An SOS-Based Blind Channel Shortening Algorithm

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***Abstract*— A new SOS-expressed blind channel shortening algorithm is proposed. It is based on a necessary and sufficient condition that guarantees detection of all possible shortening equalizers before the best one is selected. The unique existing SOS-based blind channel shortening algorithm meets the same objective but has a much higher complexity.**

I. Introduction

The cyclic prefix is a mean to avoid intersymbol interference (ISI) at the price of lowering the spectral efficiency of the OFDM system. This is especially relevant for wireless broadband communication where large propagation delays imply long cyclic prefixes. This has justified a renewed interest in channel shortening. First proposed to reduce the complexity of the Viterbi algorithm [1], channel shortening aims at devising a linear equalizer so that the combined channel-equalizer impulse response is shorter than the initial channel response.

Blind channel shortening techniques are attractive compared to supervised ones [2], [3] that require training sequences. Early blind algorithms were based on the minimization of non-quadratic cost functions of the channel output high order statistics (HOS) [4], [5] and suffer from slow and ill convergence. Blind channel shortening from the second order statistics (SOS) is made possible when multiple receive antenna are used and/or the channel output is sampled at a rate higher than the Baud rate. The channel phase information, then, appears in the channel SOS which, then, become self-sufficient to conduct channel processing tasks. Not only do SOS-based techniques guarantee global and fast convergence, but they also reach perfect shortening by means of FIR equalizers when HOS-based techniques require IIR equalizers [6]. Up to our knowledge, only one SOS-based blind channel shortening algorithm [7] has been proposed to date. The objective of this paper is to propose a new blind technique that has a lower complexity and reaches the same asymptotic performance.

Notations T, H, $*$ and $\natural$ stand for transpose, transconjugate, conjugate and the Moore pseudo-inverse, respectively. $\mathbf{0}_{a,b}$ is the $a\times b$ zero matrix. $\mathbf{I}_a$ is the $a\times a$ identity matrix. $\mathbf{P}_a$ is the $a\times a$ permutation matrix with 1s on its anti-diagonal and zeros elsewhere. $\mathbf{J}_a \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{0} & 0 \\ \mathbf{I}_{a-1} & \mathbf{0} \end{bmatrix}$ is the $a\times a$ (down) shifting matrix. We denote $\mathbf{J}_a^{-k} \stackrel{\text{def}}{=} \left(\mathbf{J}_a^T\right)^k$. Dimensions are dropped when they can be inferred from the context. $\|.\|$ denotes the Euclidean norm. $\mathbf{A}\otimes\mathbf{B}$ is the Kronecker product between matrices $\mathbf{A}$ and $\mathbf{B}$ defined such that its (i,j) block is $a_{i,j}\mathbf{B}$.

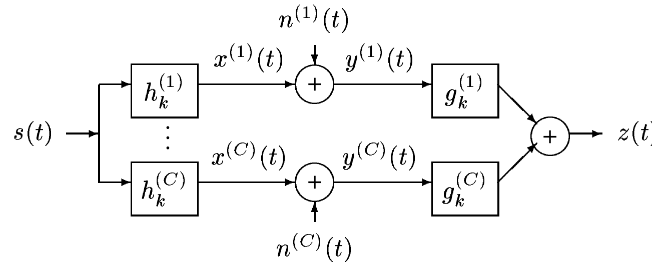

Fig. 1. Equalized SIMO channel.

II. Notation and previous results

A multiple antenna system and/or a fractional receiver is often modeled as a single-input multiple-output (SIMO) system. As depicted in Fig. 1, symbols $s(t)$ are simultaneously inputed to a set of the C sub-channels. The m-th tap of the M-order SIMO channel is defined as $\mathbf{h}_m \stackrel{\text{def}}{=} \left[h_m^1, h_m^2, \cdots, h_m^C\right]^T$, where $\left[h_0^c, \cdots, h_M^c\right]^T$ represents the impulse response associated with the c-th sub-channel. The SIMO channel order M is the largest among the orders of the c sub-channels. The SIMO channel impulse response is defined as $\mathbf{h} \stackrel{\text{def}}{=} \left[\mathbf{h}_0^T \cdots \mathbf{h}_M^T\right]^T$. The noise-corrupted input-output relationship is expressed by

