ERROR RATE ANALYSIS OF PHASE-MODULATED OFDM (OFDM-PM) IN AWGN CHANNELS

Ryan A. Pacheco and Dimitrios Hatzinakos

University of Toronto, The Edward S. Rogers Sr. Elec. and Comp. Eng. Dept.
10 King's College Road, Toronto, Ontario, CANADA M5S 3G4
{ryan,dimitris}@comm.toronto.edu

ABSTRACT

QAM transmission of OFDM signals achieves good spectral efficiency while greatly simplifying the equalization process. However, RF transmitters must be operated in their linear region and highly stable oscillators (low phase-noise) are a necessity or the BER will degrade significantly. If angle-modulation is instead used, then the RF signal has constant envelope and phase-noise has an additive effect, the result being more efficient power transmission and orthogonality is maintained with a noisy oscillator. Angle-modulation has lower spectral efficiency then QAM and angle-demodulators suffer a threshold effect: the receiver output SNR degrades significantly once below a certain input SNR. To apply angle-modulated OFDM systems in practice this threshold effect and its impact on the BER (for a given spectral efficiency) must be analyzed. This paper is the first to present such an analysis for phase-modulation (OFDM-PM) in AWGN channels.

1. INTRODUCTION

Power efficient transmission of baseband OFDM signals can be achieved with angle-modulation. However, angle-modulation can not achieve the same spectral efficiency as QAM and angle-modulated systems suffer from an SNR threshold effect: when the SNR at the input to the receiver is below a certain threshold then random phase changes of $\pm 2\pi$ (from thermal noise) cause the output SNR to significantly degrade. Rice's seminal work [1] analyzed this phenomenon, proposing to model the noise output of an FM limiter-discriminator (LD) as a combination of additive Gaussian noise and shot (impulsive or "click") noise. It is known [2, ch.10] that phase-locked loops (PLL) and the FM demodulator using feedback (FMFB) also have the same noise model but with shot noise of a lower intensity. Understanding the connection between this threshold effect and the probability of error of OFDM signals is a significant and important challenge and is the main contribution of this work. This paper will focus on phase-modulation (PM), a special case of angle-modulation, but the results can be applied to frequency-modulation or more general angle-modulation with only minor changes.

There has been recent interest in angle-modulation of OFDM because of the fact that efficient power transmission is possible. In [3] a VHF/UHF system was tested using frequency-modulation of OFDM (OFDM-FM). The analysis assumed that the frequency domain noise was Gaussian, which is not true in general. In [4] binary OFDM-PM was studied for AWGN channels assuming that the received SNR is higher then threshold. PM was also considered in [5] where phase detection and correction using oversampling was proposed. A common theme in these works is that the threshold effect is not well studied. Understanding the effects of threshold in the context of OFDM is important because at the range of received SNR that one expects a digital communication system to operate at (4 to 10 dB for 1 to 2 bps/Hz spectral efficiency in AWGN), the shot noise intensity is high enough to significantly impact receiver performance.

This paper analyzes the performance of OFDM-PM in AWGN channels in a more general way then the previous works which assumed that the shot noise is either nonexistent or that its DFT is approximately Gaussian. This leads to a better understanding of such systems because we can now predict the performance, at a given E_b/N_o and for a given spectral efficiency, instead of relying on extensive computer simulations.

Section 2 presents the signal model and discusses the bandwidth of OFDM-PM signals. The symbol error probability is analyzed in Section 3, and concluding remarks are in Section 4.

2. SIGNAL MODEL AND PROBLEM FORMULATION

The transmitted RF signal, $y(t)$, is written as $y(t) = \sqrt{2P_T}\cos(2\pi f_c t + m(t))$ where P_T is the transmitted power, f_c is the carrier frequency, and $m(t)$ is the baseband OFDM signal

$$m(t) = \frac{1}{\sqrt{N_s}} \sum_{l=-\infty}^{\infty} \sum_{k=-N_s/2}^{k=N_s/2} \alpha_k a_{l,k} \phi_k(t - lT_o). \tag{1}$$

