BLIND CHANNEL ESTIMATION FOR LINEARLY PRECODED MIMO-OFDM

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ABSTRACT

We propose a nonredundant linear precoder for MIMO-OFDM which enables blind channel estimation. Due to the structure introduced by the precoding matrix, the channel can be estimated based on general SVD. The identifiability of the proposed algorithm is guaranteed even when the channel matrices share common zeros at subcarrier frequencies. Computer experiments show that the performance of our proposed algorithm compares favorably to the training based LS algorithm.

1. INTRODUCTION

MIMO-OFDM systems, combining the OFDM technique with MIMO systems, can provide high-performance transmission of signals over wireless channels [1]. Blind channel estimation for MIMO-OFDM system has been a very active area of reach in recent years. A subspace channel estimation algorithm by exploiting the cyclostationarity of MIMO-OFDM channel outputs is presented in [2]. This algorithm aims at channel estimation for cyclic prefix-OFDM, but can not cope with the wireless channel for zero padded-OFDM. In [3], another subspace-based technique which exploits spatial diversity is introduced. But this algorithm is based on the assumption that channel transfer functions share no common zeros at the subcarrier frequencies, which may not be necessarily fulfilled. A blind subspace algorithm with the assistance of redundant linear precoding is proposed in [4]. In [5, 6], the virtual subcarriers are exploited to estimate the MIMO-OFDM channel. However, both the precoding redundancy and the virtual subcarriers impair the bandwidth efficiency.

In this paper, we propose a novel approach for blind MIMO-OFDM channel estimation. A nonredundant linear precoder is applied to each source data block before the conventional OFDM transmission. Due to the structure introduced by the precoding matrix, the channel can be estimated at the receiver based on general SVD operations. In many existing algorithms, the assumption that the channel transfer functions share no common zeros at subcarrier frequencies is necessary. However, our proposed algorithm can still work even when this assumption is not fulfilled.

The rest of this paper is organized as follows. In Section 2, we review the MIMO-OFDM system model and formulates the problem. In Section 3, we propose the blind estimation algorithm with the assistance of linear precoder. The identifiability, precoder designing, and SNR degradation are discussed in Section 4. Simulations are carried out in Section 5, and conclusions are drawn in the last section.

2. SYSTEM MODEL

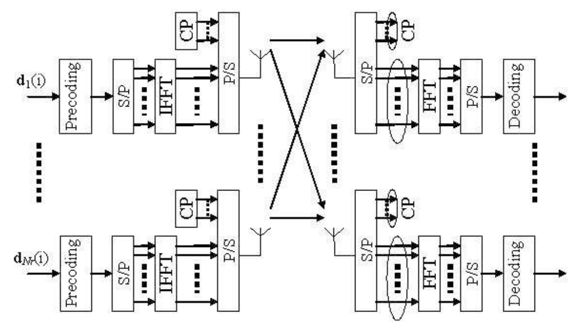

Fig. 1. MIMO-OFDM System Block Diagram

Consider the above MIMO-OFDM system equipped with N_T and N_R transmit and receive antennas respectively. Define the k^{th} block of data stream transmitted by the i^{th} ($i = 1, \cdots, N_T$) transmit antenna as

$$\mathbf{d}_i(k) \triangleq [d_i[kM], d_i[kM+1], \cdots, d_i[kM+M-1]]^T \quad (1)$$

where M is the number of subcarriers. We assume that the transmitted signals are i.i.d. with zero-mean and unit variance. A $M \times M$ precoding matrix $\mathbf{A}$ is applied to each block, mapping them as

$$\mathbf{s}_i(k) = \mathbf{A}\mathbf{d}_i(k) \quad (2)$$

The coded blocks are then transmitted through conventional MIMO-OFDM systems as shown in Fig.1. Let $h_{ji}[l], (i =$