$$\begin{aligned}\mathbf{y}(t) &\stackrel{\text{def}}{=} \left[y^{(1)}(t)\cdots y^{(C)}(t)\right]^T \\ &= \mathbf{H}\mathbf{s}_{M+1}(t) + \mathbf{n}(t),\end{aligned}$$

where $\mathbf{H} \stackrel{\text{def}}{=} [\mathbf{h}_0 \cdots \mathbf{h}_M]$ and $\mathbf{s}_l(t) \stackrel{\text{def}}{=} [s(t)\cdots s(t-l+1)]^T$ for any l. Successive outputs are stacked into the vector $\mathbf{y}_L^T(t) \stackrel{\text{def}}{=} \left[\mathbf{y}^T(t)\cdots \mathbf{y}^T(t-L+1)\right]^T$. We have

$$\mathbf{y}_L(t) = \mathbf{H}_L\mathbf{s}_{L+M}(t) + \mathbf{n}_L(t),$$

where

$$\mathbf{H}_L \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{H} & \mathbf{0} & \cdots & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H} \end{bmatrix}$$

is the $CL\times(L+M)$ channel filtering matrix, $\mathbf{n}_L(t)$ is defined similarly as $\mathbf{y}_L(t)$ and $\mathbf{0}$ is the C-dimensional zero vector. The matrix $\mathbf{H}_L$ is full column rank if [8][1]

$$L \geq M. \tag{1}$$

[1]A *non-common zero* condition is also necessary but is not mentioned here because it is almost always fulfilled by channels encountered in practice [8].

The channel SOS are expressed by the correlation matrices

$$\begin{aligned}\mathbf{R}_L^{(i)} &\stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}_L(i+i')\mathbf{y}_L^H(i')\right]\\ &= \begin{bmatrix}\Gamma_i & \cdots & \Gamma_{i+L-1}\\ \vdots & \ddots & \vdots\\ \Gamma_{i-L+1} & \cdots & \Gamma_i\end{bmatrix}\\ &= \sigma_s^2\mathbf{H}_L\mathbf{J}_{L+M}^i\mathbf{H}_L^H + \sigma_n^2\left(\mathbf{J}_L^i \otimes \mathbf{I}_C\right), \qquad (2)\end{aligned}$$

where $\Gamma_k \stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}(t+k)\mathbf{y}^H(t)\right]$. In particular, we denote $\mathbf{R}_L \stackrel{\text{def}}{=} \mathbf{R}_L^{(0)}$. Eqn. (2) is valid if the symbols are independent and identically distributed (i.i.d.) and uncorrelated from the white noise components. This condition is assumed throughout the paper. We also assume the transmitted symbols and the noise samples to be zero-mean and denote by σ_s^2 and σ_n^2 their respective powers. These assumptions are common to all existing SOS/HOS based blind channel shortening techniques.

Similar notations are used for the equalizer, depicted in Fig. 1 : $\mathbf{g}_k \stackrel{\text{def}}{=} \left[g_k^1, g_k^2, \cdots, g_k^C\right]^T$ denotes the k-th equalizer tap and $\mathbf{g} \stackrel{\text{def}}{=} \left[\mathbf{g}_0, \cdots, \mathbf{g}_{L-1}\right]^T$ the impulse response of the $(L-1)$-order SIMO equalizer. The equalizer $\mathbf{g}$ is a shortening equalizer with delay d and leading to an W-long combined channel-equalizer impulse response iff

$$\mathbf{g}^T\mathbf{H}_L = \left[\mathbf{0}_{1,d} \quad \mathbf{w}^T \quad \mathbf{0}_{1,M+L-W-d}\right] \qquad (3)$$

for some W-dimensional row vector $\mathbf{w}$ that stands for the effective combined channel-equalizer impulse response.

The SOS-based blind channel shortening algorithm in [7] is a straightforward extension of a blind channel equalization method [9]. The shortening equalizers are given by the vectors left orthogonal to [7, (26)]

$$\begin{bmatrix}\mathbf{R}_L^{(d+W)} - \sigma_n^2\left(\mathbf{J}_L^{d+W}\otimes\mathbf{I}_C\right)\\ \mathbf{U}_d^H\left(\mathbf{R}_L - \sigma_n^2\mathbf{I}\right)\end{bmatrix}^*, \qquad (4)$$

where $\mathbf{U}_d$ is the left kernel of $\mathbf{R}_L^{(d)} - \sigma_n^2\left(\mathbf{J}_L^d \otimes \mathbf{I}_C\right)$.