$N_s/2$ is the number of independent subcarriers ($a_{l,0} = 0$). $a_{l,k}$ denote complex information symbols with $E\{|a_{l,k}|^2\} = 1$, and belong to a rectangular alphabet with M_k^2 equally probable points. α_k are constants that are used to shape the spectrum of $m(t)$ and control its average power. Define b_k as the number of bits for subcarrier k, then $M_k^2 = 2^{b_k}$ and $\bar{b} = (2/N_s)\sum_{k=1}^{N_s/2} b_k$ is the average number of bits/subcarrier. The minimum Euclidean distance between any two constellation points is proportional to $\alpha_k\sqrt{6/[2^{b_k}-1]}$. Increasing α_k for a fixed b_k will increase the Euclidean distance without changing the transmitted power but will increase the bandwidth of $y(t)$, lowering the spectral efficiency and increasing the shot noise intensity. It will be shown that it is important to constrain the variance of $m(t)$ for these reasons. $\phi_k(t)$ is a windowed complex exponential with frequency kf_o, $\phi_k(t) = w_{T_o}(t)\exp(j2\pi k f_o(t - T_g))$, where f_o is the subcarrier frequency spacing. Define $T = 1/f_o$, then $T_o = T + T_g$, where T_g is the guard interval (cyclic prefix) and $w_{T_o}(t)$ is a windowing function that allows for some overlap between adjacent OFDM symbols (so to smooth the transition). The overlap consumes only a small portion of the guard interval. In this work $w_{T_o}(t)$ assumes the same form as in [6].

Since $m(t)$ must be real to phase-modulate the carrier, this requires that $\alpha_k a_{l,k}$=$\alpha_{-k}^* a_{l,-k}^*$, so the maximum number of independent complex symbols that can be sent in any signaling interval (T_o

seconds) is $N_s/2$. Thus, to achieve the same data rate as a system using QAM, twice the average number of bits/subcarrier need to be sent for the PM system. In addition, since $m(t)$ is the sum of a large number of independent subcarriers, it can be well approximated as a zero-mean Gaussian random process. Phase-noise can be included in the signal model by including an additional phase modulating term to the RF signal. However, since the bandwidth and power of phase-noise is much smaller than $m(t)$ in practical situations, it will be ignored in this work. A small amount of phase-noise can severely degrade the performance of QAM systems but not for angle-modulated OFDM because the effect is additive not multiplicative.

2.1. Signal Bandwidth

The bandwidth definition used here is the width of a rectangular bandpass filter required to pass 99% of the signal power. Obviously bandwidth is an important issue to consider, particularly when quantifying spectral efficiency, but also because the noise output of an angle-demodulator is dependent on the ratio of the bandwidth of $y(t)$, B_y, to the bandwidth of $m(t)$, B_m. Define $\sigma_m^2 = E\{m^2(t)\}$. When $\sigma_m^2 \ll 1$ then this is narrowband angle-modulation and it is known that $B_y \approx 2B_m$. Alternatively, when $\sigma_m^2 \gg 1$ then this is wideband angle-modulation and Woodward's Theorem [7, pgs. 370–371] shows that the PSD of $y(t)$ can be expressed directly in terms of the pdf of $m(t)$, which is assumed to be Gaussian, and the 99% bandwidth can be easily found. Our interest is in neither of these two cases because in the first case σ_m^2 is much too small to achieve a BER of practical interest at a reasonable SNR and in the second case the spectral efficiency is extremely low. For the intermediate condition (for example $\sigma_m = 1.1$), a simple expression for B_y is not possible, but it is possible to compute B_y given the Gaussian assumption on $m(t)$. The PSD of a sinusoid phase-modulated by a Gaussian random process can be found in [8]. That result will now be used to formulate a computational method to determine B_y for the OFDM-PM signal when equal subcarrier weighting is used (the PSD of $m(t)$ is approximately flat in this case). Using (1) it follows that $\sigma_m^2 = (2/N_s)\sum_{k=1}^{N_s/2}\alpha_k^2$, and because of equal subcarrier weighting $\sigma_m^2 = \alpha_k^2$. The normalized (unit power) PSD, $S_{y0}(f)$, of the lowpass version of $y(t)$ is [8]

$$S_{y0}(f) = \frac{e^{-\sigma_m^2}}{1-e^{-\sigma_m^2}} \sum_{i=1}^{\infty} \frac{\left(\sigma_m^2\right)^i}{i!} S_m(f) \overset{i}{*} S_m(f) \qquad (2)$$

