ESTIMATION OF RAPIDLY TIME-VARYING CHANNELS FOR OFDM SYSTEMS

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ABSTRACT

Channel estimation for OFDM systems in rapidly time-varying environments is challenging. In this paper, relying on a basis expansion channel model, we propose a scheme for estimating channel parameters varying within a transmission block. Along with the estimation scheme, we also derive the optimal pilot sequence and optimal placement of pilot tones with respect to the mean square error (MSE) of the channel estimate. It is shown that the optimal pilot sequence consists of some adjacent equipowered and equispaced subsequences that are constrained by certain phase conditions. Simulation results demonstrate the performance of the proposed scheme in rapidly time-varying scenarios.

1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has recently been applied widely in wireless communication systems due to its high data rate transmission capability, robustness to multi-path delay, and simple implementation. It has been standardized for a variety of applications, such as wireless local area networks (WLANs), digital television broadcasting, and asymmetric digital subscriber lines.

Channel estimation is crucial for data detection and channel equalization of OFDM systems. There are two classes of channel estimation methods: one is based on training symbols that are a prior known to the receiver, whereas the other is blind. Comparing with training, blind channel estimation generally requires longer data record. Hence, it is limited to slowly time-varying channels and entails high complexity. For these reasons, we focus on training-based channel estimation in this paper.

Identifying the channel based on training has been well studied. For slowly time-varying environments (e.g., the OFDM block duration is less than 10% of the channel coherence time), the channel can be approximately assumed constant or varying in a linear fashion [1] over an OFDM block, and the estimation schemes under these two assumptions can be found in [2, 3] and [4], respectively. However, for rapidly time-varying environments (e.g., the OFDM block duration is more than 10% of the channel coherence time), the above two assumptions no longer hold and give rise to an error floor. Channel estimation and the optimal training sequences design in such scenarios become critical. In [5], optimal training over doubly selective channels is proposed, but it is only for pilot symbol assisted modulation (PSAM) case. In this paper, relying on basis expansion channel model (BEM) [5, 6, 7], we propose an estimation scheme of rapidly time-varying channels for OFDM systems. Along with the estimation scheme, we also derive the optimal pilot sequence and optimal placement of the pilot tones with respect to (w.r.t.) the mean square error (MSE) of the channel estimate. It is shown that, to obtain the minimum MSE, the optimal pilot sequence must consist of some adjacent equipowered and equispaced subsequences that are constrained by certain phase conditions.

This paper is organized as follows. In Section II, we present a description of the OFDM system model based on basis expansion channel model. In Section III, we propose the channel estimation scheme and derive the optimal pilot sequence. Section IV provides simulation results, and Section V concludes the paper.

Notation: $\mathbf{I}_N$ and $\mathbf{0}_{N\times M}$ denote the $N \times N$ identity matrix and the $N \times M$ all-zero matrix, respectively. diag($\mathbf{x}$) stands for the diagonal matrix with the column vector $\mathbf{x}$ on its diagonal, $\lceil\cdot\rceil$ denotes integer ceiling.

2. SYSTEM MODEL

A discrete-time baseband OFDM system model is depicted in Fig. 1.

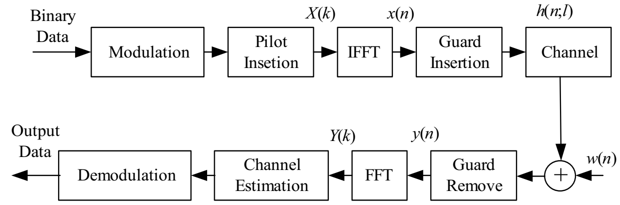

Fig. 1. Baseband OFDM system Model

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After inserting pilots to the information data sequence, IFFT block is used to transform the data sequence $\{X(k)\}$ of length N into time domain signal $\{x(n)\}$. A cyclic prefix of length g is then added to prevent intersymbol interference. We assume that $g \geq L-1$, where L is the maximum channel length. After removing the cyclic prefix, we can express the received signal as

$$y(n) = \sum_{l=0}^{L-1} h(n;l)x(n-l) + w(n), \quad 0 \leq n \leq N-1 \quad (1)$$

where $h(n;l)$ is the sampled time-varying channel impulse response, and $w(n)$ is additive white Gaussian noise with zero mean and variance σ_w^2. Then the output of the FFT at the receiver can be expressed as [4]

$$Y(m) = G(m,m)X(m) + \sum_{k=0,k\neq m}^{N-1} G(m,k)X(k) + W(m) \quad (2)$$

where $0 \leq m \leq N-1$; $Y(m)$, $X(m)$, and $W(m)$ are the FFTs of $y(n)$, $x(n)$, and $w(n)$, respectively; $G(m, k)$ is evaluated as

$$G(m,k) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h(n;l)e^{j2\pi n(k-m)/N}e^{-j2\pi kl/N}. \quad (3)$$