III. An original algorithm for blind channel shortening

We denote $\mathbf{H}_L$ and $\mathbf{R}_L$ by $\mathbf{H}$ and $\mathbf{R}$, respectively. We partition the channel filtering matrix as follows

$$\mathbf{H} = \left[\mathbf{H}^{\text{head}}\mathbf{H}^{\text{win}}\mathbf{H}^{\text{tail}}\right],$$

where $\mathbf{H}^{\text{head}}$, $\mathbf{H}^{\text{win}}$ and $\mathbf{H}^{\text{tail}}$ are made of the first d, intermediate W, and last $M+L-W-d$ columns of $\mathbf{H}$, respectively. In particular, we have

$$\mathbf{H}^{\text{win}} = \begin{bmatrix}\mathbf{h}_d & \cdots & \mathbf{h}_{d+W-1}\\ \vdots & \ddots & \vdots\\ \mathbf{h}_{d-L+1} & \cdots & \mathbf{h}_{d-L+W}\end{bmatrix},$$

where we use the convention $\mathbf{h}_m = \mathbf{0}$ if $m > M$ or $m < 0$. The sufficient and necessary condition (3) is equivalent to have

$$\mathbf{g}^T\left[\mathbf{H}^{\text{head}}\mathbf{H}^{\text{tail}}\right] = \mathbf{0}. \qquad (5)$$

The set of $(L-1)$-order shortening equalizers leading to d-delayed W-long combined channel-equalizer response form a linear subspace given by the left kernel of $\left[\mathbf{H}^{\text{head}}\mathbf{H}^{\text{tail}}\right]^*$. If the equalization parameter L satisfies (1), then $\left[\mathbf{H}^{\text{head}}\mathbf{H}^{\text{tail}}\right]$ is full column rank and its left kernel has dimension

$$K \stackrel{\text{def}}{=} (C-1)L - M + W.$$

We express the outer product of $\left[\mathbf{H}^{\text{head}}\mathbf{H}^{\text{tail}}\right]$ in terms of the channel SOS. This opens the way for a blind procedure that detects all channel shortening equalizers with the specified delay and length. We write that (5) is equivalent to have

$$\mathbf{g}^T\left[\mathbf{H}^{\text{head}}\left(\mathbf{H}^{\text{head}}\right)^H + \mathbf{H}^{\text{tail}}\left(\mathbf{H}^{\text{tail}}\right)^H\right]\mathbf{g}^* = 0,$$

where

$$\begin{aligned}&\mathbf{H}^{\text{head}}\left(\mathbf{H}^{\text{head}}\right)^H + \mathbf{H}^{\text{tail}}\left(\mathbf{H}^{\text{tail}}\right)^H\\ =\ &\mathbf{H}\mathbf{H}^H - \mathbf{H}^{\text{win}}\left(\mathbf{H}^{\text{win}}\right)^H\\ =\ &\frac{1}{\sigma_s^2}\left(\mathbf{R} - \sigma_n^2\mathbf{I}\right) - \sum_{i=0}^{W-1}\begin{bmatrix}\mathbf{h}_{d+i}\\ \vdots\\ \mathbf{h}_{d+i-L+1}\end{bmatrix}\begin{bmatrix}\mathbf{h}_{d+i}\\ \vdots\\ \mathbf{h}_{d+i-L+1}\end{bmatrix}^H.\end{aligned}$$

Terms $\left[\mathbf{h}_{d+i}^T \cdots \mathbf{h}_{d+i-L+1}^T\right]^T\left[\mathbf{h}_{d+i}^H \cdots \mathbf{h}_{d+i-L+1}^H\right]$ are (possibly zero-padded) sub-blocks of $\left[\mathbf{h}_M^T \cdots \mathbf{h}_0^T\right]^T\left[\mathbf{h}_M^H \cdots \mathbf{h}_0^H\right]$, which is, up to a row and column permutation, the channel response outer product $\mathbf{h}\mathbf{h}^H$. In [10], it was proved that

$$\sigma_s^2\mathbf{h}\mathbf{h}^H = \mathcal{R} - \left(\mathbf{J}_{M+1}^T \otimes \mathbf{I}_C\right)\mathcal{R}\left(\mathbf{J}_{M+1}\otimes\mathbf{I}_C\right),$$

