BLIND CFO ESTIMATION FOR OFDM/OQAM SYSTEMS

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ABSTRACT

In this paper we investigate the problem of blind carrier frequency offset (CFO) estimation in orthogonal frequency division multiplexing systems based on offset QAM. Specifically, by modeling the received signal as a complex Gaussian random vector and under the hypothesis of a non dispersive channel, we derive the unconditional maximum likelihood CFO estimator. Moreover, due to its computational complexity we consider, successively, the more feasible best linear unbiased estimator. The performance of the proposed algorithms is assessed via computer simulation and compared with that of recently proposed CFO estimators exploiting second order cyclostationarity.

1. INTRODUCTION

Recently, a new efficient OFDM scheme based on offset QAM has been developed. Differently from OFDM systems, exploiting rectangular pulses, OFDM/OQAM systems use more efficient pulse shaping filters apt to guarantee a sharper localization in frequency for each subcarrier in presence of dispersive channels [1]. However, as all multicarrier modulation schemes, OFDM/OQAM systems are in general more sensitive to frequency synchronization errors than single-carrier systems. Therefore, accurate carrier-frequency offset (CFO) synchronization schemes must be designed for these systems.

In [2] a blind CFO estimation algorithm has been derived by exploiting the conjugate second order cyclostationarity of the received signal. Moreover, the second order cyclostationarity of the OFDM/OQAM signal has been exploited in [3] to obtain a joint CFO and symbol timing estimator.

This paper deals with the problem of blind CFO estimation in OFDM/OQAM systems. Specifically, by assuming that the number of subcarriers is sufficiently large, the received signal is modeled as a complex Gaussian random vector (CGRV). Moreover, it is shown that the OFDM/OQAM signal results to be a noncircular (NC) process (i.e., its relation function is different from zero [4]). Therefore, by exploiting the generalized probability density function (PDF) for NC-CGRVs derived in [4] and under the hypothesis of a non dispersive channel, we derive the unconditional maximum likelihood (ML) algorithm for CFO estimation. Moreover, due to its computational complexity we propose, successively, a more feasible best linear unbiased (BLU) estimator.

As illustrated by computer simulations, in AWGN channel the ML estimator outperforms those proposed in [2] and [3], and can assure a performance very close to the corresponding Gaussian Cramér-Rao bound (GCRB). Moreover, in multipath channel the proposed BLU estimator provides the best performance.

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2. SIGNAL MODEL

Let us consider the discrete time received OFDM/OQAM signal in presence of a CFO ν and a carrier phase offset ϕ

$$r(k) = s(k)e^{j[2\pi\nu k+\phi]} + n(k)\,, \tag{1}$$

where $s(k)$ is the transmitted signal and $n(k)$ denotes the additive zero-mean circular complex white Gaussian noise statistically independent of $s(k)$. As indicated in [1] the signal $s(k)$ can be written as

$$s(k)=\sigma_s \sum_{p=-\infty}^{\infty} \left[x^R_{p,(k-pN)} + j\, x^I_{p,(k-pN)}\right]\,, \tag{2}$$

where $\sigma_s^2 \triangleq E[|s(k)|^2]$, N is the number of subcarriers and $x^R_{p,m}$ and $x^I_{p,m}$ are given by

$$\begin{aligned} x^R_{p,m} &\triangleq \frac{1}{\sqrt{2}} \sum_{l=0}^{N-1} a^R_{p,l} e^{j\frac{2\pi}{N}lm} g(m)\,, \\ x^I_{p,m} &\triangleq \frac{1}{\sqrt{2}} \sum_{l=0}^{N-1} a^I_{p,l} e^{j\frac{2\pi}{N}lm} g(m+N/2). \end{aligned} \tag{3}$$

The sequences $a_{p,l}^R$ and $a_{p,l}^I$ in (3) denote real and imaginary parts of the complex data symbols transmitted on the lth subcarrier during the pth OFDM/OQAM symbol while $g(k)$ is the real transmitted pulse shaping filter. We assume that

(AS1) The data symbols $\{a_{p,l}^R\}_{p=-\infty}^{\infty}$ and $\{a_{p,l}^I\}_{p=-\infty}^{\infty}$, $\forall l \in \{0,\ldots,N-1\}$, are statistically independent and identically distributed random variables with zero mean and unit variance.