where $S_m(f)$ denotes the normalized PSD of $m(t)$ and $S_m(f) \overset{j}{*} S_m(f)$ denotes $j-1$ convolutions of $S_m(f)$ with itself. Notice that $S_{y0}(f)$ does not contain a spectral line at zero frequency. We have removed it because it does not contribute to the information content of the transmitted signal. The PSD has been renormalized to take this into account. Define the 99% bandwidth as the value of z for which $\int_{-z}^{z} S_{y0}(f)\,\mathrm{d}f = 0.99$, The transmission bandwidth is then $B_y = 2z$. The required z is a function of σ_m^2 and $S_m(f)$. Assuming $S_m(f)$ is bandlimited with flat PSD and maximum frequency X it follows that

$$\int_{-z}^{z} S_m(f)\,\mathrm{d}f = \begin{cases} \frac{z}{X} & z \le X \\ 1 & z > X \end{cases} \qquad (3)$$

$$\int_{-z}^{z} S_m(f) * S_m(f)\,\mathrm{d}f = \begin{cases} \frac{z}{X} - \frac{z^2}{4X^2} & z \le 2X \\ 1 & z > 2X \end{cases} \qquad (4)$$

$$\int_{-z}^{z} S_m(f) \overset{j}{*} S_m(f)\,\mathrm{d}f \approx \begin{cases} \mathrm{erf}\left(z/\sqrt{2jX^2/3}\right) & z \le jX \\ 1 & z > jX \end{cases} \qquad (5)$$

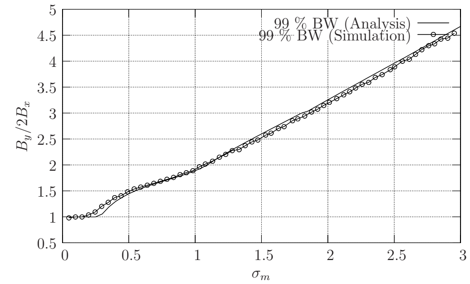

Fig. 1. Bandwidth expansion when phase-modulating a bandlimited Gaussian random process with rectangular PSD and comparison with an OFDM signal.

The approximation is very close for $j \ge 3$ because $S_m(f) \overset{j}{*} S_m(f)$ rapidly approaches a Gaussian distribution with variance $jX^2/2$ when $S_m(f)$ is rectangular [8]. Using (3)-(5) it is possible to search for the z which corresponds to the 99% bandwidth point for any given σ_m^2. The ratio $B_y/2B_m$ is an important parameter for quantifying spectral efficiency and predicting the shot noise intensity.

2.1.1. Example

Fig. 1 is a plot of $B_y/2B_m$ versus σ_m computed assuming $X = 8.125$ MHz and calculations performed every 10 kHz. Also shown is the result of simulating an OFDM signal with the following parameters: $f_o = 78.125$ kHz, $N_s/2 = 104$, $T_g = 0.8\,\mu s$, $T_{TR} = 0.1\,\mu s$. There was equal weighting for all subcarriers. A sampling frequency of $F_s = 320$ MHz was used and $f_c = F_s/4$. The 99% bandwidth point was estimated using the Welch method for PSD estimation (rectangular window, 50% overlap, 4096 point FFT). The estimate was done for both $m(t)$ and $y(t)$ using an identical procedure. Results were averaged over 10^3 independent trials with 50 symbols each time. In the simulations the spectral line at f_c was ignored and the 99% power point was found using the remaining indices. It is observed that the analysis in the preceding section provides a very close approximation to the 99% BW estimated using the Welch method when uniform subcarrier weighting is used. $\sigma_m = 1.1$ will be used throughout because it conveniently gives $B_y/2B_x \approx 2$, meaning that the OFDM-PM system will use roughly twice the transmission bandwidth of the QAM system.

2.2. Received Signal and Angle-Demodulator Output

After ideal bandpass filtering, assuming an AWGN channel, the received signal, $r(t)$, is

$$r(t) = y(t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \qquad (6)$$

where $n_c(t)$ and $n_s(t)$ have flat PSD with height N_o and bandwidth $B_y/2$. The signal is then input to an angle-demodulator, implemented as either an LD, PLL or FMFB. These are the most common angle-demodulators. It has been found both in theory and practice that the baseband output of such devices, $r_b(t)$, can be accurately modeled by [1]