where

$$\mathcal{R} \stackrel{\text{def}}{=} \mathbf{R}^\circ\left(\mathbf{R}_{M+1} - \sigma_n^2\mathbf{I}\right)^\natural\left(\mathbf{R}^\circ\right)^H. \qquad (6)$$

and the Hankel-structured matrix $\mathbf{R}^\circ$ is defined as [10, (3.10)]

$$\mathbf{R}^\circ \stackrel{\text{def}}{=} \begin{bmatrix}\Gamma_0 - \sigma_n^2\mathbf{I} & \Gamma_1 & \cdots & \Gamma_M\\ \Gamma_1 & \Gamma_2 & & \mathbf{0}\\ \vdots & & & \vdots\\ \Gamma_M & \mathbf{0} & \cdots & \mathbf{0}\end{bmatrix}.$$

We, hence, have

$$\begin{aligned}&\sigma_s^2\left[\mathbf{h}_M^T \cdots \mathbf{h}_0^T\right]^T\left[\mathbf{h}_M^H \cdots \mathbf{h}_0^H\right]\\ =\ &\left(\mathbf{P}_{M+1}\otimes\mathbf{I}_C\right)\left[\mathcal{R} - \left(\mathbf{J}_{M+1}^T\otimes\mathbf{I}_C\right)\mathcal{R}\left(\mathbf{J}_{M+1}\otimes\mathbf{I}_C\right)\right]\\ &\times\left(\mathbf{P}_{M+1}\otimes\mathbf{I}_C\right)\\ =\ &\left(\mathbf{P}_{M+1}\otimes\mathbf{I}_C\right)\mathcal{R}\left(\mathbf{P}_{M+1}\otimes\mathbf{I}_C\right)\\ &-\left(\mathbf{J}_{M+1}\otimes\mathbf{I}_C\right)\mathcal{R}\left(\mathbf{J}_{M+1}^T\otimes\mathbf{I}_C\right) \qquad (7)\end{aligned}$$

where $\left(\mathbf{P}_{M+1}\otimes\mathbf{I}_C\right)\mathcal{R}\left(\mathbf{P}_{M+1}\otimes\mathbf{I}_C\right)$ is a row and column permuted version of $\mathcal{R}$; and $\left(\mathbf{J}_{M+1}\otimes\mathbf{I}_C\right)\mathcal{R}\left(\mathbf{J}_{M+1}^T\otimes\mathbf{I}_C\right)$ is a shifted version of $\mathcal{R}$. Let the CL-square Hermitian matrix

$$\mathbf{A} \stackrel{\text{def}}{=} \mathbf{R} - \sum_{i=0}^{W-1}\sigma_s^2\begin{bmatrix}\mathbf{h}_{d+i}\\ \vdots\\ \mathbf{h}_{d+i-L+1}\end{bmatrix}\begin{bmatrix}\mathbf{h}_{d+i}\\ \vdots\\ \mathbf{h}_{d+i-L+1}\end{bmatrix}^H, \qquad (8)$$

where the terms in the sum above are sub-blocks of (7). All possible shortening equalizers are given by the column span of the K-dimensional kernel of $\mathbf{A}$. We can also compute $\mathbf{A}$ as

$$\sigma_s^2\sum_{i=0,\cdots,d-1,d+W,\cdots,M+L}^{W-1}\left[\mathbf{h}_i^T \cdots \mathbf{h}_{i-L+1}^T\right]^T\left[\mathbf{h}_i^H \cdots \mathbf{h}_{i-L+1}^H\right],$$

but the sum above involves much more terms than in (8) because, in practice, L is large and W is small. If we let the $CL \times K$ matrix $\mathbf{N}$ be the kernel of $\mathbf{A}$, then the iff condition (5) is equivalent to write

$$\mathbf{g} = \mathbf{N}^* \mathbf{x}, \tag{9}$$

where $\mathbf{x}$ is an arbitrary K-dimensional vector.