$1, \cdots, N_T, j = 1, \cdots, N_R, l = 0, \cdots, L)$ denote the time domain channel impulse response between the i^{th} transmit antenna and j^{th} receive antenna, where L is the channel order. The frequency domain channel matrix for each transmit-receive antenna pair can be defined as

$$\mathcal{H}_{ji} \triangleq diag\left([H_{ji}[0], \cdots, H_{ji}[M-1]]\right) \tag{3}$$

where $H_{ji}[m] = \sum_{l=0}^{L} h_{ji}[l] e^{-j\frac{2\pi}{M}ml}$, $m = 0, \cdots, M-1$, $i = 1, \cdots, N_T$, $j = 1, \cdots, N_R$. The received signals after removing CP and FFT demodulation at the j^{th} receive antenna is given by

$$\check{\mathbf{s}}_j = \sum_{i=1}^{N_T} \mathcal{H}_{ji}\mathbf{s}_i + \mathbf{v}_j \tag{4}$$

where $\mathbf{v}_j$ is the zero mean white Gaussian noise vector with variance $\sigma^2\mathbf{I}$. Note that we have removed the symbol index k in Eqn.(4) to simplify the discussion.

3. BLIND CHANNEL ESTIMATION

Without loss of generality, we focus on the wireless channels associated with the j^{th} receive antenna. Consider the correlation matrix of the received signal

$$\begin{aligned}\mathbf{R}_{\check{\mathbf{s}}_j} &\triangleq E\{\check{\mathbf{s}}_j\check{\mathbf{s}}_j^H\} \\ &= E\{(\sum_{i_1=1}^{N_T} \mathcal{H}_{ji_1}\mathbf{s}_{i_1} + \mathbf{v}_j)(\sum_{i_2=1}^{N_T} \mathcal{H}_{ji_2}\mathbf{s}_{i_2} + \mathbf{v}_j)^H\} \\ &= \sum_{i=1}^{N_T} \mathcal{H}_{ji}\mathbf{A}\mathbf{A}^H\mathcal{H}_{ji}^H + \sigma^2\mathbf{I}\end{aligned} \tag{5}$$

Since $\mathcal{H}_{ji}$ is diagonal, it is not difficult to verify that

$$\mathcal{H}_{ji}\mathbf{A}\mathbf{A}^H\mathcal{H}_{ji}^H = \mathbb{H}_{ji}\mathbb{H}_{ji}^H \odot \mathbf{A}\mathbf{A}^H \tag{6}$$

where $\odot$ means the element-by-element multiplication, and $\mathbb{H}_{ji}$ is the column vector defined by

$$\mathbb{H}_{ji} \triangleq [H_{ji}[0], \cdots, H_{ji}[M-1]]^T \tag{7}$$

Assume $\mathbf{A}$ is full rank, while $\mathbf{A}\mathbf{A}^H$ has unit diagonal entries and no zero entries. Hence, we can perform an element-by-element division of $\mathbf{R}_{\check{\mathbf{s}}_j}$ with $\mathbf{A}\mathbf{A}^H$

$$\breve{\mathbf{R}}_j \triangleq \mathbf{R}_{\check{\mathbf{s}}_j}./(\mathbf{A}\mathbf{A}^H) = \sum_{i=1}^{N_T} \mathbb{H}_{ji}\mathbb{H}_{ji}^H + \sigma^2\mathbf{I} \tag{8}$$

where $./$ is a symbol of element-by-element division. Define

$$\mathbb{H}_j \triangleq [\mathbb{H}_{j1}, \cdots, \mathbb{H}_{jN_T}] \in \mathbb{C}^{M\times N_T} \tag{9}$$

Then Eqn.(8) can be rewritten as

$$\breve{\mathbf{R}}_j = \mathbb{H}_j\mathbb{H}_j^H + \sigma^2\mathbf{I} \tag{10}$$

We assume $\mathbb{H}_j$ is full column rank, then the singular-value decomposition (SVD) of $\breve{R}_j$ can be used to estimate the channel matrix. Let the SVD of $\breve{\mathbf{R}}_j$ given in Eqn.(10) denoted as

$$\breve{\mathbf{R}}_j = [\mathbf{U}_s\mathbf{U}_0]\left(\begin{bmatrix} \mathbf{\Sigma}_s & \\ & \mathbf{0}\end{bmatrix} + \sigma^2\mathbf{I}\right)[\mathbf{V}_s\mathbf{V}_0]^H \tag{11}$$

Then $\mathbb{H}_j$ can be estimated by

$$\hat{\mathbb{H}}_j = \mathbf{U}_s\Sigma_s^{\frac{1}{2}} = \mathbb{H}\mathbf{Q}_j \tag{12}$$

where $\mathbf{Q}_j$ is the constant unitary ambiguity matrix.

