COMBINED FREQUENCY AND TIME DOMAIN CHANNEL ESTIMATION IN MOBILE MIMO-OFDM SYSTEMS

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ABSTRACT

This paper proposes a combined frequency and time domain channel estimation method for MIMO OFDM systems. Initial channel estimation is performed by first estimating the channel response in frequency domain, exploiting a set of dedicated pilot carriers, followed by an interpolation step. In order to reduce the interpolation error and improve the bit error rate performance, we propose to refine the channel estimates in time domain using the equalized signal from the frequency domain processing. Simulations have been carried out using the spatial channel model proposed under the 3GPP framework. The proposed method has been tested in a wide range of mobile speeds in conjunction with several standard MIMO equalizers. The results show that the proposed estimator outperforms widely used time and frequency domain channel estimation approaches.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques are promising candidates for future mobile wireless communications systems since they provide high spectral efficiency by exploiting the spatial component and can therefore obtain much higher capacities than conventional single-input single-output (SISO) systems.

Multicarrier transmission such as orthogonal frequency division multiplexing (OFDM) plays an important role in future beyond 3G wireless communication systems. A key benefit of OFDM is its ability to turn a frequency-selective channel into a set of parallel narrowband channels. This leads to simple equalization in case of SISO systems because transmission becomes free of intersymbol interference (ISI). Hence, the equalization in the SISO case may be accomplished using single-tap equalizers. For the MIMO case, the equalization of the received signal is more complicated and one may have to resort to a suitable multiuser detection (MUD) technique [1, 2, 3, 4] to combat the interference arising from the different MIMO channels.

In order to achieve a good receiver performance, the MUD detector (or MIMO-OFDM equalizer) requires accurate channel estimates. In mobile wireless systems, the channel is both time and frequency selective. As the frequency selectivity is handled by the OFDM transmission itself, the receiver is required to track the timeselective channel [5]. High data rate and high mobility scenarios require advanced processing at the receiver in order to achieve sufficiently low bit-error rates (BER).

To aid the channel estimation, we will assume throughout the paper that pilot data is available. There are basically two approaches considered in the literature: *block-type* (or training symbol based) and *comb-type* (or pilot tone multiplexing) [6]. The block-type training, assumes that a whole OFDM symbol is periodically transmitted, i.e., pilot data available on all subcarriers. In between train-

ing symbols, the estimator needs to operate in a decision-directed mode to track the channel. For the comb-type approach, a set of carriers are dedicated for pilot data. The channel estimates for the data carriers are obtained by interpolation using truncated DFT matrices [7, 8]. For the MIMO extension of the one-shot SISO channel estimator [7], the comb-type pilot structure requires that each transmit antenna has its own dedicated pilot carriers [9].¹ For high speed scenarios, the comb-type pilot structure has the advantage of avoiding advanced channel tracking, e.g., by using channel prediction required in block-type approaches [10]. The main drawback of the comb-type pilot structure is the performance degradation due to interpolation errors [11].

In the following, we propose a new channel estimation method in order to improve the channel estimates fed to a chosen MIMO-OFDM equalizer. For this purpose, we consider a combined frequency and time domain approach. In this work initial channel estimation is performed in FD using comb-type pilot channels. This initial channel estimate is refined by performing a second channel estimation in time domain. The proposed TD channel estimator utilizes the symbol estimates from the equalizer constructed from the FD channel estimates. As a consequence, the proposed TD stage exploits the available tone-multiplexed pilot necessary for high-speed scenarios.