In rapidly time-varying environments, since we can express $h(n;l)$ as [5]

$$h(n;l) = \sum_{q=0}^{Q} h_q(l)e^{j2\pi(q-Q/2)n/N} \quad (4)$$

(3) can be rewritten as

$$G(m,k) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1}\sum_{q=0}^{Q} h_q(l)e^{j2\pi n(q-Q/2+k-m)/N}e^{-j2\pi lk/N}. \quad (5)$$

where $Q = 2\lceil f_{\max}NT_s\rceil$, $f_{\max}$ is the maximum Doppler spread, T_s is the sample interval. Similar to [5], we assume that the coefficients $h_q(l)$ are zero-mean, complex Gaussian random variables with variance $\sigma_{q,l}^2$.

Since $G(m, k)$ is nonzero only with $q-Q/2 + k - m \in \mathbb{Z}$, (5) can be further expressed as

$$G(m,k) = \begin{cases} \sum_{l=0}^{L-1} h_q(l)e^{-j2\pi lk/N}, & k = (m-q+Q/2) \bmod N \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

It can be seen from (6) that given m, each $q \in \{0, \ldots, Q\}$ is corresponding to a certain $k \in \{0, \ldots, N-1\}$ which leads to $G(m, k) \neq 0$. By defining

$$k_q^m = (m-q+Q/2) \bmod N \quad (7)$$

we can rewrite (2) as

$$\begin{aligned} Y(m) &= X(m)\sum_{l=0}^{L-1} h_{Q/2}(l)e^{-j(2\pi/N)lm} \\ &+ \sum_{q=0,q\neq Q/2}^{Q}\left(X(k_q^m)\sum_{l=0}^{L-1} h_q(l)e^{-j(2\pi/N)lk_q^m}\right) + W(m) \end{aligned} \quad (8)$$

Note that $k_{Q/2}^m = m$.

3. CHANNEL ESTIAMTION AND PILOT SEQUENCE DESIGN

3.1. Channel Estimation

As shown (4), estimation of $h(n;l)$ amounts to estimating $(Q+1)L$ coefficients grouped in the $(Q+1)L \times 1$ vector

$$\bar{\mathbf{h}} = [\mathbf{h}_0^T, \ldots, \mathbf{h}_Q^T]^T \quad (9)$$

where $\mathbf{h}_q = [h_q(0), \ldots, h_q(L-1)]^T$, $0 \leq q \leq Q$. Thus, by defining $1 \times L$ row vector $\mathbf{F}(k) = [1, \ldots, \exp(-j2\pi k(L-1)/N)]$, we can express $Y(m)$ as a function of $\bar{\mathbf{h}}$, i.e.,

$$\begin{aligned} Y(m) &= X(m)\mathbf{F}(m)\mathbf{h}_{Q/2} + \sum_{\substack{q=0\\ q\neq Q/2}}^{Q} X(k_q^m)\mathbf{F}(k_q^m)\mathbf{h}_q + W(m) \\ &= \left[X(k_0^m)\mathbf{F}(k_0^m) \cdots X(k_Q^m)\mathbf{F}(k_Q^m)\right]\bar{\mathbf{h}} + W(m). \end{aligned} \quad (10)$$