By repeating the above procedures to each receive antenna, all the channels can by estimated up to different unitary matrices, which need to be "synchronized". Hereby, we introduce another method to estimate the remaining channels so that this "synchronizing" can be avoided.

Consider the cross correlation matrix between k^{th} and j^{th} receive antenna for $k = 1, \cdots, N_R$ and $k \neq j$

$$\mathbf{R}_{\check{\mathbf{s}}_{kj}} = \sum_{i=1}^{N_T} \mathcal{H}_{ki}\mathbf{A}\mathbf{A}^H\mathcal{H}_{ji}^H = \sum_{i=1}^{N_T} \mathbb{H}_{ki}\mathbb{H}_{ji}^H \odot \mathbf{A}\mathbf{A}^H \tag{13}$$

Similar to Eqn.(8), we perform an element-by-element division of $\mathbf{R}_{\check{\mathbf{s}}_{kj}}$ with $\mathbf{A}\mathbf{A}^H$, hence

$$\breve{\mathbf{R}}_{kj} \triangleq \mathbf{R}_{\check{\mathbf{s}}_{kj}}./(\mathbf{A}\mathbf{A}^H) = \mathbb{H}_k\mathbb{H}_j^H \tag{14}$$

Multiply $\breve{\mathbf{R}}_{kj}$ with the pseudo-inverse of $\hat{\mathbb{H}}_j^H$, where $\hat{\mathbb{H}}_j$ is the estimated channel shown in Eqn.(12). Thus we have

$$\begin{aligned}\hat{\mathbb{H}}_k &= \breve{\mathbf{R}}_{kj}(\hat{\mathbb{H}}_j^H)^\dagger \\ &= \mathbb{H}_k\mathbb{H}_j^H(\mathbf{Q}_j^H\mathbb{H}_j^H)^\dagger \\ &= \mathbb{H}_k\mathbf{Q}_j\end{aligned} \tag{15}$$

where $\dagger$ denotes the pseudo inverse. By repeating the above procedure to all the $N_R - 1$ remaining receive antennas, the channel matrices can be estimated up to the same unitary matrix $\mathbf{Q}_j$. Therefore we can summarize our estimation algorithm as follows

1. Select a receive antenna j and calculate the self correlation matrix $\mathbf{R}_{\check{\mathbf{s}}_j}$.
2. Perform element-by-element divide with $\mathbf{A}\mathbf{A}^H$.
3. Apply SVD to $\mathbb{H}_j$, and check whether rank($\mathbb{H}_j$)$= N_T$. If yes, continue; otherwise, choose another receive antenna, and go to step 1.
4. Estimate $\mathbb{H}_j$ (up to an unitary matrix $\mathbf{Q}_j$) as Eqn.(12).
5. Calculate the cross correlation matrix $\mathbf{R}_{\check{\mathbf{s}}_k j}$ for all $k \neq j$.
6. Estimate $\mathbb{H}_k$ (up to the same unitary matrix $\mathbf{Q}_j$) by multiplying $\mathbf{R}_{\check{\mathbf{s}}_{kj}}$ with the pseudo inverse of $\hat{\mathbb{H}}_j$.

4. DISCUSSION

4.1. Identifiability

The proposed algorithm can identify any channel up to a unitary ambiguity matrix as long as $\mathbf{A}\mathbf{A}^H$ has unit diagonal entries and no zero entries, and $\mathbb{H}_j$ is full column rank. Note that we need only one out of N_R channel matrices to be full column rank. Once $\mathbb{H}_j$ is estimated, the other $\mathbb{H}_k$ ($k \neq j$) can be estimated regardless of the rank of $\mathbb{H}_k$. Moreover, the traditional blind channel estimation algorithms for MIMO- or SIMO-OFDM usually require the channel transfer functions do not share common zeros at the subcarrier frequencies [2]. However, this assumption may not be satisfied necessarily. On the other hand, according to the uniqueness of the SVD, the identifiability of our proposed algorithm is guaranteed even when this assumption is not fulfilled.