2. MIMO OFDM SIGNAL MODEL

The MIMO OFDM system under consideration has N subcarriers per antenna, and the number of antennas at the transmitter and receiver are denoted N_t and N_r , respectively. The transmitted block $\mathbf{x}_{i,cp}(k) \in \mathbb{C}^{(N+L)\times 1}$ from antenna *i* at time *k* is given by

$$\mathbf{x}_{i,cp}(k) = \mathbf{T}_{cp}\mathbf{x}_i(k) = \mathbf{T}_{cp}\mathbf{F}_N\mathbf{a}_i(k), \qquad (1)$$

where \mathbf{T}_{cp} is the $(N + L) \times N$ cyclic prefix (CP) insertion matrix, $\mathbf{x}_i(k) = \mathbf{F}_N \mathbf{a}_i(k)$ is the IDFT of the modulated symbols $\mathbf{a}_i(k) \in \mathbb{C}^N$, and \mathbf{F}_N denotes the $N \times N$ IDFT matrix [12].

Assuming that the transmission channel $\mathbf{h}_{ij}(k)$ between transmit antenna *i* and receive antenna *j* remains constant over at least one OFDM block, the received signal after cyclic prefix removal at antenna *j* may be written as

$$\mathbf{r}_{j}(k) = \sum_{i=1}^{N_{t}} \tilde{\mathbf{H}}_{ij}(k) \mathbf{x}_{i}(k) + \mathbf{n}_{j}(k), \qquad (2)$$

¹Solutions that enable the antennas to share pilot carriers include orthogonal training sequences (orthogonal in time) or space-time coding. However, these solutions would require the channel to remain constant over more than one OFDM symbol.

where $\tilde{\mathbf{H}}_{ij}(k)$ is the $N \times N$ circulant channel matrix for the MIMO branch (i, j), and $\mathbf{n}_i(k)$ is the additive noise, assumed here to be circular complex Gaussian with $\mathbf{E}[\mathbf{n}_j(k)\mathbf{n}_j^H(k)] = \sigma_n^2 \mathbf{I}_N$. By applying a DFT to Equation (2), the circulant matrix $\tilde{\mathbf{H}}_{ij}(k)$ is diagonalized:

$$\tilde{\mathbf{r}}_j(k) = \sum_{i=1}^{N_t} \mathbf{D}_{ij}(k) \mathbf{a}_i(k) + \tilde{\mathbf{n}}_j(k)$$
(3)

where $\mathbf{D}_{ij}(k)$ is a diagonal matrix that contains the frequency response of the channel \mathbf{h}_{ij} at each subcarrier frequency.

By using the commutativity property for convolution we get the following alternative form for the received signal at each antenna:

$$\mathbf{r}_{j}(k) = \sum_{i=1}^{N_{t}} \mathbf{X}_{i}(k) \mathbf{h}_{ij}(k) + \mathbf{n}_{j}(k), \quad j = 1, \dots, N_{r} \quad (4)$$

where $\mathbf{X}_i(k)$ is the $N \times L$ circulant matrix formed by the modulated symbols $\mathbf{x}_i(k)$.

3. CHANNEL ESTIMATION

This section proposes a combined frequency and time domain channel estimation method. The main idea of the approach, depicted in Figure 1, is to benefit from the robustness of the pilot-aided frequency domain method, and from the reduced variance of the time domain method. An initial channel estimation and equalization is made in frequency domain based on dedicated pilot carriers. Thereafter, the equalized signals from the initial stage are used to build a time domain channel estimator in order to reduce the effect of interpolation errors introduced by the frequency domain approach. In the following subsections, we describe the frequency and time domain estimators used in the block-diagram of Figure 1.

3.1. Frequency-domain channel estimation

For the frequency domain channel estimation, we will assume a comb-type pilot arrangement as described in [9], where each transmitter is allocated P pilot carriers. Therefore, a total of $N_t P$ pilot carriers need to be reserved for pilot data.