From (10) we can see that, different from the time-invariant case, the received signal $Y(m)$ is not only affected by $X(m)$ (i.e., $X(k_{Q/2}^m)$), but also affected by other Q transmitted data. Thus, if $X(m)$ is selected as the pilot, in order to estimate $\bar{\mathbf{h}}$, those Q transmitted data affecting the same received signal $Y(m)$ should also be selected as the pilots. Assuming that corresponding to $q = Q/2$, there are P pilots placed at tones $m(1), \ldots, m(P)$ (i.e., $X(k_{Q/2}^{m(1)}), \ldots, X(k_{Q/2}^{m(P)})$ are pilots), we can form $P \times (Q+1)L$ system of linear equations

$$\tilde{\mathbf{Y}} = \begin{bmatrix} X(k_0^{m(1)})\mathbf{F}(k_0^{m(1)}) & \cdots & X(k_Q^{m(1)})\mathbf{F}(k_Q^{m(1)}) \\ \vdots & \ddots & \vdots \\ X(k_0^{m(P)})\mathbf{F}(k_0^{m(P)}) & \cdots & X(k_Q^{m(P)})\mathbf{F}(k_Q^{m(P)}) \end{bmatrix}\bar{\mathbf{h}} + \tilde{\mathbf{W}} \quad (11)$$

where $\tilde{\mathbf{Y}} = [Y(m(1)), \ldots, Y(m(P))]^T$ and $\tilde{\mathbf{W}} = [W(m(1)), \ldots, W(m(P))]^T$. As mentioned earlier, since $\{X(k_q^{m(p)})\}_{p=1}^P$, $q \in \{0, \ldots, Q\}$ are all pilots, the number of all pilot tones is $M = (Q+1)P$, and their positions $\{k_q^{m(p)}\}$ can be obtained through (7). It implies that once the positions of pilots corresponding to $q = Q/2$ are given, the positions of pilots corresponding to other q can then be determined.

By defining $P\times L$ matrix $\tilde{\mathbf{F}}_q = [\mathbf{F}^T(k_q^{m(1)}), \ldots, \mathbf{F}^T(k_q^{m(P)})]^T$, and $P \times 1$ vector $\tilde{\mathbf{X}}_q = [X(k_q^{m(1)}), \ldots, X(k_q^{m(P)})]^T$, we can express (11) in a compact form

$$\begin{aligned} \tilde{\mathbf{Y}} &= [\mathrm{diag}(\tilde{\mathbf{X}}_0)\tilde{\mathbf{F}}_0, \ldots, \mathrm{diag}(\tilde{\mathbf{X}}_Q)\tilde{\mathbf{F}}_Q]\bar{\mathbf{h}} + \tilde{\mathbf{W}} \\ &= \mathbf{A}\bar{\mathbf{h}} + \tilde{\mathbf{W}} \end{aligned} \quad (12)$$

From (12) we know that the pilot sequence actually consists of $Q+1$ adjacent subsequences that are of the same length P. Each subsequence is denoted by $\tilde{\mathbf{X}}_q$ and with the set of pilot tones $\mathcal{K}_q = \{k_q^{m(1)}, \ldots, k_q^{m(P)}\}$. In other words, if denote the pilot sequence by $\tilde{\mathbf{X}}$ and the set of pilot tones by $\mathcal{K}$, we have $\tilde{\mathbf{X}} = [\tilde{\mathbf{X}}_0^T, \ldots, \tilde{\mathbf{X}}_Q^T]^T$ and $\mathcal{K} = \{\mathcal{K}_0, \ldots, \mathcal{K}_Q\}$.

According to (12), the least squares (LS) estimate of $\bar{\mathbf{h}}$ can then be obtained as

$$\hat{\bar{\mathbf{h}}} = \mathbf{A}^\dagger\tilde{\mathbf{Y}} = \bar{\mathbf{h}} + \mathbf{A}^\dagger\tilde{\mathbf{W}}. \quad (13)$$

We require $P \geq (Q+1)L$ (i.e., $M \geq (Q+1)^2L$) such that the P

$\times (Q+1)L$ matrix $\mathbf{A}$ is of full column rank $(Q+1)L$.

After $\bar{\mathbf{h}}$ is estimated, the time-varying channel impulse response $h(n; l)$ can finally be acquired through (4).