(AS2) The sequences $\{a_{p,l}^R\}_{p=-\infty}^{\infty}$ and $\{a_{p,l}^I\}_{p=-\infty}^{\infty}$, $\forall l \in \{0,\ldots,N-1\}$, belong to a PAM constellation with $\mathrm{E}[(a_{p,l}^R)^2] = \mathrm{E}[(a_{p,l}^I)^2] = 1$.

(AS3) The number of subcarriers N is sufficiently large so that the OFDM/OQAM signal $s(k)$ can be modeled as a complex Gaussian process.

From the assumptions (AS1)-(AS3), we can easily derive the following results:

Result 1 The autocorrelation function of the transmitted signal is equal to

$$E\left[s(k)s^*(m)\right] = \sigma_s^2\delta(k-m-\beta N) \times \sum_{q=0;\ \text{even}\ q}^{N-1} e^{j\frac{2\pi}{N}qk}\int_0^1 G(\nu)G^*\left(\nu-\frac{q}{N}\right)e^{j2\pi\nu(m-k)}\mathrm{d}\nu, \beta\in\mathbb{Z}\,, \tag{4}$$

where $\delta(k)$ is the Kronecker delta and $G(\mu)$ is the discrete time Fourier transform of the pulse-shaping filter $g(k)$ assumed to be an even function with unit energy.

Result 2 The relation function (or the conjugate correlation function) of the transmitted signal $s(k)$ is given by

$$E\left[s(k)s(m)\right] = \sigma_s^2\delta(k+m-\beta N) \times \sum_{q=0;\ \text{odd}\ q}^{N-1} e^{j\frac{2\pi}{N}qk}\int_0^1 G(\nu)G^*\left(\nu-\frac{q}{N}\right)e^{j2\pi\nu(m-k)}\mathrm{d}\nu, \beta\in\mathbb{Z}. \tag{5}$$

From result 1 (result 2) one can easily verify that the transmitted OFDM/OQAM signal results to be unconjugate (conjugate) second order cyclostationary. We can note, however, that into the widely used case of a pulse shaping filter given by a square-root raised cosine pulse with a rolloff parameter ρ, the OFDM/OQAM signal is stationary with respect to its unconjugate correlation but cyclostationary with respect to its conjugate correlation. Therefore, the choice of a pulse shaping filter $g(k)$ given by a square-root raised cosine pulse can destroy the unconjugate second order cyclostationarity unless, as shown in [3], different subcarrier transmit powers (subcarrier weighting) or a form of periodic transmitter precoding are employed.

The **result 1** has been used in [3] for joint CFO and timing estimation while the **result 2** particularized to a square-root raised cosine pulse $g(k)$ has been considered in [2] to obtain a CFO synchronization algorithm. In the following we will use both the results to derive the unconditional ML CFO estimator.

3. ML CFO ESTIMATOR

In this section the ML CFO estimator for OFDM/OQAM systems is derived by maximizing the log-likelihood function (LLF) for the vector of unknown parameters $\boldsymbol{\lambda} \triangleq [\nu,\phi]^T$. Precisely, let us consider the observation vector $\boldsymbol{r}$ of total length $W=\eta N$ and given by

$$\boldsymbol{r} = \boldsymbol{\Psi}(\boldsymbol{\lambda})\boldsymbol{s} + \boldsymbol{n}\,, \tag{6}$$

where $\boldsymbol{\Psi}(\boldsymbol{\lambda}) \triangleq \mathrm{diag}\left\{e^{j\phi},\ldots,e^{j[2\pi\nu(\eta N-1)+\phi]}\right\}$ is the $W\times W$ diagonal matrix parameterized by the vector of unknown parameters $\boldsymbol{\lambda}$. Moreover, $\boldsymbol{s}\triangleq[s(0),\ldots,s(W-1)]^T$ is the transmitted OFDM/OQAM vector, while $\boldsymbol{n}$ denotes the noise vector modeled as a zero-mean circular CGRV with covariance matrix $E[\boldsymbol{n}\boldsymbol{n}^H] = \sigma_n^2\boldsymbol{I}_W$, with $\boldsymbol{I}_n$ the $n\times n$ identity matrix. Finally, the observations vector $\boldsymbol{r}\triangleq[r(0),\ldots,r(W-1)]^T$ is assumed to be a zero-mean NC-CGRV whose covariance matrix is equal to

