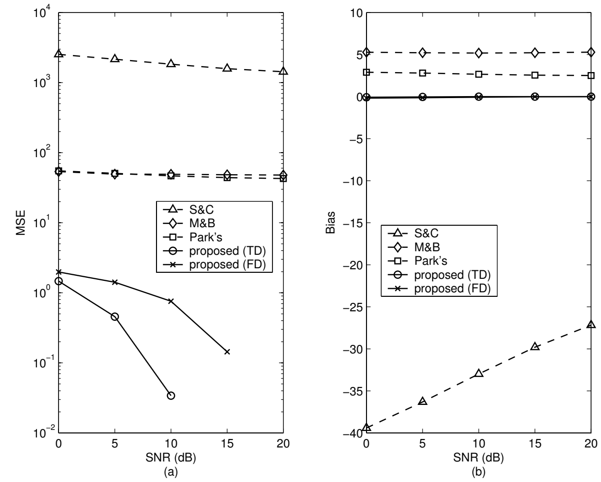

Fig. 2. Performance of timing estimation versus SNR: (a) MSE, (b) Bias, $v = 80km/hr$.

where $\beta^2 = \sum_{l=0}^{J-1} \sigma_l^2$, σ_{1st}^2 and σ_{2nd}^2 represent the 1^{st} and 2^{nd} largest channel path power respectively, C_{min} and C_{max} denote the minimal and maximum value of $C_p(\tau)$. For $K = 2^m$, there are $2^{m+1} \cdot m!/2$ bipolar Golay sequences of length K [4]. Given a channel power delay profile, we can easily obtain pairs of Golay sequence satisfying (15) by trail search from $2^{m+1} \cdot m!/2$ sequences or just choosing the pairs with the largest merit factors [4].

3.3. Frequency Synchronization

Notice that θ_e can be written as the sum of its fractional and integer parts, i.e. $\theta_e = f_e + i_e$. We use $X(\widehat{d}_{opt})$ to correct the fractional frequency error. Similar as the method in [1], f_e can be determined by

$$\widehat{f}_e = \frac{N}{2\pi K} \angle(X(\widehat{d}_{opt})) \quad (16)$$

The acquisition range of using (16) is $[-\frac{1}{2}, \frac{1}{2}]$. If we divide the training symbol into more segments, we can get larger estimation range. However, the estimation accuracy will be accordingly decreased. The fractional part of θ_e can be compensated by multiplying the received samples $r(n)$ with $e^{-j2\pi n\widehat{f}_e/N}$, that is

$$r'(n) = e^{-j2\pi n\widehat{f}_e/N} r(n) \simeq e^{j2\pi n i_e/N} w(n) + \rho'(n) \quad (17)$$

where $\rho'(n) = e^{-j2\pi n\widehat{f}_e/N} \rho(n)$. The integer part i_e can be easily estimated by the maximum-likelihood method [5] shown below,

$$\widehat{i}_e = arg \max_i \left\{ \left| \sum_{n=0}^{N-1} r'(n) s^*(n) e^{-j2\pi n i/N} \right|^2 \right\} \quad (18)$$

where $i \in [-\frac{N}{4}, \frac{N}{4}]$. The difference between the methods in this paper and in [5] is that we only search on the integer points which greatly reduces the computational complexity. Finally, the total frequency error can be estimated as

$$\widehat{\theta}_e = \widehat{f}_e + \widehat{i}_e \quad (19)$$

4. NUMERICAL RESULTS AND DISCUSSION

4.1. Simulation Parameters

The new method using FD preamble and TD preamble are evaluated. For FD training, the number of subcarriers N is 1024, the two bipolar Golay sequences are first fed into $N/2$-point FFT. For TD training, the two sequences are transmitted directly. To mitigate the multipath effect, 102 guard samples is added in front of the training symbol. The total bandwidth of the transmitted signal is 5MHz and the carrier frequency is 2.4GHz.

The multipath channel consists of 17 Rayleigh fading taps with $\tau_{max} = 76$. The fading channel has an exponential power delay profile and the ratio of the first fading tap to the last fading tap is set equal to 20dB. The maximum Doppler spread is 356Hz (i.e. the terminal speed is 160km/hr). For a given SNR, the results below are averaged over in 10000 simulation runs.