3.2. Optimal Pilot Sequence Design

From (13), the MSE of the LS estimate is given by

$$\begin{aligned}\text{MSE} &= \frac{1}{(Q+1)L}\,\mathrm{E}\left\{\left\|\hat{\bar{\mathbf{h}}}-\bar{\mathbf{h}}\right\|^2\right\} \\ &= \frac{1}{(Q+1)L}\,\mathrm{tr}\{\mathbf{A}^{\dagger}\mathrm{E}\{\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\}\mathbf{A}^{\dagger H}\} \\ &= \frac{\sigma_w^2}{(Q+1)L}\,\mathrm{tr}\{(\mathbf{A}^H\mathbf{A})^{-1}\}\end{aligned} \tag{14}$$

It has been shown in [3] that the minimum MSE will be achieved if $\mathbf{A}^H\mathbf{A} = \mathcal{P}\,\mathbf{I}_{(Q+1)L}$, where $\mathcal{P}$ is a fixed power. Thus, according to this condition, we will derive the optimal pilot sequence and optimal placement of the pilot tones in the following.

Let us first rewrite $\mathbf{A}^H\mathbf{A}$ as

$$\mathbf{A}^H\mathbf{A} = \begin{bmatrix}\mathbf{B}_{0,0} & \cdots & \mathbf{B}_{0,Q} \\ \vdots & \ddots & \vdots \\ \mathbf{B}_{Q,0} & \cdots & \mathbf{B}_{Q,Q}\end{bmatrix} \tag{15}$$

where $\mathbf{B}_{q,s}$ is the (q, s)th $L \times L$ sub-matrix, which is given by

$$\mathbf{B}_{q,s} = \tilde{\mathbf{F}}_q^H \mathrm{diag}(\tilde{\mathbf{X}}_q)^H \mathrm{diag}(\tilde{\mathbf{X}}_s)\tilde{\mathbf{F}}_s\,. \tag{16}$$

As mentioned before, to obtain the minimum MSE of the estimate, we require $\mathbf{A}^H\mathbf{A} = \mathcal{P}\,\mathbf{I}_{(Q+1)L}$, i.e.,

$$\mathbf{B}_{q,s} = \begin{cases}\mathcal{P}\mathbf{I}_{\mathrm{L}}, & \text{if } q = s \\ \mathbf{0}_{L\times L}, & \text{if } q \neq s.\end{cases} \tag{17}$$

In fact, (17) shows the constraint between two pilot subsequences.

Now, we will consider the case $q = s$ in (17), which shows the condition that each pilot subsequence itself should satisfy. Let $\mathcal{P}_q^p$ denote the power on the $k_q^{m(p)}$ th pilot tone. We then obtain

$$\mathbf{B}_{q,q} = \tilde{\mathbf{F}}_q^H \mathrm{diag}\left\{\mathcal{P}_q^1,\ldots,\mathcal{P}_q^P\right\}\tilde{\mathbf{F}}_q = \mathcal{P}\mathbf{I}_{\mathrm{L}}\,. \tag{18}$$

After calculating the matrix multiplication in (18), we can obtain $2L-1$ equations

$$\sum_{p=1}^{P}\mathcal{P}_q^p\cdot e^{-j(2\pi/N)\xi\cdot k_q^{m(p)}} = \begin{cases}0, & \xi = 1-L,\ldots,-1,1,\ldots,L-1 \\ \mathcal{P}, & \xi = 0.\end{cases} \tag{19}$$

For a minimum number of pilot tones needed in the subsequence, the above condition is satisfied if and only if $\mathcal{P}_q^p = \mathcal{P}/P$ and $k_q^{m(p)} = k_q + (p-1)N/P$, where $k_q \in \{0, \ldots, N/P-1\}$ is some offset. It means that the pilots belong to the same subsequence must be uniformly placed and with the same transmitted power $\mathcal{P}/P$, i.e., each pilot subsequence must be equipowered and equispaced. In practical systems, for cheap, fast, and simple implementation of the DFT, the number of subcarriers N is usually chosen to be a power of 2. Since P should divide N, it should also be a power of 2. Hence, keeping in mind that $P \geq (Q+1)L$, we generally set $P = 2^{\lceil \log_2((Q+1)L)\rceil}$.