So far, a procedure is developed that detects all possible shortening equalizers, including undesirable ones, those for which $\mathbf{w}$ in (3) is arbitrarily small. These should be easily avoided by applying the following selection criterion. The *best* equalizer is chosen such that it maximizes the norm $\|\mathbf{w}\|$ of the effective combined channel-equalizer response, under the constraint that $\|\mathbf{x}\|$ in (9) remains constant [11], [7]. We write

$$\begin{aligned} \|\mathbf{w}\|^2 &= \|\mathbf{H}^T \mathbf{g}\|^2 \\ &= \frac{1}{\sigma_s^2} \mathbf{x}^H \mathbf{N}^T \left(\mathbf{R}^* - \sigma_n^2 \mathbf{I} \right) \mathbf{N}^* \mathbf{x}. \end{aligned}$$

When $\mathbf{N}$ is obtained by EVD of $\mathbf{A}$, we have $\mathbf{N}^H \mathbf{N} = \mathbf{I}$. The maximization above is, then, equivalent to choose $\mathbf{x}$ as the conjugate of the eigen vector associated with the largest eigenvalue of the K-square Hermitian positive definite matrix

$$\mathbf{B} \stackrel{\text{def}}{=} \mathbf{N}^H \mathbf{R} \mathbf{N}. \tag{10}$$

Because $\mathbf{N}$ is orthogonal to $\mathbf{A}$, $\mathbf{B}$ is also equal (up to a multiplication by σ_s^2) to

$$\mathbf{N}^H \left(\sum_{i=0}^{W-1} \begin{bmatrix} \mathbf{h}_{d+i} \\ \vdots \\ \mathbf{h}_{d+i-L+1} \end{bmatrix} \begin{bmatrix} \mathbf{h}_{d+i} \\ \vdots \\ \mathbf{h}_{d+i-L+1} \end{bmatrix}^H \right) \mathbf{N}. \tag{11}$$

Both expressions (10) and (11) of $\mathbf{B}$ are equivalent in the exact SOS case and may differ only slightly in practice, as verified by simulations. Finally, the performance of the shortened channel can differ significantly and unpredictably depending on the pre-selected delay d and length W of the shortened impulse response. Of course, the optimal (w.r.t. the criterion above) set of parameters d and W can be obtained, but the computational cost would, then, be prohibitive [2].

Prior to comparing the respective performances of the proposed and existing algorithms (Sec. IV), we can compare them from the complexity point-of-view. First, the noise power, required to compute $\mathcal{R}$, is obtained as a by-product of the EVD of $\mathbf{R}_{M+1}$. It is computed at no computational cost as the average of the lowest $(C-1)(M+1) - M$ eigenvalues of $\mathbf{R}_{M+1}$. On the contrary, for the existing algorithm which processes shifted correlation matrices only, a supplementary EVD of $\mathbf{R}_{M+1}$ is required to estimate the noise power. Tab. I illustrates the comparison from the complexity point-of-view between the existing and proposed algorithms. For practical systems for which the number of sub-channels is typically low, the complexity of the existing algorithm is about four times that of the proposed algorithm[2].

[2] A less complex version of the existing algorithm is presented in [7]. We do not consider this version because, as commented in [7], it leads to a biased estimate even in the exact SOS case.

EVD/SVD of …	Prop. alg.	Exist. alg.
$\mathbf{R}_{M+1}$	$O\left[C^3 M^3\right]$	$O\left[C^3 M^3\right]$
$\mathbf{A}$	$O\left[(C-2)^3 M^3\right]$	
$\mathbf{B}$	$O\left[(C-2)^3 M^3\right]$	$O\left[(C-2)^3 M^3\right]$
$\mathbf{R}_{M+1}^{(d)}$		$O\left[C^3 M^3\right]$
$\mathbf{R}_{M+1}^{(d+W)}$		$O\left[C^3 M^3\right]$
(4)		$O\left[C^3 M^3\right]$
Total	$O\left[\left(2(C-2)^3 + C^3\right) M^3\right]$	$O\left[\left((C-2)^3 + 4C^3\right) M^3\right]$

TABLE I
COMPLEXITY COMPARISON FOR $L = M + 1$, $M \gg 1$ AND $W \ll L$.