4.2. Timing Synchronization Performance

We compare our new scheme with the methods proposed in [1] (S&C method), [2] (M&B method), and [3] (Park's method). Fig.1 shows the timing metric for different methods under $SNR = 0dB$, and the terminal speed equals to $v = 80km/hr$. The timing metric trajectory of the S&C method has a large plateau. M&B method, assigning different signs to the segments of training symbols (for this experiment, $L = 4$), can greatly reduce the timing metric plateau. However, in dispersive channel the timing metric is affected by multipath effects and the correlation property among the segments is perturbed thus causing estimation bias. For Park's method, its timing metric has an impulse-like trajectory. Through time-variant multipath channels, the Hermitian symmetric property of Park's training symbol is distorted thus resulting in biased timing estimation. The proposed method also has an impulse-like timing metric trajectory. However, the LSS correlator can suppress the effect of channel distortion so that the timing metric can identify the correct timing position even in the dispersive fading channels. Fig.2-3 shows the mean squared error (MSE) and bias for the timing estimation under different channel conditions. It can be seen that reduction of timing metric plateau

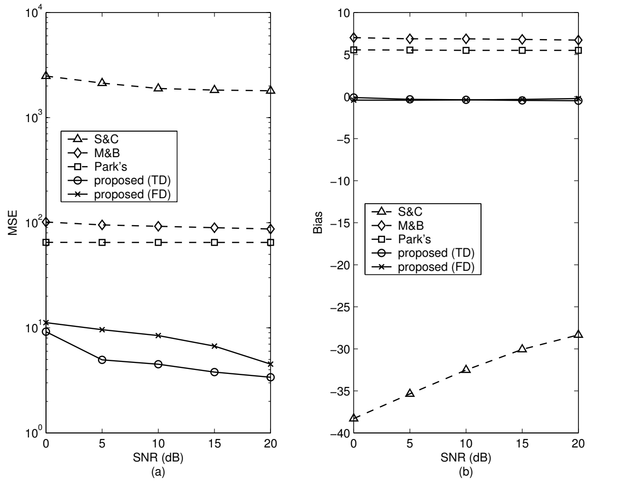

Fig. 3. Performance of timing estimation versus SNR: (a) MSE, (b) Bias, $v = 160km/hr$.

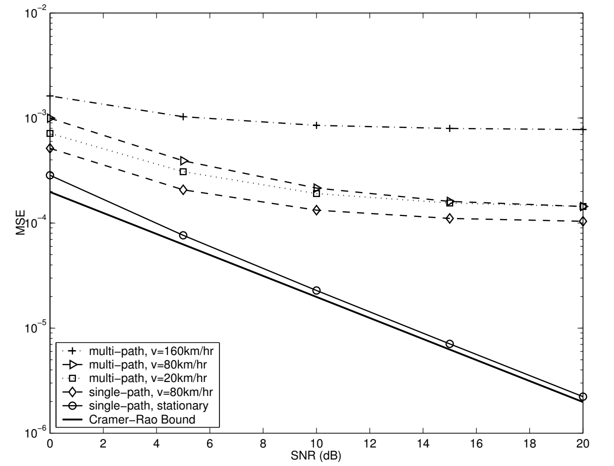

Fig. 4. MSE of frequency estimator under different channel conditions.

Table 1. Computational Load

Method	Real products	Real additions
S&C	$3N+3$	$3N-1$
Park's	$3N+3$	$3N-1$
M&B	$6N-4N/L+3$	$4N-N/L+2L-1$
proposed method	$5N+3$	$4.5N-1$

yields more accurate results. The proposed method has much smaller MSE than other methods. The bias of the proposed method is almost zero. We also observe that TD training has better performance than FD training. This is because the TD LSS has better AACF property than that of FD LSS.

The computational complexity of timing estimations can be assessed by the number of real products and additions used in $\Lambda(d)$. A complex product uses 4 real products and 3 real additions. The computational loads for different methods are listed in Table.1. For M&B method, L is at least 4 so that its computational load is the highest. The proposed method requires computing $\gamma(n, K)$ before the LSS correlator, so the complexity is higher than S&C and Park's method.

4.3. Frequency Synchronization Performance

The frequency acquisition range of proposed method is $\pm N/4$. In comparison, the acquisition range of S&C, M&B and Park's method are ± 0.5, $\pm L/2$ and ± 2 respectively.

In non-dispersive channel, the fractional frequency offset estimation is the maximum-likelihood estimator. For high SNR, this method almost meets the Cramer-Rao bounds (CRB) [1] defined as $var[\widehat{\theta}_e] \geq \frac{1}{\pi^2(N/2)SNR}$. The proposed method has similar frequency estimation performance as S&C, M&B and Park's method in terms of MSE. Thus in Fig.4, we only plot the MSE performance of the proposed method. The frequency error in the simulation is set to 1.6 subcarrier spacing. The CRB is also indicated in the figure for comparison. In timing-vary channel, the curves exhibit a floor due to the coherence loss between the segments of the training symbol. In the dispersive environment, the proposed method is an biased estimator which also has large MSE due to the multipath effect.

5. CONCLUSION

A new preamble structure and synchronization scheme are presented. The proposed timing estimator has an impulse-like timing metric trajectory which yields a much smaller timing estimation MSE. Even at low SNR and in severe multipath fading channels the estimation bias is nearly zero. Fractional and integer part of frequency offset are estimated separately. The acquisition range can reach up to $\pm N/4$ subcarrier spacing without the loss of accuracy. The proposed method are suitable for initial synchronization for both OFDM and single carrier systems.

6. REFERENCES

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