Next, we will look at the case $q \neq s$ in (17), which indicates the constraint condition between two different pilot subsequences. Assume that the pilot subsequences have already satisfied (19). After calculating matrix multiplexing at the right hand side of (16), we can finally obtain the (r,t)th element of $\mathbf{B}_{q,s}$ as

$$[\mathbf{B}_{q,s}]_{r,t} = (\mathcal{P}/P)\sum_{p=1}^{P} e^{j\theta_{q,s}^{m(p)}}\cdot e^{j(2\pi/N)(r\cdot k_q^{m(p)} - t\cdot k_s^{m(p)})} \tag{20}$$

where $\theta_{q,s}^{m(p)}$ denotes phase difference between $X(k_q^{m(p)})$ and $X(k_s^{m(p)})$. By defining $\Delta_{q,s} = q - s$, we can obtain $k_q^m - k_s^m = -\Delta_{q,s}$ or $-\Delta_{q,s} \pm N$ according to (7). Thus, (20) can be rewritten as

$$[\mathbf{B}_{q,s}]_{r,t} = (\mathcal{P}/P)e^{-j(2\pi/N)t\Delta_{q,s}}\sum_{p=1}^{P} e^{j\left(\theta_{q,s}^{m(p)} + (2\pi/N)(r-t)k_q^{m(p)}\right)}. \tag{21}$$

It is clear from (21) that the second part of (17), i.e., $\mathbf{B}_{q,s} = \mathbf{0}_{L\times L}$ with $q \neq s$, is satisfied when

$$\sum_{p=1}^{P} e^{j\left(\theta_{q,s}^{m(p)} + (2\pi/N)(r-t)k_q^{m(p)}\right)} = 0,\ \forall r,t \in \{0,\ldots,L-1\} \qquad \forall q,s, \in \{0,\ldots,Q\},\ \text{with } q \neq s. \tag{22}$$

The above condition is satisfied if and only if $\theta_{q,s}^{m(p)} = (2\pi/N)k_q^{m(p)}\beta$, where $\beta \in \mathbb{Z} \setminus \{1-L, \ldots, L-1\}$.

In summary, the optimal pilot sequence $\tilde{\mathbf{X}}$ can be designed as follows.

1. Select P as $P = 2^{\lceil \log_2((Q+1)L)\rceil}$.
2. Choose an equispaced set $\{m(1),\ldots, m(P)\}$ as $\mathcal{K}_{Q/2}$. One possible choice is $m(p) = p + N/P$.
3. Calculate $\mathcal{K}_q = \{k_q^{m(1)},\ldots,k_q^{m(P)}\}$ with $q \in \{0,\ldots,Q\}$ and $q \neq Q/2$ according to $\mathcal{K}_{Q/2}$. Then, the optimal placement of pilot tones can be obtained by $\mathcal{K} = \{\mathcal{K}_0, \ldots, \mathcal{K}_Q\}$.
4. Choose an arbitrary sequence of length P as the subsequence $\tilde{\mathbf{X}}_{Q/2}$. Let $\theta_{Q/2}^p$ denote the phase of $X(k_{Q/2}^{m(p)})$ (i.e., $X(m(p))$). Then, the phase of $X(k_q^{m(p)})$ in the subsequence $\tilde{\mathbf{X}}_q$ must be selected as $\theta_{Q/2}^p + (2\pi/N)m(p)\beta_q$, where β_q should satisfy 1) $\beta_q \in \mathbb{Z} \setminus \{1-L, \ldots, L-1\}$; and 2) $\beta_q - \beta_s \in \mathbb{Z} \setminus \{1-L, \ldots, L-1\}$ with $\forall q, s, \in \{0, \ldots, Q\} \setminus \{Q/2\}$ and $q \neq s$. One possible choice is $\beta_q = (q - Q/2)\,L$.

Then, let the modulus of any pilot subsequence $\tilde{\mathbf{X}}_q$ be $\sqrt{\mathcal{P}/P}$. The optimal pilot sequence can finally be acquired as $\tilde{\mathbf{X}} = [\tilde{\mathbf{X}}_0^T,\ldots,\tilde{\mathbf{X}}_Q^T]^T$.