Let i_P denote the set specifying the P pilot carriers of transmit antenna i. Let $\tilde{\mathbf{r}}_{j,i_P}(k) \in \mathbb{C}^P$ denote the received signal at antenna j on the pilot carriers of transmit antenna i. Since no other antenna transmits data on the carriers specified by i_P , the received signal on those carriers at antenna j $\tilde{\mathbf{r}}_{j,i_P}(k) \in \mathbb{C}^P$ reduces to the SISO case

$$\tilde{\mathbf{r}}_{j,i_p}(k) = \mathbf{A}_{i_p}(k)\tilde{\mathbf{h}}_{j,i_p}(k) + \tilde{\mathbf{n}}_{j,i_p}(k).$$
(5)

Here $\mathbf{A}_{i_p}(k)$ is a diagonal matrix with the pilot data from transmit antenna *i*. The channel response on the pilot carriers can be obtained via the least-squares (LS) solution:

$$\tilde{\mathbf{h}}_{i_p j}(k) = \left[\mathbf{A}_{i_p}^H(k)\mathbf{A}_{i_p}(k)\right]^{-1}\mathbf{A}_{i_p}^H(k)\tilde{\mathbf{r}}_{j,i_p}(k)$$
(6)

The whole frequency domain channel response is obtained through interpolation using truncated DFT matrices

$$\tilde{\mathbf{h}}_{ij}(k) = \mathbf{F}_{i_P,N-P} \mathbf{F}_{i_P,P}^+ \tilde{\mathbf{h}}_{i_P j}(k)$$
(7)

where $\mathbf{F}_{i_P,N-P}$ refers to the truncated DFT matrix of antenna *i* containing the *P* rows of \mathbf{F}_N specified by i_P and $\mathbf{F}_{i_P,P}$ contains the *P* rows of the truncated matrix $\mathbf{F}_{i_P,N-P}$.

One of the major drawbacks of the channel estimation scheme outlined above is the performance degradation due to interpolation errors [11]. In the next section, we propose to refine the MIMO channel estimate by TD processing.

3.2. Time-domain channel estimation

This section details the time-domain channel estimator that will be used in conjunction with the frequency-domain channel estimator described in the previous subsection. The idea is to improve the channel estimates obtained in frequency domain by using the equalized MIMO signal as a building block for the time domain channel estimation.



Fig. 1. Combined frequency and time domain channel estimation.

Stacking the received OFDM blocks at each antenna in vector $\mathbf{r}(k) = \left[\mathbf{r}_1^T(k) \cdots \mathbf{r}_{N_r}^T(k)\right]^T$ gives us

$$\mathbf{r}(k) = \mathbf{X}(k)\mathbf{h}(k) + \mathbf{n}(k)$$
(8)

with matrix $\mathbf{X}(k) \in \mathbb{C}^{NN_r \times LN_r N_t}$ and vector $\mathbf{h}(k) \in \mathbb{C}^{LN_r N_t}$ are given by [5]

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{X}_1(k) & \cdots & \mathbf{X}_{N_t}(k) & \mathbf{0}_{N \times L} & \cdots \\ \mathbf{0}_{N \times L} & \cdots & \mathbf{0}_{N \times L} & \mathbf{X}_1(k) & \cdots & \mathbf{X}_{N_t}(k) \\ \vdots & & \ddots & \vdots \end{bmatrix}$$
(9)

and

$$\mathbf{h}(k) = [\mathbf{h}_{11}^T(k) \cdots \mathbf{h}_{N_t 1}^T(k) \cdots \mathbf{h}_{1N_r}^T(k) \cdots \mathbf{h}_{N_t N r}^T(k)]^T$$
(10)

For a known matrix $\mathbf{X}(k)$, the least-squares (LS) estimate (zeroforcing estimate) of $\mathbf{h}(k)$ is given by

$$\hat{\mathbf{h}}_{LS}(k) = \mathbf{R}_{\mathbf{X}}^{-1} \mathbf{X}^{H}(k) \mathbf{r}(k)$$
(11)

where $\mathbf{R}_{\mathbf{X}} = \mathbf{X}^{H}(k)\mathbf{X}(k)$ is the $LN_{r}N_{t} \times LN_{r}N_{t}$ input-signal correlation matrix. Note that matrix $\mathbf{R}_{\mathbf{X}}$ has the block-diagonal form diag $[\mathbf{B}(k), \ldots, \mathbf{B}(k)]$ with matrix $\mathbf{B}(k) \in \mathbb{C}^{N_{t}L \times N_{t}L}$ given by