$$\bar{\boldsymbol{C}} \triangleq E\left\{\begin{bmatrix}\boldsymbol{r}\\ \boldsymbol{r}^*\end{bmatrix}\left[\boldsymbol{r}^H,\boldsymbol{r}^T\right]\right\} = \begin{bmatrix}\boldsymbol{C_r} & \boldsymbol{R_r}\\ \boldsymbol{R_r}^H & \boldsymbol{C_r}^*\end{bmatrix}\,, \tag{7}$$

where

$$\boldsymbol{C_r} \triangleq E[\boldsymbol{r}\boldsymbol{r}^H] = \boldsymbol{\Psi}(\boldsymbol{\lambda})\left[\underbrace{E[\boldsymbol{s}\boldsymbol{s}^H]}_{\sigma_s^2\boldsymbol{C_s}} + \sigma_n^2\boldsymbol{I}_W\right]\boldsymbol{\Psi}^*(\boldsymbol{\lambda}) \tag{8}$$

while

$$\boldsymbol{R_r} \triangleq E[\boldsymbol{r}\boldsymbol{r}^T] = \boldsymbol{\Psi}(\boldsymbol{\lambda})\underbrace{E[\boldsymbol{s}\boldsymbol{s}^T]}_{\sigma_s^2\boldsymbol{R_s}}\boldsymbol{\Psi}(\boldsymbol{\lambda}) \tag{9}$$

is the so-called relation matrix.

The W-dimensional NC-CGRV $\boldsymbol{r}$ is characterized by the joint PDF [4]

$$f(\boldsymbol{r},\boldsymbol{r}^*;\boldsymbol{\lambda}) = \frac{1}{k}\exp\left\langle -\frac{1}{2}\left[\boldsymbol{r}^H\boldsymbol{r}^T\right]\bar{\boldsymbol{C}}^{-1}\begin{bmatrix}\boldsymbol{r}\\ \boldsymbol{r}^*\end{bmatrix}\right\rangle\,, \tag{10}$$

where the constant $k\triangleq\pi^W\sqrt{\det\left\{\bar{\boldsymbol{C}}\right\}}$ does not depend on the parameter vector to estimate. By using the properties of inversion of block matrices and dropping an additive constant independent of the parameters to estimate, we obtain that the LLF for the vector of parameters of interest can be written as

$$\begin{aligned}\Lambda(\boldsymbol{\lambda}) &= \log\left[f(\boldsymbol{r},\boldsymbol{r}^*;\boldsymbol{\lambda})\right]\\ &= \frac{1}{2(\sigma_n^2+\sigma_s^2)}\Re\Big\{-\boldsymbol{r}^H\boldsymbol{\Psi}(\boldsymbol{\lambda})\boldsymbol{P_s}^{-1}\boldsymbol{\Psi}^*(\boldsymbol{\lambda})\boldsymbol{r}\\ &\quad+\alpha\,\boldsymbol{r}^T\boldsymbol{\Psi}^*(\boldsymbol{\lambda})\left[\alpha\boldsymbol{C_s}+(1-\alpha)\boldsymbol{I}_W\right]^{-1}\boldsymbol{R_s}\boldsymbol{P_s}^{-1}\boldsymbol{\Psi}^*(\boldsymbol{\lambda})\boldsymbol{r}\Big\},\end{aligned} \tag{11}$$

where $\Re\{\cdot\}$ denotes real part,

$$\boldsymbol{P_s}{=}\alpha\boldsymbol{C_s}{+}(1{-}\alpha)\boldsymbol{I}_W{-}\alpha^2\boldsymbol{R_s}\left[\alpha\boldsymbol{C_s}+(1-\alpha)\boldsymbol{I}_W\right]^{-1}\boldsymbol{R_s}$$

and

$$\alpha \triangleq \frac{\sigma_s^2/\sigma_n^2}{1+\sigma_s^2/\sigma_n^2} = \frac{SNR}{1+SNR}. \tag{12}$$