$$r_b(t) = m^{'}(t) + \frac{n_s^{'}(t)}{\sqrt{2P_T}} + i(t) \qquad (7)$$

where $'$ is used to denote the action of differentiation on the respective signal and $i(t)$ is shot noise written as $i(t) = \sum_{j=-\infty}^{\infty} c_j \delta(t - t_j)$ where c_i is a random variable that is $\pm 2\pi$ with equal probability. The number of impulses/second is a Poisson distributed random variable. The average number of impulses/second, N_i, will be called the intensity of the shot noise. The probability that exactly k impulses occur in a T second interval ($T = 1/f_o$) is then $e^{-N_i T}(N_i T)^k/k!$ [7]. Depending on the type of angle demodulator used, the intensity of the shot noise will vary. Generally speaking, it increases with decreasing SNR and increasing ratio $B_y/2B_m$. To process the demodulated OFDM signal, $r_b(t)$ is first filtered with an antialiasing filter with bandwidth B_m and sampled with a period $T_s > 1/2B_m$. The discrete-time baseband model then becomes $r_b[n] = \tilde{m}[n] + v[n] + \tilde{i}[n]$ where $\tilde{m}[n] = m^{'}(nT_s)$, $v[n]$ is Gaussian noise with PSD

$$V(w) = \frac{1}{2P_T}\frac{N_o}{T_s}\left(\frac{w}{T_s}\right)^2 \quad |w| \leq 2\pi B_m T_s \tag{8}$$

and $\tilde{i}[n] = \sum_j c_j h(nT_s - t_j)$, where $h(t) = \sin(2\pi B_m t)/\pi t$. An approximate baseband model which is very useful for analytic and simulation purposes is to write

$$\tilde{i}[n] = \sum_{j=-\infty}^{\infty} 2B_m c_j \delta[n - n_j] \tag{9}$$

This approximation assumes that the impulses occur at only integer instants of the sampling period, and that the sampling period is short enough such that the probability of more then one impulse occurring is negligible. Thus, at each sampling instant $\tilde{i}[n]$ can take one of three values $\{\pm 4\pi B_m, 0\}$. It is assumed that $E\left[\tilde{i}[n]\tilde{i}[m]\right] = (4\pi B_m)^2 p\delta[n-m]$, where p is the probability of an impulse (either positive or negative). This will be called the probability of anomaly. It is assumed that the sampling period is short enough such that $N_i T_s e^{-N_i T_s} \approx N_i T_s$. Thus, $p \approx N_i T_s$.

2.3. Probability of Anomaly

The shot noise intensity depends on the operating SNR, bandwidth expansion factor, method of angle-demodulation (LD, PLL, FMFB) and statistics of the modulating signal (sinusoidal, Gaussian, etc.). The LD will generally have higher intensity then the PLL or FMFB. Define SNR as $\beta = P_T/(2B_m N_o)$, and spectral efficiency as $\eta = N_s\bar{b}/(2T_o B_y)$. Since $P_T T_o = N_s \bar{b} E_b/2$, where E_b is the energy/bit, then $2B_m\beta/B_y = E_b\eta/N_o$. The shot noise intensity for the LD can be shown to be [2, ch.10]

$$N_i = \frac{B_y}{2\sqrt{3}}\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_o}\eta}\right) + \frac{2f_o}{\sqrt{\pi N_s}}\sqrt{\sum_{k=1}^{N_s/2} k^2\alpha_k^2 \exp\left(-\frac{E_b}{N_o}\eta\right)} \tag{10}$$

It is known that the PLL and FMFB have lower intensity, but this depends on the specific structure such as the order of the PLL, or the VCO gain of the FMFB. [9] describes a means to compute N_i for any PLL, and [2, pg. 360 (10.14-6)] contains equations describing performance of the FMFB receiver once certain parameters are specified. Wozencraft and Jacobs in [10] conjectured that an approximate lower bound to the probability of anomaly of FMFB receivers is

$$p \approx \frac{1}{\sqrt{2\pi\beta}}\left(\frac{B_y}{B_m} - 5\right)\exp\left(-\frac{\beta}{2}\right) \tag{11}$$

We take $p = 0$ for $B_y/B_m \leq 5$. (10) and (11) will be used to plot upper and lower bounds respectively when illustrating OFDM-PM performance in section 3.2.

3. PROBABILITY OF ERROR ANALYSIS

This section derives a closed-form expression for the symbol error probability at each subcarrier in the OFDM-PM system. Performance of OFDM in impulsive channels was considered in [11] but the statistics of this channel are very different and the results in [11] can not be generalized to this situation.