$$\mathbf{B}(k) = \begin{bmatrix} \mathbf{X}_{1}^{H}(k)\mathbf{X}_{1}(k) & \cdots & \mathbf{X}_{1}^{H}(k)\mathbf{X}_{N_{t}}(k) \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{N_{t}}^{H}(k)\mathbf{X}_{1}(k) & \cdots & \mathbf{X}_{N_{t}}^{H}(k)\mathbf{X}_{N_{t}}(k) \end{bmatrix}$$
(12)

and the channel estimation can be decoupled into N_r parallel problems of size $N_t L \times N_t L$. If we exploit the Toeplitz structure of matrix $\mathbf{B}(k)$ [10], the Levinson algorithm [13] can be used to efficiently solve for the channel estimates at each antenna. If L is small, $\mathbf{B}^{-1}(k)$ can also be pre-computed and applied directly at each antenna branch. Alternatively, an MMSE variant could be used that includes the noise covariance matrix in the inverse in order to balance between noise attenuation and channel estimation. We end this section by noting that the time domain channel estimator above requires knowledge of the transmission matrix $\mathbf{X}(k)$. In practice, $\mathbf{X}(k)$ is only known exactly at the transmitter. Instead, and estimate $\hat{\mathbf{X}}(k)$ is obtained using the equalized output signal, see Figure 1. For this purpose, the next section briefly reviews well-known MIMO equalizers.

4. FREQUENCY-DOMAIN EQUALIZER

If the frequency-domain channel coefficients of each MIMO channel are known, i.e., $\mathbf{F}_N^H \mathbf{h}_{ij}(k)$, channel equalization is carried out by applying a suitable multiuser detection technique [1, 2, 3, 4]. Let $\tilde{\mathbf{r}}^{(n)}(k) = [\tilde{r}_1^{(n)}(k), \ldots, r_{N_r}^{(n)}(k)]^T$ denote the the vector containing the received signals at each receive antenna on subcarrier *n*. Then, for each subcarrier we can write the received signal vector as

$$\tilde{\mathbf{r}}^{(n)}(k) = \tilde{\mathbf{H}}^{(n)}(k)\mathbf{a}^{(n)}(k) + \tilde{\mathbf{n}}^{(n)}(k), \ n = 1, \dots, N$$
(13)

where the components of the $N_r \times N_t$ matrix $\tilde{\mathbf{H}}^{(n)}(k)$, $H_{ij}^{(n)}$, denote the channel response of the MIMO branch (i, j) on subcarrier n. In our simulations presented in the next section, we implement the MMSE equalizer (also considered in [5]), the parallel interference canceller (PIC), and the optimal maximum likelihood (ML) sequence detector [1, 2, 3, 4]. The computational complexity of the optimal equalizer for the MIMO OFDM system is in the order of $\mathcal{O}(\mathcal{M}^{N_t})$ per subcarrier, where \mathcal{M} is the modulation constellation size. For example, for a 2 × 2 system employing QPSK or 8-PSK modulation, the ML detector may be a rather feasible solution.

5. SIMULATIONS

The performances of different estimators were evaluated in an 2×2 OFDM system with N = 256 subcarriers. The performances of pure frequency-domain (FD) in (6) and frequency-domain plus time-domain (FDTD) processing were compared with the MIMO extension of the pure time-domain (TD) processing with channel prediction presented in [10]. The carrier frequency is $f_0 = 2$ GHz, the available bandwidth is B = 5MHz, and the QPSK modulation is employed. Different frequency equalizers are implemented to investigate their effect on the channel estimator performances.