The joint ML estimator is obtained by searching the value of the vector $\boldsymbol{\lambda}$ that maximizes the LLF. To proceed we keep the parameter ν fixed and let ϕ vary. In these conditions the function $\Lambda(\boldsymbol{\lambda})$ in (11) achieves a maximum for

$$\begin{aligned}\hat{\phi}_{ML}(\nu) = \ & \tfrac{1}{2}\arg\Big\{\boldsymbol{r}^T\boldsymbol{\Upsilon}^*(\nu)\left[\alpha\boldsymbol{C_s}{+}(1{-}\alpha)\boldsymbol{I_W}\right]^{-1} \\ & \times\boldsymbol{R_s}\boldsymbol{P_s}^{-1}\boldsymbol{\Upsilon}^*(\nu)\boldsymbol{r}\Big\},\end{aligned} \tag{13}$$

where $\boldsymbol{\Upsilon}(\nu) \triangleq \operatorname{diag}\left\{1,\ldots,\, e^{j2\pi\nu(\eta N-1)}\right\}$. Then, accounting for (11) and (13), the ML CFO estimator is given by

$$\begin{aligned}\hat{\nu}_{ML} = & \arg\max_{\tilde{\nu}}\Big\langle -\Re\left\{\boldsymbol{r}^H\boldsymbol{\Upsilon}(\tilde{\nu})\boldsymbol{P_s}^{-1}\boldsymbol{\Upsilon}^*(\tilde{\nu})\boldsymbol{r}\right\} \\ & +\alpha\left|\boldsymbol{r}^T\boldsymbol{\Upsilon}^*(\tilde{\nu})\left[\alpha\boldsymbol{C_s}{+}(1{-}\alpha)\boldsymbol{I}_W\right]^{-1}\boldsymbol{R_s}\boldsymbol{P_s}^{-1}\boldsymbol{\Upsilon}^*(\tilde{\nu})\boldsymbol{r}\right|\Big\rangle,\end{aligned} \tag{14}$$

where $\tilde{\nu}$ is a trial value for CFO. It can be shown that the ML CFO estimator $\hat{\nu}_{ML}$ gives ambiguous estimates unless $|\nu| \leq 1/(2N)$. This is also the acquisition range of the CFO estimator proposed in [2] and it is double of that of the estimator proposed in [3] in the absence of subcarrier weighting.

4. A REDUCED COMPLEXITY CFO ESTIMATOR

The ML CFO estimator (14) requires a time-consuming maximization procedure with respect to the continuous parameter $\tilde{\nu}$. To overcome this problem, we propose a BLU estimator that provides a closed form expression for the CFO estimate. Since this estimator exploits the relation function of the OFDM/OQAM signal it will be referred to in the following as NC-BLU estimator. Let us consider the term

$$B(m) = \frac{1}{N-1}\sum_{k=1}^{N-1} r(k+mN)r(N-k+mN) \tag{15}$$

and let us substitute the expression (1) of the received signal into (15). Then, neglecting the noise×noise term, we have

$$\begin{aligned}B(m) = & \ \zeta\, e^{j[2\pi\nu N(2m+1)+2\phi]} \\ & \times\left[1{+}\frac{2}{(N{-}1)\zeta}\sum_{k=1}^{N-1} s(k+mN)w(N-k+mN)\right],\end{aligned} \tag{16}$$

where, under the hypothesis of a zero-mean circular noise, the random variable $w(k) \triangleq n(k)e^{-j[2\pi\nu k+\phi]}$ is statistically coincident with $n(k)$. Moreover, the quantity ζ defined as

$$\zeta \triangleq \frac{1}{N-1}\sum_{k=1}^{N-1} s(k+mN)s(N-k+mN)\,,$$

for $N \gg 1$ can be approximated as

$$\zeta \simeq \frac{\sigma_s^2}{(N{-}1)}\sum_{k=1}^{N-1}\sum_{\substack{q=0\\ \text{odd } q}}^{N-1} e^{j\frac{2\pi}{N}qk}\int_0^1 G(\nu)G^*\Big(\nu{-}\frac{q}{N}\Big)e^{j2\pi\nu(N-2k)}\mathrm{d}\nu. \tag{17}$$

Details about this approximation are reported in [8].