IV. SIMULATIONS

A series of simulations is conducted to test the proposed algorithm and compare it to the existing one. The coefficients h_k^c of the SIMO channel impulse response ($C = 4$ and $M = 9$) are generated as normalized and centered i.i.d. complex-valued Gaussian-distributed random variables. Unit-variance QPSK symbols are generated. The channel SNR is defined as

$$\text{SNR} \stackrel{\text{def}}{=} \frac{\mathbf{E}\left[\|\mathbf{x}(t)\|^2\right]}{\mathbf{E}\left[\|\mathbf{n}(t)\|^2\right]} = \frac{\sigma_s^2}{\sigma_n^2} \frac{\|\mathbf{h}\|^2}{C}.$$

Unless otherwise stated, 200 snapshots are generated for each run, the SNR equals 20 dB, a shortening equalizer with $L = 12$ taps is computed, and the targeted shortened channel is such that $W = 4$ and $d = 4$.

In practice, the channel SOS are estimated from a limited number, say T, of snapshots. As a consequence, strictly speaking, $\mathbf{A}$ is not rank-deficient. We compute $\mathbf{N}$ as the set of eigen vectors associated to the lowest K eigenvalues of $\mathbf{A}$. For the same reason, the combined channel-equalizer response may not have the perfect shape of (3), so that we do have

$$\mathbf{g}^T \mathbf{H}_L = \left[\mathbf{w}_{\text{head}}^T \ \mathbf{w}^T \ \mathbf{w}_{\text{tail}}^T \right]$$

for some possibly weak but non-zero vectors $\mathbf{w}_{\text{head}}$ and $\mathbf{w}_{\text{tail}}$, respectively d and $M + L - W - d$ dimensional. As a performance measure of the shortening equalizers, we refer to the shortening SNR (SSNR) [2], [4] which is the ratio of the energy in the consecutive W coefficients to the energy in the remaining coefficients, and is defined as follows

$$\text{SSNR} \stackrel{\text{def}}{=} \frac{\|\mathbf{w}\|^2}{\|\mathbf{w}_{\text{head}}\|^2 + \|\mathbf{w}_{\text{tail}}\|^2}.$$

For every run, the SSNR is measured and finally averaged over the 1000 runs.

Results, plotted in Fig. 2 and Fig. 3, clearly show that the two algorithms have roughly the same performance for practical values of the sample size and the noise level. Under uncomfortable observation conditions (low SNR and/or small sample size), the proposed algorithm slightly outperforms the existing one. Actually, Fig. 3 shows the proposed algorithm to be particularly robust to the observation noise.

W.r.t Fig. 4, the performance of the proposed algorithm degrades as the desired length of shortened response increases, and so, to the point of performing poorer than the existing algorithm. In fact, when W increases, more terms appear in the sum (8). These are sub-blocks of the same matrix and, hence, error propagation takes place.

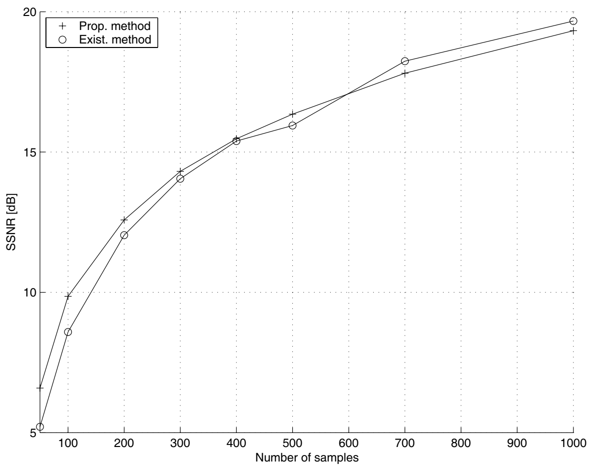

Fig. 2. Effect of the number of snapshots T.

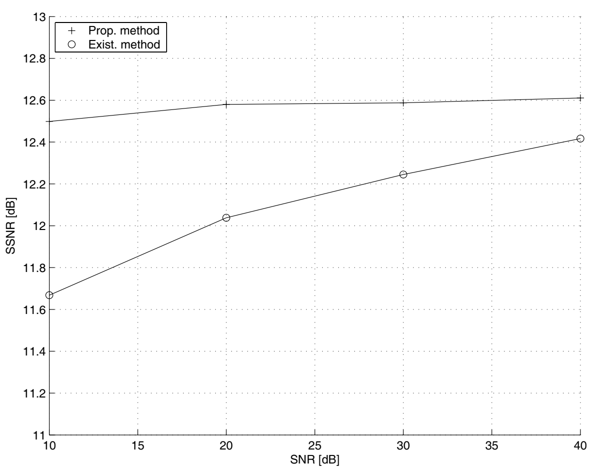

Fig. 3. Effect of the noise level.