Define $N = T/T_s$ ($N > N_s$), then an N-DFT of $r_b[n]$ over the appropriate interval is used to recover $a_{l,k}$. Using $l = 0$ without loss of generality,

$$R_b[k] = \frac{N}{\sqrt{N_s}}\alpha_k a_{0,k}\cdot[j2\pi k f_o] + \overbrace{V[k] + \tilde{I}[k]}^{Z[k]} \tag{12}$$

where $V[k]$ and $\tilde{I}[k]$ are the N-DFTs of $v[n]$ and $\tilde{i}[n]$ respectively. Assuming $V[k]$ and $\tilde{I}[k]$ are independent random variables, it can be shown that their combined pdf is

$$\begin{aligned} f_{Z[k]} &= [1-p]^N C(0,\sigma_k^2) + \frac{p}{2}[1-p]^{N-1}\sum_{j=0}^{N-1} C(\pm\mu_N^{jk},\sigma_k^2) + \\ &\left(\frac{p}{2}\right)^2 [1-p]^{N-2}\sum_{j=0}^{N-2}\sum_{m=j+1}^{N-1} C(\pm\mu_N^{jk} \pm \mu_N^{mk},\sigma_k^2) + \cdots \\ &\cdots + \left(\frac{p}{2}\right)^N C(\pm\mu_N^{0k} \pm \mu_N^{1k} \pm \cdots \pm \mu_N^{(N-1)k},\sigma_k^2) \end{aligned} \tag{13}$$

where $C(a,b)$ denotes a complex Gaussian pdf with mean a and variance b, $\sigma_k^2 = E\left[|V[k]|^2\right] = N\cdot V(2\pi k/N)$, and $\mu_N^{ab} = 4\pi B_m \exp(-i2\pi ab/N)$. The nth term ($n = 0,\cdots,N$) in $f_{Z[k]}$ contains exactly $\binom{N}{n}2^n$ components. This pdf is circularly symmetric, thus the probability of symbol error for subcarrier k, P_k, can be derived considering just the real (or imaginary) component. It can be shown that

$$P_k = \frac{4(M_k-1)}{M_k}P_{Z_r[k]} - \left(\frac{2(M_k-1)}{M_k}\right)^2 P_{Z_r[k]}^2 \tag{14}$$

where $P_{Z_r[k]} = P\left(Z_r[k] > d/2\right)$,

$$d^2 = \frac{N^2}{N_s}\alpha_k^2\left(\frac{2\pi k}{T}\right)^2\frac{6}{[2^{b_k}-1]} \quad , \quad \sigma_k^2 = \frac{N^2 N_o}{2P_T T}\left(\frac{2\pi k}{T}\right)^2 \text{ and}$$

$$\begin{aligned} P_{Z_r[k]} &= \frac{[1-p]^N}{2}\mathrm{erfc}\left(\sqrt{\Gamma}\right) + \\ &\frac{1}{2}\frac{p}{2}[1-p]^{N-1}\sum_{j=0}^{N-1}\mathrm{erfc}\left(\sqrt{\Gamma}\left(1 \pm \zeta\cdot\cos\left(\frac{2\pi kj}{N}\right)\right)\right) + \cdots \\ &\cdots + \frac{1}{2}\left[\frac{p}{2}\right]^N \mathrm{erfc}\left(\sqrt{\Gamma}\left(1 \pm \zeta\cdot\Re\left\{\sum_{j=0}^{N-1} W_N^{jk}\right\}\right)\right) \end{aligned} \tag{15}$$

where $\zeta = \sqrt{\frac{2N_s^3\left[2^{b_k}-1\right]}{3N^2k^2\alpha_k^2}}$, $\Gamma = \frac{3}{2}\frac{E_b}{N_o}\frac{\bar{b}\alpha_k^2\gamma}{\left[2^{b_k}-1\right]}$, $\gamma = 1/(1+T_g/T)$, and $W_N^{jk} = \exp(-i2\pi jk/N)$. This is the exact symbol error probability for subcarrier k ($k = 1,\cdots,N_s/2$). Assuming the discrete-time baseband model is accurate and that p is known, then using (15) in (14) makes it possible to predict the performance of OFDM-PM systems for any E_b/N_o and any kind of angle-demodulator. (15) shows that the probability of error is not uniform over the subcarriers. Impulses have a more pronounced effect on symbols at lower frequencies than higher, which can be seen by the presence of $1/k^2$ in the second and third terms of (15) (through ζ). If an impulse occurs then the lower frequency bits will likely be erased but a reasonable error correcting code can help to recover these lost bits.