Let us now consider the vector $\boldsymbol{y} \in \mathbb{R}^{(\eta-1)\times 1}$ whose elements are defined as

$$y(m) \triangleq \arg\left[B(m)B^*(m-1)\right], \quad m \in \{1,\ldots,\eta-1\}. \tag{18}$$

At high SNR values and under the constraint $|\nu| < 1/(4N)$, as shown in [5], the quantity $y(m)$ can be approximated by

$$y(m) \simeq 4\pi\nu N + \Im[\beta(m) + \beta^*(m-1)]\,, \tag{19}$$

where $\Im\{\cdot\}$ denotes imaginary part and

$$\beta(m) \triangleq \frac{2}{\zeta(N-1)}\sum_{k=1}^{N-1} s(k+mN)w(N-k+mN)\,. \tag{20}$$

Thus, the estimation problem can be reduced to a linear model and following [6] (p. 136) the NC-BLU frequency offset estimator can be expressed as

$$\hat{\nu}_{NC-BLU} = \frac{1}{4\pi N}\left[\frac{\boldsymbol{y}^T\boldsymbol{C}_y^{-1}\boldsymbol{1}_{\eta-1}}{\boldsymbol{1}_{\eta-1}^T\boldsymbol{C}_y^{-1}\boldsymbol{1}_{\eta-1}}\right] \tag{21}$$

where $\boldsymbol{C}_y$ is the covariance matrix of the vector $\boldsymbol{y}$ and $\boldsymbol{1}_n$ is an n-dimensional column vector of all ones. After some calculations, the NC-BLU estimator assumes the form

$$\hat{\nu}_{NC-BLU} = \frac{3}{2\pi N\,\eta(\eta^2-1)}\sum_{m=1}^{\eta-1}\left[y(m)\,m(\eta-m)\right]. \tag{22}$$

It is of interest to remark that the closed-form NC-BLU CFO estimator in (22), whose acquisition range is $|\nu| < 1/4N$, unlike the ML estimator does not require the knowledge of the SNR value and of the pulse-shaping filter $g(k)$.

5. NUMERICAL RESULTS AND COMPARISON

In this section the performance of ML and NC-BLU CFO estimators is assessed via computer simulations and compared with that of Bölcskei's algorithm proposed in [3] and that of the estimator proposed by Ciblat and Serpedin (CS estimator) in [2]. The actual values of the CFO and the carrier phase have been fixed at $\nu = 1/(4W)$ and $\phi = \frac{\pi}{8}$, respectively, while 10^3 trials were used to obtain the performance plot. Moreover, the pulse-shaping filter $g(k)$ is a square-root raised cosine with a rolloff parameter $\rho = 0.6$. Thus, the transmitted OFDM/OQAM signal does not exhibit unconjugate second order cyclostationarity. Therefore, to exploit the Bölcskei's estimator subcarrier weighting is used.

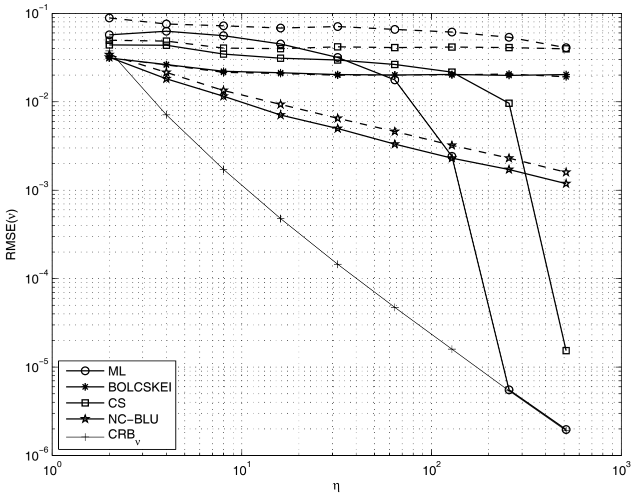

Fig. 1. RMSE of CFO estimators in AWGN (solid lines) and in multipath channel (dashed lines) as a function of the number of observed symbols η, for $N = 4$ and $SNR = 20$ dB.