V. Conclusion

An orthogonalization property that stems directly from the definition of a shortening equalizer is obtained then expressed in terms of the channel SOS. A straightforward blind shortening technique is, hence, defined that contrasts with the much more complex existing technique, the unique SOS-based one proposed so far. Both techniques are guaranteed to reach the optimal asymptotic performance, and, in practice, exhibit comparable results.

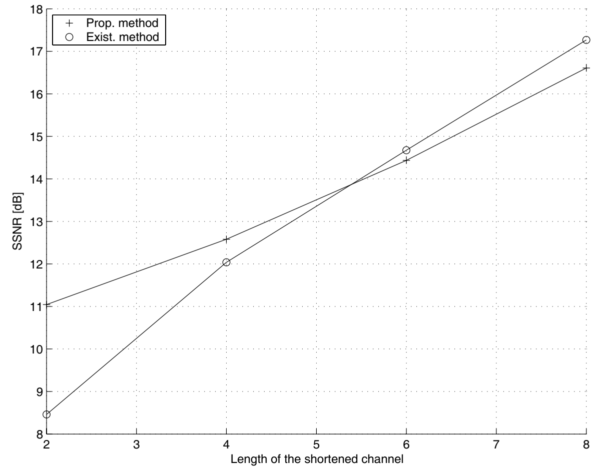

Fig. 4. Effect of the length W of the shortened channel.

Acknowledgment

The work reported in this paper has formed part of the Wireless Enablers area of the Core 3 Research Programme of the Virtual Centre of Excellence in Mobile & Personal Communications, Mobile VCE, www.mobilevce.com, whose funding support, including that of EPSRC, is gratefully acknowledged. Fully detailed technical reports on this research are available to Industrial Members of Mobile VCE.

References

[1] D. D. Falconer and R. F. Magee, "Adaptive channel memory truncation for maximum likelihood sequence estimation," *Bell Sys. Tech. Journal*, pp. 1541-1562, Nov. 1973.

[2] P. J. W. Melsa, R. C. Younce and C. E. Rohrs, "Impulse response shortening for discrete multitone tranceivers," *IEEE Trans. Commun.*, vol. 44, pp. 1662-1672, Dec. 1996.

[3] G. Arslan, B. L. Evans and S. Kiaei, "Equalization for discrete multitone tranceivers to maximize bit rate," *IEEE Trans. Signal Processing*, vol. 49, pp. 3123-3135, Dec. 2001.

[4] J. Balakrishnan, R. K. Martin and C. R. Johnson, Jr., "Blind, adaptive channel shortening by sum-squared auto-correlation minimization (SAM)," *IEEE Trans. Signal Processing*, vol. 51, pp. 3086-3093, Dec. 2003.

[5] R. K. Martin, J. Balakrishnan, W. A. Sethares and C. R. Johnson, Jr., "A blind adaptive TEC for multicarrier systems," *IEEE Signal Processing Lett.*, vol. 9, pp. 341-343, Nov. 2002.

[6] R. K. Martin, C. R. Johnson, Jr., M. Ding and B. L. Evans, "Infinite length results for channel shortening equalizers," in *Proc. IEEE SPAWC*, 2003, pp. 150-154.

[7] T. Miyajima and Z. Ding, "Second-order statistical approaches to channel shortening in multicarrier systems," *IEEE Trans. Signal Processing*, vol. 52, pp. 3253-3264, Nov. 2004.

[8] L. Tong, G. Xu and T. Kailath, "Blind identification and equalization based on second-order statistics : a time domain approach", *IEEE Trans. Inform. Theory*, vol. 40, pp. 340-349, Mar. 1994.

[9] Z. Ding, I. B. Collings and R. Liu, "A new blind zeroforcing equalizer for multichannel systems," in *Proc. IEEE ICASSP*, 1998, vol. VI, pp. 3177-3180, .

[10] Z. Ding, "Matrix outer product decomposition method for blind multiple channel identification," *IEEE Trans. Signal Processing*, vol. 45, pp. 3053-3061, Dec. 1997.

[11] K. Abed-Meraim and Y. Hua, "Blind equalization methods in colored noise field," in *Proc. Information, Decision and Control*, 1999, pp. 477-481.