3.1. Gaussian Approximation

Since $E\{|\tilde{I}[k]|^2\} = Np(4\pi B_m)^2$, one might argue (using the CLT), that the distribution of $\tilde{I}[k]$ can be approximated with $C(0, Np(4\pi B_m)^2)$ and therefore $Z[k] \sim C(0, \sigma_k^2 + Np(4\pi B_m)^2)$. In this case it can be easily shown that $P_{Z_r[k]} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\Gamma \frac{1}{\left[1+\frac{E_b}{N_o}\frac{\bar{b}\gamma N_s^2}{Nk^2}p\right]}}\right)$. Which demonstrates a similar trend as before, higher frequency subcarriers have more protection from impulses, but simulations will show it to be a crude approximation to the probability of error at E_b/N_o of interest.

3.2. Example

To illustrate the relative performance of OFDM-PM against QAM the same simulation parameters as in Section 2.1.1 are used with $\sigma_m = 1.1$, uniform subcarrier weighting, and 4 bits/subcarrier ($b_k = 4,\ k = 1, \cdots, Ns/2$). This means that the OFDM-PM system will have the same data rate as a QAM system using 2 bits/subcarrier, but half the spectral efficiency since $\sigma_m = 1.1$ corresponds to $B_y/2B_x \approx 2$ (Fig. 1). The symbol error probability is shown in Fig. 2. Results were averaged over $4 * 10^4$ independent trials. The FMFB lower bound obtained using (11) is plotted with the curve labeled "OFDM-PM (LB)". Three curves are used to show the performance of OFDM-PM systems demodulated using an ideal limiter-discriminator. The first, labeled "OFDM-PM (LD)" is the result of simulating an ideal LD using a high sampling frequency ($F_s = 640$ MHz, $f_c = F_s/4$) to approximate a continuous-time system and see whether the baseband model together with (10) accurately describe OFDM-PM using an LD, which it does since it is identical with the curve labeled "Baseband Model". The curves labeled "Analysis" and "CLT Approx." plots the results of Section 3 and 3.1 respectively. Since N in this case is very large ($N = 256$) it is not possible to evaluate all $\binom{N}{n}2^n$ different combinations for all possible n. We used the first four terms in (15) to plot the performance, which is more than enough whenever $pN \leq 1$. When pN is large then the CLT can be used, but the probability of error is so high in this case it is of limited practical interest. The error for each subcarrier is shown in Fig. 3. This illustrates how the symbol error is greater for lower frequency subcarriers, which was predicted by the analysis. Eventually the probability of anomaly is so low ($E_b/N_o > 12$ dB) that the error is equal for all subcarriers because the channel is now AWGN.

4. CONCLUDING REMARKS

This paper has studied the performance of phase-modulated OFDM in AWGN channels. The symbol error probability is derived which allows one to predict the performance at any E_b/N_o once the probability of anomaly is known. The analysis has shown that anomalies cause more errors for low frequency subcarriers then high frequency.

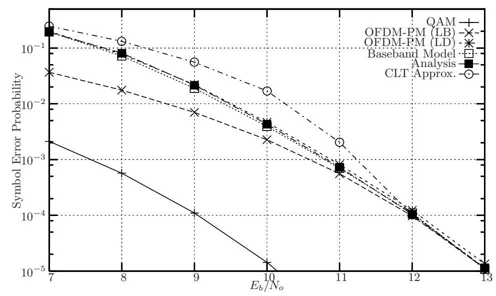

Fig. 2. Symbol Error Probability Comparison, $\sigma_m = 1.1$

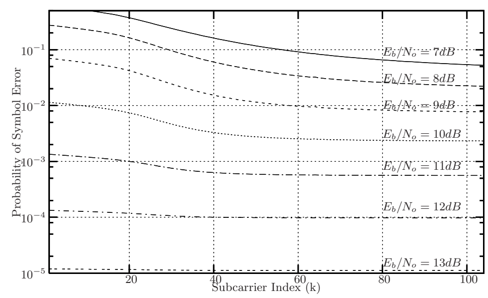

Fig. 3. Symbol Error Probability for Individual Subcarriers

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