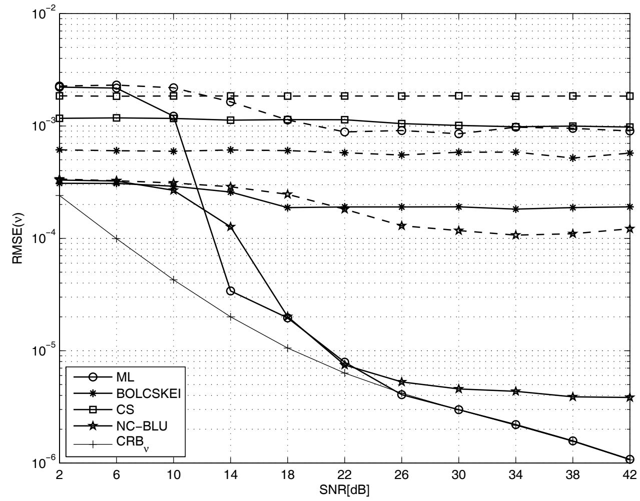

Fig. 2. Performance of CFO estimators in AWGN (solid lines) and in multipath channel (dashed lines) as a function of SNR for $N = 128$ subcarriers and for $\eta = 16$.

In Fig.1 we compare the root mean squared error (RMSE) of the considered estimators as a function of the number of observed symbols η for an OFDM/OQAM systems with $N = 4$ subcarriers, $SNR = 20$dB, in AWGN (solid lines) and in multipath channel (dashed lines). In each experiment the multipath channel $h(l)$ has been modeled to consist of $N_m+1=4$ independent Rayleigh-fading taps with $E[|h(l)|^2]=Ce^{-\frac{l}{2}}, l \in \{0, \ldots, N_m\}$ and $\sum_{l=0}^{N_m} E[|h(l)|^2] = 1$. Moreover, the channel is fixed in the observation window but independent from one run to another. The GCRB [7] reported in [8] is also shown as a comparison. The results show that in AWGN channel and for $\eta \geq 2^8$ the proposed ML estimator attains the corresponding GCRB. Nevertheless, for a smaller number of observed symbols, the presence of outliers during the coarse search leads to a performance degradation. Instead, the NC-BLU estimator assures sufficiently accurate CFO estimates also for a small observation window. Moreover, in multipath channel, the performance of the NC-BLU estimator is practically unchanged with respect to the AWGN case while ML and CS algorithms present a significant performance degradation due to the poor estimate in the coarse search.

In Fig.2 we report the RMSE of the considered estimators as a function of SNR for an observation window of length $W = 16N$ and N=128 subcarriers. The results show that in AWGN channel the proposed ML estimator achieves the GCRB at sufficiently high values of SNR. Moreover, the NC-BLU estimator presents for low SNR values the best performance while the CS estimator, demanding much more large observation intervals, reveals a severe performance degradation in the whole range of considered SNR values. Finally, in multipath channel the proposed NC-BLU estimator assures the lowest RMSE.

6. REFERENCES

[1] P. Siohan, C. Siclet and N. Lacaille, “Analysis and design of OFDM/OQAM systems based on filterbank theory,” *IEEE Trans. Signal Processing*, vol. 50, pp. 1170-1183, May 2002.

[2] P. Ciblat and E. Serpedin, “A fine blind frequency offset estimator for OFDM/OQAM systems,” *IEEE Trans. Signal Processing*, vol. 52, pp. 291-296, January 2004.

[3] H. Bölcskei, “Blind estimation of symbol timing and carrier frequency offset in wireless OFDM systems,” *IEEE Trans. Commun.*, vol. 49, pp. 988-999, June 2001.

[4] B. Picinbono, “Second-order complex random vectors and normal distributions,” *IEEE Trans. Signal Processing*, vol. 44, pp. 2637-2640, October 1996.

[5] U. Mengali and M. Morelli, “Data-aided frequency estimation for bust digital transmission,” *IEEE Trans. Commun.*, vol. 45, pp. 23-25, Jan 1997.

[6] S.M. Kay, “Fundamentals of statistical signal processing: estimation theory,” *Prentice Hall*, 1993.

[7] J.P. Delmas and H. Abeida, “Stochastic Cramér-Rao bound for noncircular signals with application to DOA estimation,” *IEEE Trans. Signal Processing*,vol. 52, pp. 3192-3199, November 2004.

[8] T. Fusco and M. Tanda, “Blind frequency-offset estimation for OFDM/OQAM systems,” submitted to *IEEE Trans. Signal Processing*.