In our simulations we have employed the Spatial Channel Model (SCM) [14, 15]. The model was developed under the 3GPP framework and is the latest model used in MIMO simulations. Two environments have been used, depending on the speed of the mobile station: Suburban Macrocell for speeds greater than 80 km/h, and Urban Microcell for speeds less than 80 km/h [14]. The channel is time and frequency selective Rayleigh fading with spatially correlated propagation paths (L = 6 distinct paths per MIMO branch), according to the SCM model [14].

The BER curves presented are the result of averaging 1000 OFDM symbol blocks for 50 different channel realizations. A lower bound for the BER performance of the estimators is given by using the ideal channel state information (CSI), i.e., perfectly known channel.

For each of the N_t transmit antennas, L + 1 pilot symbols are placed at equally spaced subcarriers to achieve the optimal performance [8, 7]. The training channels are independent parallel channels, hence the estimation and tracking are performed in a decoupled way as described in Section III.

Figure 2 shows the BER versus SNR when a linear MMSE frequency equalizer is used. We note the better performance of the FD channel estimation compared to the TD estimation. The proposed FDTD method is seen to improve the results further. The



Fig. 2. BER versus E_b/N_o using pure FD channel estimation, combined FDTD channel estimation, and using CSI, vehicle speed 100km/h. A linear MMSE equalizer is used.

poor performance of the TD channel estimator is due to the decisionfeedback errors associated with the predictor stage. In order to improve the BER results and also reduce feedback errors (for pure TD processing), it is necessary to increase the number of receive antennas or to employ nonlinear equalizers (considered next).

In Figure 3, the BER versus SNR is shown for the cases of PIC and ML frequency equalizers. The performance of the FD is now the worst while the performances of the TD and the FDTD are very similar. The improvement in the TD case is caused by the reduced decision-feedback error rate in the predictor accredited the switch to a more suitable frequency equalizer.

Figures 4 and 5 show the BER versus speed. In Figure 4 the speed was ramped from 5 to 80 km/h and the environment used for MIMO channel generation was Micro Urban. In Figure 5 the speed was ramped from 80 to 200 km/h and the environment used for MIMO channel generation was Macro Suburban. The results of the pure TD channel estimation with channel prediction are plotted for the cases of MMSE and PIC equalizers. It is clear from the figure that for high speeds the prediction starts to degradate due to the decision-feedback errors. In contrast to this TD method, frequency domain processing with dedicated pilot carriers does not suffer from the problem of error propagation since channel estimation and equalization are performed at the same time instance. This is reflected in the robustness to speed in the FD based channel estimation. The difference in performance for the speed 80km/h in Figures 4 and 5 is due to the different channel parameters the SCM model uses for the different environments [14].

6. CONCLUSIONS

This paper proposed a new approach to channel estimation for MIMO OFDM systems that takes advantage of both frequency and time domain processing. The channel estimation is first carried out on dedicated pilot carriers followed by a time-domain refinement of the estimates. As a consequence, performance degradation due to interpolation errors in the pure frequency-domain processing case can be reduced. Furthermore, the benefit of having pilots available makes the estimator reliable for high mobile speeds as no prediction stage is needed as is often the case in pure time-domain processing. The performance of the proposed method was evaluated through simulations using channel models developed in the 3GPP standardization. The results showed an improvement of up to 4dB in comparison to widely used pure frequency and time-domain methods.



Fig. 3. BER versus E_b/N_o using pure FD channel estimation , combined FDTD channel estimation, and using CSI, vehicle speed 100km/h. Equalization is performed using the parallel interference canceller (PIC) and the maximum likelihood (ML) equalizers.



Fig. 4. BER versus speed in a Micro Urban environment using different FD equalizers in a 2×2 MIMO OFDM system with QPSK modulation using pure FD channel estimation, combined FDTD channel estimation, and using CSI, SNR = 25 dB.

7. REFERENCES

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Fig. 5. BER versus speed in a Macro Suburban environment using different FD equalizers in a 2×2 MIMO OFDM system with QPSK modulation using pure FD channel estimation, combined FDTD estimation, and using CSI, SNR = 25 dB.

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