4.2. Precoder Design

As discussed in Section 3, the precoding matrix $\mathbf{A}$ should satisfy that $\mathbf{A}\mathbf{A}^H$ has unit diagonal elements and no zero elements. Assume $\mathbf{P}$ is an arbitrary $M \times M$ full rank and symmetric matrix with unit diagonal elements. Denote the SVD of $\mathbf{P}$ as

$$\mathbf{P} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{16}$$

Since $\mathbf{P}$ is full rank, then $\mathbf{U}\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{V}^H$ is also full rank. Thus, the precoding matrix can be designed as

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{V}^H \tag{17}$$

Without loss of generality, we can enforce all of the non-diagonal elements of $\mathbf{P}$ being $p, (p \neq 1)$. Under this situation, the precoding matrix $\mathbf{A}$ is a circulant matrix. It should be noted that the precoder proposed in this section is only one of the possible precoders. The optimization of the precoder design is yet to be studied.

4.3. SNR Analysis

In this subsection, we focus on the SNR degradation caused by the precoder. We assume that the channel is perfectly estimated, while the signal is detected by the zero forcing criterion. In the conventional MIMO-OFDM system without precoding, the detected signal of the i^{th} user is given by

$$\hat{\mathbf{d}}_i(k) = \mathbf{d}_i(k) + \mathcal{H}_i^{\dagger}\mathbf{v}(k) = \mathbf{d}_i(k) + \mathbf{e}_i(k) \tag{18}$$

where $\mathbf{e}(k) = \mathcal{H}_i^{\dagger}\mathbf{v}(k)$ is the detection error, $\mathcal{H}_i^{\dagger}$ stands for the $((i-1)N_T+1)^{th}$ to $(iN_T)^{th}$ rows of the pseudo inverse of the channel matrix $\mathcal{H}$, and $\mathcal{H}$ is the block matrix with the $(j,i)^{th}$ block being $\mathcal{H}_{ji}$, which is defined by Eqn.(3). Thus, the SNR of the i^{th} user according to the m^{th} subcarrier is given by

$$\mathrm{SNR_u}(i,m) = \frac{E\{|d_{i,m}(k)|^2\}}{E\{|e_{i,m}(k)|^2\}} = \frac{\beta(i,m)}{\gamma(i,m)} \tag{19}$$

where $d_{i,m}(k)$ and $e_{i,m}(k)$ are the m^{th} element of $\mathbf{d}_i(k)$ and $\mathbf{e}_i(k)$ respectively. On the other hand, the detected and decoded signal in the precoded MIMO-OFDM system is

$$\hat{\mathbf{d}}_i(k) = \mathbf{d}_i(k) + \mathbf{A}^{-1}\mathbf{e}_i(k) \tag{20}$$

Hence, the SNR is modified as

$$\begin{aligned} \mathrm{SNR_c}(i,m) &= \frac{E\{|d_{i,m}(k)|^2\}}{\sum_{n=0}^{M-1}\left(|a'_{m,n}|^2 E\{|e_{i,n}(k)|^2\}\right)} \\ &= \frac{\beta(i,m)}{\sum_{n=0}^{M-1}\left(|a'_{m,n}|^2\gamma(i,n)\right)} \end{aligned} \tag{21}$$

where $a'_{m,n}$ is the $(m,n)^{th}$ element of $\mathbf{A}^{-1}$.

Lemma 1

$$\frac{\min_m \mathrm{SNR_u}(i,m)}{\lambda_m} \leqslant \mathrm{SNR_c}(i,m) \leqslant \frac{\max_m \mathrm{SNR_u}(i,m)}{\lambda_m} \tag{22}$$

where λ_m is the m^{th} diagonal element of $(\mathbf{A}^H\mathbf{A})^{-1}$.

proof: One can verify that $\sum_{n=0}^{M-1}|a'_{m,n}|^2 = \lambda_m$, which is the m^{th} diagonal element of $(\mathbf{A}^H\mathbf{A})^{-1}$, thus we have

$$\lambda_m\gamma_{min}(i,m) \leqslant \sum_{n=0}^{M-1}\left(|a'_{m,n}|^2\gamma(i,n)\right) \leqslant \lambda_m\gamma_{max}(i,m) \tag{23}$$

Substitute Eqn.(23) to Eqn.(21), we have

$$\frac{\beta(i,m)}{\lambda_m\gamma_{max}(i,m)} \leqslant \mathrm{SNR_c}(i,m) \leqslant \frac{\beta(i,m)}{\lambda_m\gamma_{min}(i,m)} \tag{24}$$

which is equivalent to Eqn.(22).