4. SIMULATIONS

In the simulations, we consider a QPSK-OFDM system with subcarriers $N = 128$, carrier frequency $f_0 = 5$ GHz and sampling period $T_s = 10.5$ μs. The maximum mobile speed is set to $v_{\max}$ = 160 km/hr and the maximum channel length is L = 5. It has been shown in [6, 7] that BEM can approximate Jakes' model accurately. Thus, in this paper, each channel tap is generated by BEM. With the system parameters, we found that Q = 2. Similar to [5], all the channel coefficients $h_q(l)$ are generated as independent, standardized, complex Gaussian random deviates. The multipath intensity profile is selected as $\phi_c(\tau) = \exp(-0.1\tau / T_s)$, $\forall q$, and the Doppler power spectrum is chosen as $S_c(f) = (\pi\sqrt{f_{\max}^2 - f^2})^{-1}$ when $f \le f_{\max}$; otherwise, $S_c(f) = 0$, $\forall l$. The variance of $h_q(l)$ is defined as $\sigma_{q,l}^2 = \gamma\phi_c(lT_s)S_c(q/(NT_s))$, where $\gamma = (\sum_{l,q}\phi_c(lT_s)S_c(q/(NT_s)))^{-1}$ denotes the normalizing factor. The performance of the system is measured in terms of the MSE and the bit error rate (BER) versus SNR based on channel estimate. In all simulations, we select P as P = 16 which leads to the minimum number of pilot tones, i.e., M = 48, and choose the set $\mathcal{K}_{Q/2}$ = {2, 10, 18, 26, 34, 42, 50, 58, 66, 74, 82, 90, 98, 106, 114, 122} which leads to $\mathcal{K}$ = {1, 2, 3, 9, 10, 11, 17, 18, 19, 25, 26, 27, 33, 34, 35, 41, 42, 43, 49, 50, 51, 65, 66, 67, 73, 74, 75, 81, 82, 83, 89, 90, 91, 97, 98, 99, 105, 106, 107, 113, 114 ,115, 121, 122, 123}.

Fig. 2 and Fig. 3 compare the MSE and the BER performance of the proposed channel estimation scheme with the optimal pilot sequence derived in this paper and the equipowered random pilot sequence, respectively. The BER performance with perfect channel knowledge is also shown in Fig. 3 for reference. As shown in the figures, our channel estimation approach is effective in rapidly time-varying scenarios, and using the optimal pilot sequence derived in the paper outperforms using random pilot sequence. For instance, we can see about 6.5 dB gain in SNR for the optimal pilot sequence over the random pilot sequence at all MSEs and 7.5 dB gain in SNR at BER = 10^{-2}.

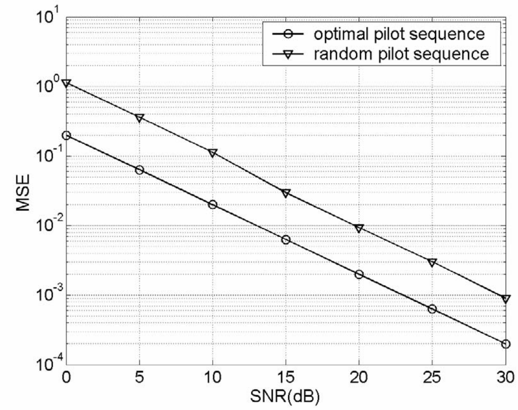

Fig. 2 MSE of the proposed estimation scheme with random and optimal pilot sequences

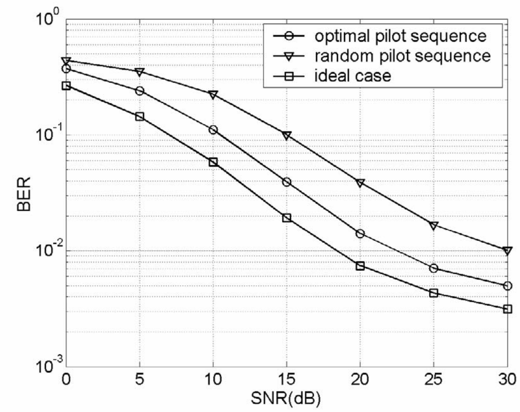

Fig. 3 BER of the proposed estimation scheme with random and optimal pilot sequences

5. CONCLUSIONS

In this paper, relying on the basis expansion channel model, we proposed a pilot-based estimation scheme of fast fading channel for OFDM systems. To obtain the minimum MSE of the channel estimate, we also derived the optimal pilot sequence as well as the optimal placement of pilot tones. It is shown that the optimal pilot sequence consists of some adjacent subsequences, which are equipowered, equispaced, and constrained by certain phase conditions.

6. REFERENCES

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