If the circulant precoding matrix mentioned in Section 4.2 is in use, we can verify that $\lambda_m = [1/(1+(M-1)p)+(M-1)/(1-p)]/M$ for $m = 0, \cdots, M-1$. Since λ_m is a function of p, then the SNR and hence the BER performance can be controlled by carefully selecting p.

5. SIMULATION RESULTS

In this section, we provide some simulation results to illustrate the performance of the proposed estimator. The simulated OFDM system is modeled containing 64 subcarriers, i.e. $M = 64$. Each OFDM frame consists of 68 symbols including the CP of length 4, i.e. $M_g = 4$. The system is equipped with 2 transmit antennas and 2 receive antennas, and the channel model used is a 3-tap FIR filter with tap coefficients independently chosen from a white Gaussian process. As a comparison, we also simulate the training based LS channel estimation algorithm proposed in [7].

To evaluate the channel estimation error, we employed the normalized-root-mean-square-error (NRMSE), which is defined as

$$\mathrm{NRMSE} = \sqrt{\frac{1}{N_R N_T N_M} \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} \sum_{t=1}^{N_M} \frac{\| \hat{\mathbb{H}}_{ji}^{(t)} - \mathbb{H}_{ji} \|^2}{\| \mathbb{H}_{ji} \|^2}} \tag{25}$$

where N_M is the number of Monte Carlo runs for each channel realization. $\hat{\mathbb{H}}_{ji}^{(t)}$ is the estimation of channel $\mathbb{H}_{ji}$ from the t^{th} run. We simulate 30 channel realizations, each for 100 Monte Carlo runs.

Fig.2 illustrate the NRMSE as a function of SNR. We can see that our proposed estimator can achieve a lower NRMSE than the training-based LS algorithm at moderate or low SNR. The figure also indicates that NRMSE performance of the proposed method can be controlled by carefully selecting p. (This property of the proposed algorithm is not discussed due to the space limitation.)

Fig. 3 illustrates the BER as a function of SNR. As shown in the figure, the proposed estimation algorithm can achieve a BER performance close to the LS estimator if p is small. As discussed in Section 4.3, when p is small enough, the SNR degradation caused by the precoding is suppressed. Thus, the proposed BER performance can improved. However, this will cause the NRMSE performance degradation. Thus, we need to balance them.

6. CONCLUSION

We presented a novel blind channel estimation method for MIMO-OFDM system where the source data is linearly precoded. With the assistance of a nonredundant linear precoder, the channel can be estimated blindly by exploiting the correlation matrix of the received signal. The proposed algorithm can identify the channel even when the channel transfer functions share zeros at subcarrier frequencies. Simulations show that the proposed algorithm compares favorably to the training based LS algorithm in both NRMSE and BER performance. The performance of the proposed algorithm could be further improved by optimizing the precoder, which is an open question.

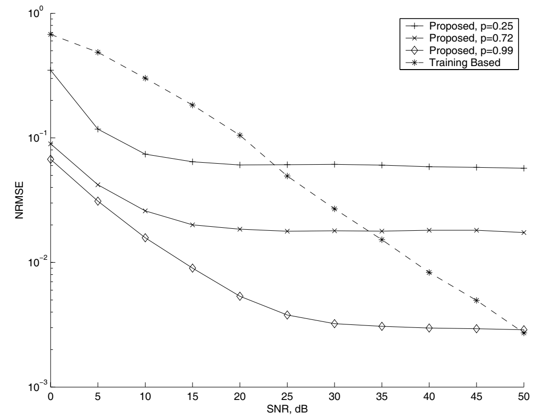

Fig. 2. NRMSE performance as a function of SNR

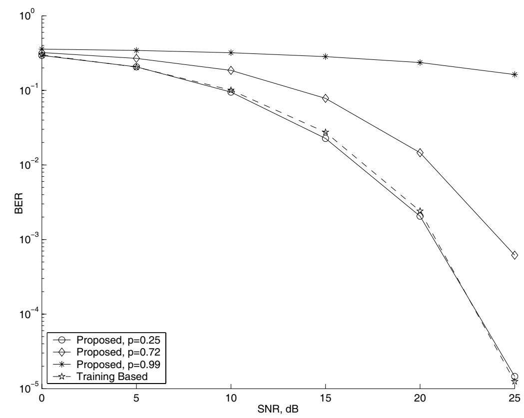

Fig. 3. BER performance as a function of SNR

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